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ELECTRICAL CIRCUITS AND MACHINERY

VOLUME II

ALTERNATING CURRENTS

WORKS OF
PROFESSOR J. H. MORECROFT

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ELECTRICAL CIRCUITS AND MACHINERY

JOHN H. MORECROFT
Professor of Electrical Engineering, Columbia University
AND
FREDERICK W. HEHRE
Associate Professor of Electrical Engineering, Columbia University

VOLUME II

ALTERNATING CURRENTS

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PREFACE

This second volume of the text on electrical circuits and machinery deals with alternating currents in the manner in which continuous currents were treated in Volume I.

The theory and calculation of alternating-current circuits takes up a somewhat larger portion of this volume than was the case for continuous currents, as the fundamental principles of alternating currents are more difficult to master; especially is this true of polyphase relations. As the polyphase circuit is so generally used in alternating-current work, the authors have devoted more than the average amount of space to analyzing, in detail, voltage and current relations and power measurements. While the treatment is not sufficiently exhaustive for the practicing electrical engineer, it will be found as complete as the average student can master in the time generally allowed.

It seems that many students fail to obtain a real grasp of the current and voltage relations of polyphase networks, merely committing to memory the various necessary formulæ without ever really comprehending their derivation. Much of this trouble arises from the failure to lay sufficient stress upon the elementary ideas of voltage and current representation by rotating vectors. Unless some definite notation for voltages and currents is chosen and adhered to, it is impossible to learn polyphase relations except as a feat of memory. It will be seen that, in this text, a reasonable notation scheme, which permits a logical calculation of all polyphase relations, is constantly used.

The discussion of types and actions of alternating-current instruments is not generally given in text books of this kind; but, as it seems to the authors that this topic should surely be treated here, a reasonably complete discussion of the more common types of meters has been given in a chapter devoted to that subject. It very frequently happens that a student is entirely ignorant of the principle and applicability of an instrument which he is using in his laboratory testing; to many a college graduate, an alternating-current meter is a brown wooden box with two binding posts.

Armature reaction is probably the one characteristic of alternating-current machinery which is of more importance than any other; the elementary ideas of electrical machines having been mastered in a course of continuous currents, the question of armature reaction becomes perhaps the first real difficulty which the student encounters. The simple scheme, originated by one of the authors some years ago, of using the circular locus of single-phase armature reaction as a basis of the discussion, is amplified in this text so that the reader can see the justification for its use with an actual distributed winding. By picturing the magnetomotive force of the alternator as it is built up from the individual contributions of the various coils, for different instants of the cycle, its action on the magnetic field from the field poles can easily be seen.

Inasmuch as this armature-reaction discussion is at once applicable to the synchronous motor, and as it completely explains the action of the stator of an induction motor in setting up its rotating field, the authors have thought it worth while to give a proportionately large amount of space to this topic.

As the transmission line is becoming, every day, a more important branch of the electrical engineer's work, a chapter dealing with the elementary considerations of power transmission has been added.

J. H. M.
F. W. H.

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ALTERNATING CURRENT CIRCUITS AND MACHINERY

CHAPTER I

FUNDAMENTAL PRINCIPLES. REACTIONS IN ALTERNATING-CURRENT CIRCUITS

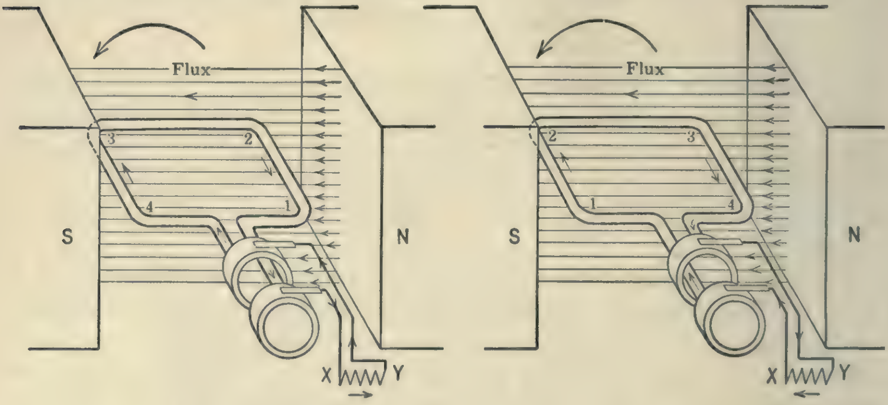
1. Advantages in the Use of Alternating Currents.—The use of electrical power is at present almost universal and is continually growing; for commercial purposes electrical power is derived either from coal or from the fall of water from a higher to a lower level. It may be generated, by suitable machinery, either in continuous-current form or in alternating-current form.

Comparing the generation of electrical power in the two forms, we find that continuous-current generators may be built with capacities as high as 5000 kw. at voltages as high as perhaps 500 volts, and in smaller capacities with voltages as high as perhaps 1500 volts. Alternating-current generators may be built with capacities of 50,000 kw. or more, at voltages as high as 15,000 volts. The advantages to be derived from large capacity in a generating unit are that the larger the unit, the more efficient it is and the lower is its cost and weight per kw. of capacity.

The efficiency, and possible distance, of electrical power transmission increases as the voltage employed increases. Not only is it possible to generate alternating currents at higher voltages than continuous currents, but there is also available, with alternating currents, the transformer, a comparatively cheap and very reliable stationary piece of apparatus, which makes it possible to transform alternating currents up or down, from one voltage to another, with efficiencies as high even as 99 per cent. By the the use of high voltages, it is possible to transmit power by alternating currents over distances that would otherwise be too great.

It is now planned to generate power in large plants located as near the coal mines as possible, and to transmit it at about 200,000 volts to all of the large Eastern cities as far north as Boston. All of the large, efficient, power plants burning coal, and all available water power plants now located in this area would be tied into the system, making it possible to abandon a great many small and inefficient plants. Such a power project, impossible with continuous currents or other means, would result in a much more efficient use of the coal; it would guarantee power at any point in the event of the failure of any local plant in the system; it would reduce very materially the amount of coal that would have to be hauled for power purposes.

The use of transformers also makes for more efficient distribution of power; current at 2200 volts may be delivered at the customer's premises over comparatively light wires or cables and then transformed to 110, 220, or 440 volts as desired. With continuous current, distribution mains, carrying the voltage desired by the customer, must be laid in the streets, and because of the relatively low voltages, 110 to 220 volts, such distribution mains must be capable of carrying large currents and are therefore of large cross-section, heavy, and costly.



(a) (b)

FIG. 1.—A coil turning at uniform speed in a parallel field of uniform strength, will generate an alternating voltage of sine form. The current in an attached load will flow successively in both directions as the coil makes one revolution.

Alternating-current motors and other apparatus are quite satisfactory for most purposes, exceptions being electroplating and electric railways, where continuous current is generally used. Although the alternating-current motor does not have the same flexibility as to speed control as the continuous-current motor, this disadvantage is in a measure offset by the fact that alternating-current motors are available in much larger capacities than continuous-current motors, and in general the care and maintenance of an a.c. motor is less than that of a c.c. motor.

2. Generation of a Sine Wave of Voltage.—If a single coil of wire, with its ends connected to slip rings, is rotated in a uniform field, as in Fig. 1, an alternating e.m.f. will be generated. As either coil-side moves up in front of the north pole, the voltage generated in it will be directed toward the observer; while moving down in front of the south pole, the voltage will be directed away from the observer, as the direction of the motion has been reversed and the field has the same direction as before. Continued rotation of the coil will evidently generate a voltage which alternates, or periodically changes its direction.

If an external circuit is connected to the brushes resting on the slip rings, this alternating voltage must produce a current, the direction of which at any instant is determined by the direction of the impressed voltage. Thus, in Fig. 1*a*, the direction of current in the external circuit is from *X* to *Y* and in Fig. 1*b* from *Y* to *X*; such a current, periodically reversing its direction, is called an alternating current.

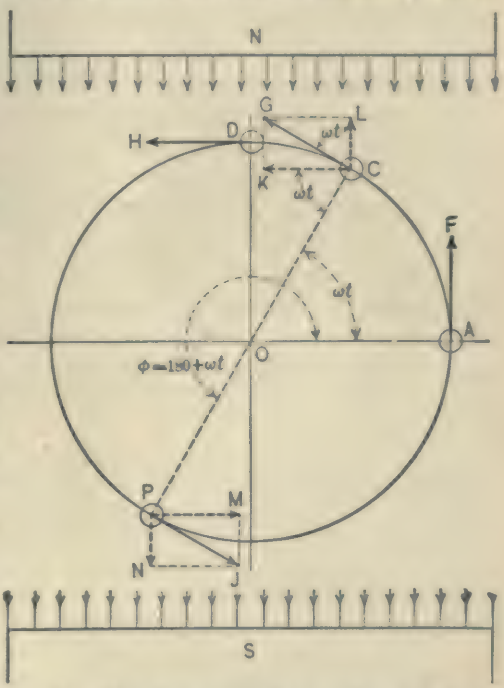


Fig. 2.—This diagram of the motion of one of the coil-sides of Fig. 1, shows how a sine wave of e.m.f. is generated.

The exact form of the voltage generated by the rotating coil may be determined by consideration of Fig. 2, in which only the coil-side 1–2 of Fig. 1 is represented by the small circles; the two coil-sides being 180° apart, and a uniform magnetic field having been assumed, whatever value of voltage is generated in one coil-side will also be generated in the other in the opposite direction with respect to the shaft, or in the same direction around the coil. The path of a coil-side will be the circle *ACDP*; the speed of rotation of the coil being uniform, the direction of motion and the peripheral velocity of the coil-side at any instant may be represented by equal vectors drawn tangential to the circle, as *AF*, *CG*, *DH*.

The voltage generated by a conductor moving in a magnetic field is proportional to the rate at which it "cuts" the lines of force. The velocity of the coil-side at any instant may be resolved into a horizontal component, *CK*, *PM*, etc., and a vertical component, *CL*, *PN*, etc., and it will be seen that the rate of cutting of the lines by a coil-side will be directly proportional to its horizontal velocity. When the coil-side is at A it is moving parallel to the direction of the field, the horizontal velocity of the conductor is zero, and so no voltage is generated. When the coil-side is at D it is moving at right angles to the field, so that its vertical velocity is zero and its horizontal velocity a maximum. The coil in this position is therefore cutting the lines of force at its greatest rate and hence generating the highest, or maximum, value of voltage, $E_{\max}$, of which it is capable. The vector DH may therefore also represent the maximum instantaneous value of the voltage generated during one revolution of the coil-side.

Considering the same phenomena from the standpoint of "interlinkages," as is frequently done, we reach the same conclusions. When the coil is in the horizontal position, OA, it is linking with the maximum possible flux; as it rotates, this interlinked flux decreases, gradually at first and then more rapidly, until the coil reaches the vertical position OD in which no flux is linking with the coil. The amount of flux linking with the coil for any position is proportional to the area of the coil projected on the horizontal axis OA, and this projected area is varying at its maximum rate when the coil is going through its vertical position OD. Hence, from this viewpoint also, we reach the conclusion that the coil is generating a maximum voltage when it is in the vertical position.

If time is measured from the instant when coil-side 1–2 is at A and the angular velocity of the coil is ω radians per second, at time t seconds, when the coil-side is at C, it will have moved through an angle ωt radians (which may, of course, also be given in degrees). The horizontal velocity of the coil-side at C is CK.

Since

$$\angle AOC = \angle OCK = \angle GCL = \omega t,$$

it follows that

$$CK = CG \sin \omega t,$$

and since

$$CG = DH = E_{\max},$$

we may write that

$$e = E_{\max} \sin \omega t, \qquad (1)$$

where e = the instantaneous value of voltage, or the voltage generated at any instant during the revolution of the coil,

$E_{\max}$ = the maximum value of the voltage generated during a complete revolution of the coil,

ωt = the angle made by the coil with the horizontal, at the instant considered.

When the coil-side 1–2 is at position P, having moved through ϕ degrees or $180° + \omega t$ degrees, Eq. (1) takes the form

$$e = E_{\max} \sin \phi = E_{\max} \sin (180° + \omega t).$$

The sine of $(180° + \omega t)$ is the same as $(-\sin \omega t)$, so that we have

$$e = -(E_{\max} \sin \omega t),$$

the negative sign denoting that the direction of the voltage is opposite to what it was before.

3. Construction of a Sine Wave.—The actual shape of a sine wave may be obtained by construction, as follows: Draw a circle and divide its circumference into a number of equal parts, as in Fig. 3. Extend the horizontal line and divide a portion into equal distances, the same in number as the distances into which the circumference was divided. The divisions $a'b'$, $b'c'$, etc., will then represent the same time intervals as the arcs ab, bc, etc.

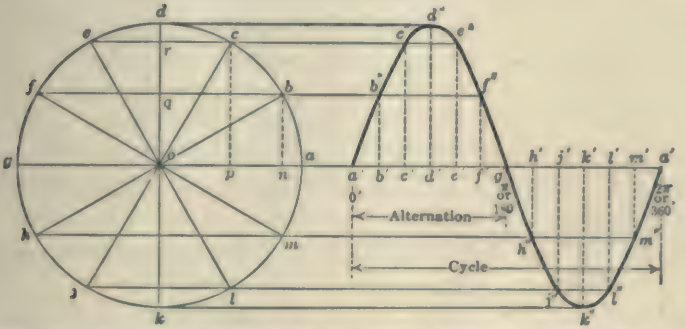


FIG. 3.—Showing how a sine wave is constructed from the vertical components of suitably spaced radii.

From simple trigonometry, we know that $\sin aob = \frac{bn}{ob}$, $\sin aoc = \frac{cp}{oc}$, etc., and since ob, oc, etc., are all equal to the radius of the circle, the sines of the respective angles are proportional to bn, cp, etc.

Now lay off the distances bn, cp, od, etc., on perpendiculars through their respective points b', c', d', etc., giving the points b'', c'', d'', etc.; a smooth curve through the latter points will be the sine wave desired.

It is to be noted further that, as the maximum value of the resulting sine wave is equal to the radius of the circle, any instantaneous value of the sine wave may be determined by projecting, upon the vertical axis, the radius in the corresponding instantaneous position. This is indicated at oq, or, etc.

An alternating voltage of sine form therefore increases from zero in one direction up to a maximum value, decreases again to zero, reverses its direction, and repeats the same series of values in the opposite sense.

4. Cycle. Frequency.—An alternating voltage, then, is one that periodically changes its direction, assuming alternately positive and negative values. Each revolution of the coil in Fig. 1 results in a complete *cycle* of events, which is represented by one positive and one negative loop of the sine wave of Fig. 3. One-half a cycle is called an *alternation;* in Fig. 3 the time represented by the distance $a'a'$ is one cycle (or period) of the alternating voltage represented. The number of cycles (or periods) passed through by the voltage in one second is called the *frequency* (or periodicity); a voltage with a frequency of 60 cycles, for example, passes through 60 cycles per second. The frequency is sometimes expressed, however, in alternations per minute, and we might say that this 60-cycle voltage had a frequency of 7200 alternations per minute.

In Fig. 1, the frequency of the voltage in cycles per second generated by the coil is the same as the number of revolutions per second, since there are only two field poles. If the same coil were rotated in a four-pole field, two cycles per revolution would result and the frequency of the voltage would be double the r.p.s. It is evident that in Eq. (1) the angle ωt must be expressed in electrical radians, and not in mechanical radians, there being 2π electrical radians per pair of poles; there are thus 4π electrical radians per revolution in a four-pole machine.

If the coil of a two-pole machine rotates f times per second, its angular speed is evidently $2\pi f$, so that Eq. (1) becomes

$$e = E_{\max} \sin 2\pi f t, \quad . \quad . \quad (2)$$

in which f is the frequency in cycles per second.

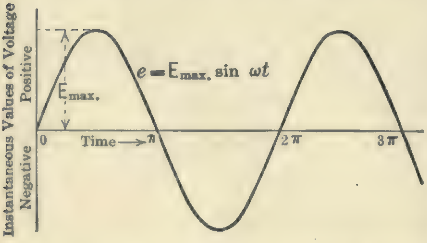


Fig. 4.—The conventional representation of an alternating voltage makes it a pure sine wave, as shown here.

5. Vector Representation of a Sine Wave of Voltage.—We have seen that a single coil, rotating at constant speed in a uniform field, will generate a sine wave of voltage which may be represented by a sine wave as in Fig. 4. The same idea is conveyed by the equation

$$e = E_{\max} \sin 2\pi f t.$$

We may also represent an harmonic voltage by a rotating vector which has a length equal to the maximum value of the voltage generated and a speed of rotation equal to 2π times the frequency of the alternating voltage.

In Fig. 5 the vector is considered as rotating in counter-clockwise direction with angular velocity ωt, where $\omega = 2\pi f$. Time is reckoned as zero when the vector OA lies along OX, so that at the end of the time t the vector has swept through the angle ωt. At this time the instantaneous value of the voltage, which the vector diagram is supposed to show, is $OB = OA \sin \omega t$; and as the length of OA has been chosen equal to the maximum value of the voltage, it is evident that the projection of the vector OA on the vertical axis represents, at any time, the instantaneous value of the voltage expressed analytically by the equation $e = E_{\max} \sin \omega t$.

Suppose that we wish to know the instantaneous value, at the instant when $\omega t = 30°$, of a voltage of sine form, the maximum value of which is 100 volts. We may obtain the desired answer either by measuring the ordinate at 30° on a sine wave whose maximum value is 100, or by laying off a radius, 100 units long, making an angle of 30° with the horizontal, and measuring its vertical projection. It is easiest, however, to use Eq. (1), substitution in which gives

$$e = 100 \sin 30° = 100 \times 0.5 = 50 \text{ volts.}$$

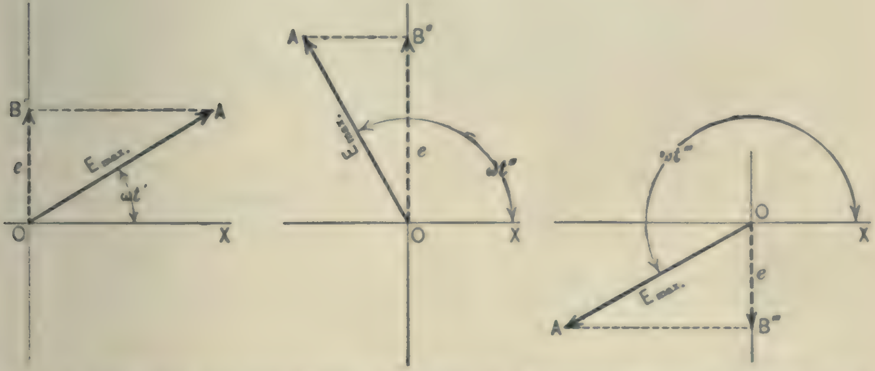


FIG. 5.—Showing how the vertical projection of a uniformly rotating vector may be used to represent a sine wave.

Had the value for an angle $\omega t = 585°$ been required, we would have $e = 100 \sin 585°$. Now an angle of 585° is the same as the angle 585°–360°, or 225°, which lies in the third quadrant and is equal to 180°+45°. Further, we know that the sine of 225° is equal to minus the sine of 45°. We have then that $e = 100 \times (-\sin 45°) = 100 \times (-0.707) = -70.7$ volts.

The instantaneous value of the voltage is then 70.7 volts, but its direction is opposite to that taken as positive.

6. Upper Harmonics Affect Wave Shape.—The shape of the curve representing an alternating e.m.f. is sometimes very important. In nearly all commercial circuits, the form of the voltage wave will be very nearly the simple sine curve as shown in Fig. 4, but for various reasons there will always be more or less departure from the simple sine form, caused by the presence of sine waves of higher frequency than the fundamental.

The voltage wave generated by a commercial alternating-current generator, which has approximately the form of a 60-cycle sine wave, is likely to be distorted by the presence of other sine waves of frequencies of 180 cycles per second, 300 cycles per second, etc. These higher frequency voltages are called *upper harmonics,* being some multiple of the fundamental wave; the 180-cycle wave is the third harmonic of the 60-cycle wave, the 300-cycle wave is the fifth harmonic, etc.

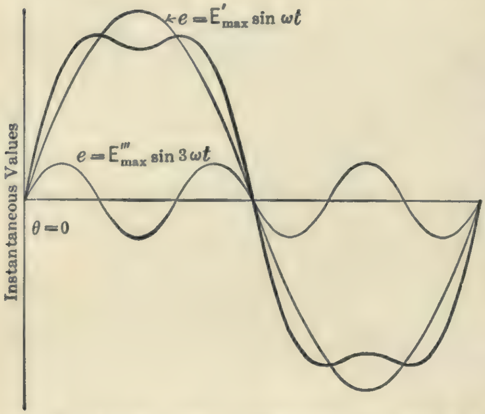


FIG. 6.—A complex voltage wave made up of a fundamental and third harmonic; the third harmonic is taken in such phase as to make the complex wave comparatively "flat-topped."

In all ordinary cases of distorted voltage waves, *only the odd harmonics appear;* this is because the machines in which they are generated are mechanically and magnetically rigid and symmetrically built. In the ordinary machine, whatever conditions a conductor finds in moving under one pole, it finds in exactly the same sequence under all other poles.

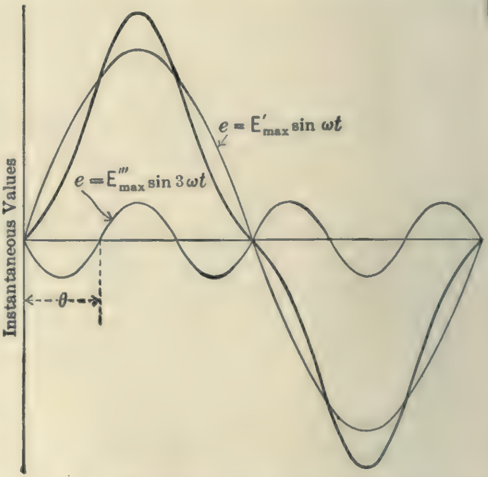


FIG. 7.—A complex wave made up of a fundamental and third harmonic; the third harmonic is here shown in a phase 180° from that shown in Fig. 6, resulting in an e.m.f. wave more "peaked" than a pure sine wave.

In Figs. 6 and 7 are shown two typical wave forms which are made up of a fundamental and a third harmonic; in Fig. 6 the third harmonic starts with positive values; in Fig. 7 it begins with equal negative values. The resultant wave in Fig. 6 is a *flat-topped* wave, while in Fig. 7 it is *peaked.* Fig. 8 shows the e.m.f. wave of an alternating-current generator which the manufacturers guaranteed to give a sine wave; this shows how poorly a machine may sometimes be designed; it is made up of the fundamental with pronounced fifth and seventh harmonics. Film *a* shows the voltage of one phase of the machine and film *b* shows the line voltages when it is connected as a three-phase generator, the three phases being interconnected in the *Y* scheme (see section 39).

It will be noticed in Figs. 6 and 7 that the sequence of values during

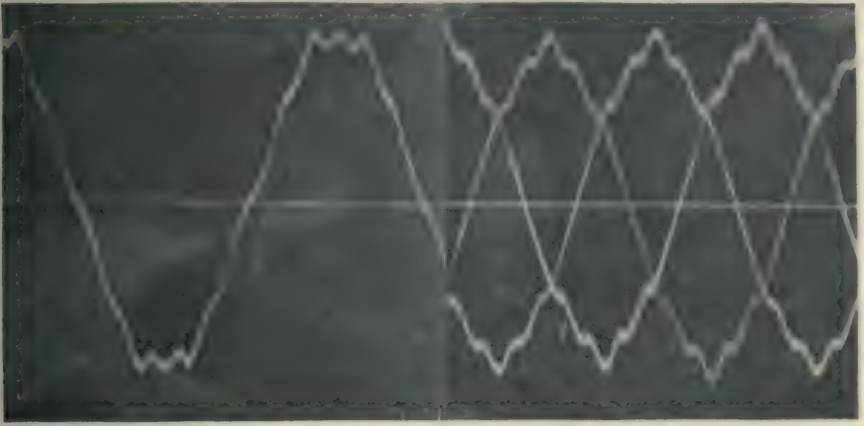

(*a*) (*b*)

FIG. 8.—An oscillogram of a commercial alternator; in *a* is given the voltage of one phase of the machine and in *b* are shown the three line voltages when the machine is Y-connected.

the negative half of the cycle is exactly similar to that of the positive half; the two halves of the cycle are similar in a peculiar way. In Fig. 9 is shown one complete cycle of a distorted alternating wave form; if the loop *B* is moved back on the time axis to *B′*, then *A* and *B′* are symmetrical with respect to the time axis. Practically all alternating current and voltage waves have this characteristic "mirror symmetry"; consideration of Fig. 10, which is made up of a fundamental and second harmonic will indicate that this mirror symmetry can exist only when no even harmonics of the fundamental are present.

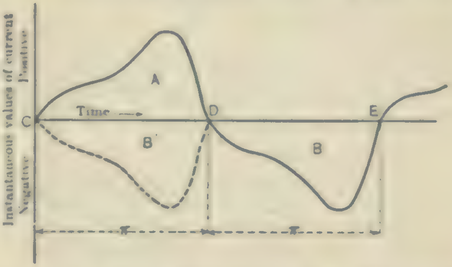


FIG. 9.—A wave form typical of the exciting current of a transformer; it has a peculiar symmetry in that loop *B* moved backward 180° seems to be the reflection of loop *A*. There are no even harmonics in such a wave form.

The harmonics in a distorted voltage wave may have any possible relative position with respect to the fundamental. In Fig. 11 is shown how a wave form similar to that of Fig. 9 may be made up of a fundamental and a third harmonic, the latter starting at its maximum positive value when the fundamental starts from zero. It is to be noticed that this "lop-sided" wave is made up of just the same components as the two symmetrical waves shown in Figs. 6 and 7; the change in form has been brought about by a shift in the phase of the third harmonic.

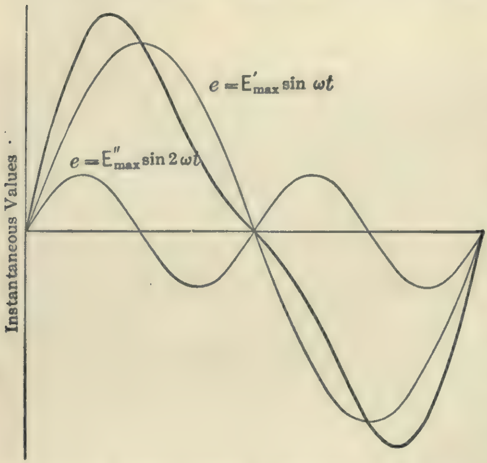


FIG. 10.—A wave form made up of a fundamental and second harmonic; it will be noticed that this wave does not have the symmetry described in Fig. 9.

7. Frequencies in Common Use.—The standard frequencies in use in this country for lighting and power purposes are 60 cycles and 25 cycles; 50 and 40 cycles are, however, also used to some extent. In the early days of alternating-current engineering, a frequency of 133 was adapted as standard; it is now no longer used. Sixty cycles is the lowest frequency that should be employed for lighting and is standard for this purpose; if a lower frequency than this is used, the resultant flickering of the lamps becomes tiring to the eyes. For the transmission of power, 25 cycles is more suitable, giving less line drop and better voltage regulation.

For the transmission of messages by wireless telephony and telegraphy a frequency of many thousand cycles per second is used; machines have been built for this work which generate as high as 200,000 cycles per second.

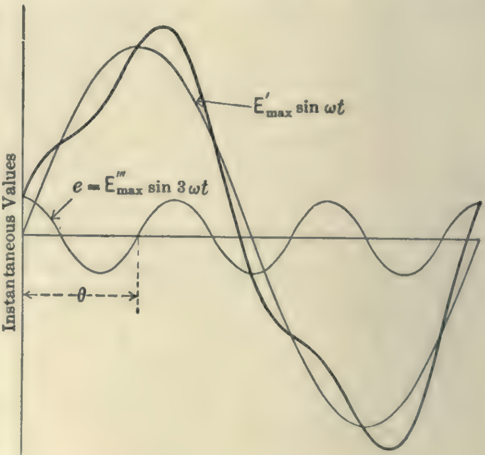


FIG. 11.—A wave made up of the same components as those of Figs. 6 and 7; this wave still has the symmetry which indicates absence of even harmonics but has been made "lop-sided" by changing the phase of the third harmonic.

8. Alternating Currents.—An alternating current flows when an alternating e.m.f. is applied to a closed circuit. Whatever has been said so far about alternating voltages may be applied directly to an alternating current of electricity. We find these currents periodically changing their direction of flow and assuming alternately positive and negative values, the sequence of values assumed during a negative half of the cycle being exactly similar to that of the positive. In other words, an alternating-current wave also possesses "mirror symmetry"; Fig. 9 is actually representative of the current wave of an unloaded transformer.

Nearly all commercial currents are simple sine waves; any departure from the simple sine-wave form is due to the presence of higher frequency currents, each of which is of sine form. As in the case of distorted voltage waves, only currents which are odd harmonics of the fundamental will appear to produce a distorted current wave, except in some very special cases, where the current waves do not have the mirror symmetry described in section 6.

9. Actual Amount of Motion of the Electrons.—It must not be thought that the electrons in a wire carrying alternating current move back and forth from one end of the circuit to the other; such is far from being the case. The electron drift, which constitutes the current, is very slow, the electrons on the average moving along the conductor only a very small fraction of 1 cm. per second; in fact, even if the wire is carrying so much current that the resultant I^2R loss causes it to become red hot, the electrons are drifting along the conductor at an average velocity of less than 1 cm. per second.

It appears, then, that with a current reversing 60 times per second, and with consequent to-and-fro drift of the electrons this number of times per second, the average to-and-fro motion of an electron in a copper wire carrying a 60-cycle current is perhaps 0.0001 cm.; less than this for small current densities and possibly one hundred times as much for a wire carrying sufficient current to bring it to the melting point. Although these distances are extremely small, it is to be pointed out that there is plenty of chance for collisions of electrons and molecules, and consequent production of heat, because even when an electron moves only 0.0001 cm. it passes by about 10,000 molecules of the copper.

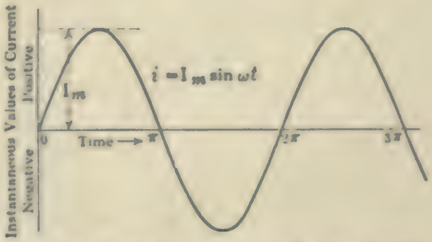


Fig. 12.—Alternating currents are conventionally taken as pure sine waves, the same as alternating voltages.

10. Representations of a Sine Wave of Current.—A sine wave of current may be represented in the same three ways that were used to indicate a sine wave of voltage. In Fig. 12 is drawn out a sine wave of current, having a maximum value, $I_{\max}$. This sine wave of current may evidently be indicated by the equation

$$i = I_{\max} \sin 2\pi ft, \qquad (3)$$

where i is the instantaneous value of the current at any time t;
$I_{\max}$ is the maximum value of the current;
f is the frequency.

The same sine wave of current may be indicated by a vector diagram, as in Fig. 13, the rotating radius having a length equal to $I_{\max}$. The vector OA is rotating in counter-clockwise direction with angular velocity $\omega t = 2\pi ft$, time being reckoned as zero when the vector OA lies along the horizontal axis Ox. The angle passed over at the end of time t is then equal to $\omega t = 2\pi ft$. As in the voltage vector diagrams, the vertical projection OB of the rotating vector OA gives the instantaneous value of the current.

11. Effective Value of an Alternating Current.—As the magnitude of an alternating current varies from instant to instant, assuming all values between its maximum positive and maximum negative values, it is apparent that we must establish some method of determining what is the *effective numerical value of such a current.* The heating effect of a current, being independent of the direction of flow, forms a convenient basis of comparison of alternating and continuous current. In fact, the unit alternating current is chosen as that current which will produce the same heating effect as unit continuous current. On this basis, an *alternating current is said to have an effective value of one ampere when it produces heat in a certain resistance at the same average rate as heat is produced in the same resistance by one ampere of continuous current.*

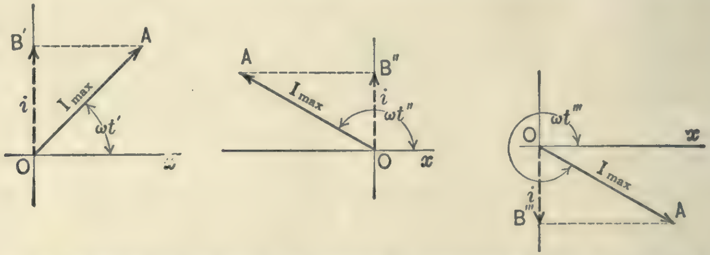


Fig. 13.—An alternating current is generally represented by a rotating vector; the instantaneous value of the current is equal to the vertical projection of the rotating vector.

The heating effect of any electric current is proportional at any instant to the square of its value at that instant. Therefore, during a given period of time, the average heating of an alternating current will depend upon the average of the squares of all instantaneous values during that length of time. The effective value of the current is therefore found by squaring all the instantaneous values first, averaging them, and then taking the square root of this average. In other words the effective value of an alternating current is that value, which, when squared, will be equal to the average of the squares of all the instantaneous values of the current. This value of the current is expressed in some texts by the letters R.M.S., meaning "root mean square." It is this number of amperes which would be indicated by an ammeter capable of measuring both continuous and alternating currents, such as a meter of the hot-wire type.

12. Effective Value of a Sine Wave.—Since any sine wave of current may be indicated by the equation

$$i = I_{\max} \sin \omega t,$$

the instantaneous value of the current at any time is equal to a constant multiplied by sine ωt. Different sine waves of current are thus distinguished one from the other only by their maximum values, and it follows that any ratio between effective value and maximum value for one sine wave will hold for all others.

TABLE I

Angle in Degrees	Sine	Sine Squared
1	.01745	.000305
3	.05233	.002738
5	.08715	.007596
	....	....
....	....	....
43	.68200	.465124
45	.70711	.500000
47	.73135	.534876
....	....	....
....	....	....
85	.99619	992404
87	99863	997262
89	.99985	999695
Sum	28 64789	22.500000

One way of finding the effective value of a sine wave is to take a sufficient number of equally spaced ordinates and find the square root of the average square. If the values of the sine of the odd angles in degrees, beginning with 1° and ending with 89°, are squared as is indicated in Table I, it will be found that the sum of the squared values is practically 22.5, which for 45 squared values gives an average squared value of 0.500. We would have obtained the same answer if we had averaged the values over 180° instead of 90° because of the symmetry of the curve. The effective value of a sine wave (the maximum of which evidently is 1.0) is then $\sqrt{0.5}$ or 0.707. For any other sine wave, the ratio of effective to maximum value is also 0.707 (or $\frac{1}{2}\sqrt{2}$). A current of sine-wave form, the maximum value of which is 1 ampere, has, therefore, an effective value of 0.707 ampere; if such a current is passed through an alternating-current ammeter, the meter will indicate 0.707 ampere. Such a current would be spoken of as a current of 0.707 ampere.

For a sine-wave current of any magnitude whatever, we see that its effective value is 0.707 of its maximum value, i.e.,

$$I = 0.707 I_{\text{max}} \quad \text{or} \quad = \tfrac{1}{2}\sqrt{2} I_{\text{max}}, \quad . \ . \ . \ . \ . \quad (4)$$

or

$$I_{\text{max}} = 1.414 I = \sqrt{2} I, \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (5)$$

where I is the effective value in amperes.

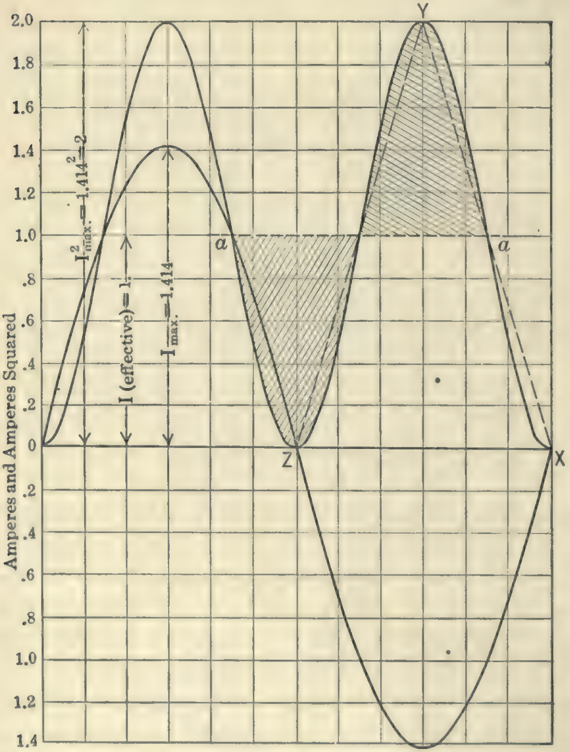


FIG. 14.—A curve diagram to bring out the significance of an "effective" value of one ampere.

The idea of the effective value of an alternating current may be looked at in a somewhat different way. In Fig. 14 is plotted a current of sine-wave form, of 1.0 ampere (effective value); its maximum value is then 1.414 amperes. The curve of instantaneous values squared is also shown, and since the square of any instantaneous value, either positive or negative, is also positive, it is found to be a curve of double frequency, the maximum value being $(1.414)^2$, or 2.

The heating effect of the current will then be proportional to the area between this curve and the horizontal axis OX. Now, it can readily be proved mathematically that the current-squared curve is actually a sine curve, its axis of symmetry being the broken line aa. From this it follows that the shaded areas, above and below the line *aa*, are equal, so that the distance from *OX* to *aa* represents the average heating effect of a current of 1.0 ampere, and the square root of this distance gives the effective value of the current, as $\sqrt{1}=1$ ampere.

By inspection it is evident that the current-squared curve between the points *ZY* and *YX* is symmetrical about the lines *ZY* and *YX* respectively, and hence the area under the current-squared curve from *Z* to *X* is the same as the area of the triangle *ZYX*. But, as the average height of a triangle is one-half its altitude, which, in this case is 2, the average height of the triangle, which is the same as the average height of the current-squared curve, is 1, as stated above.

Had a current of 10 amperes been originally assumed, its maximum value would have been 14.14 amperes and the maximum value of the current-squared curve would have been 200. The average heating would have been represented by 100, and the square root of this average value ($\sqrt{100}=10$), checks the effective value of the current assumed.

13. Effective Value by Integration.—The exact determination of the ratio of the effective value of a sine curve to its maximum value must be carried out by calculus. Since the effective value of an alternating current is to be numerically equal to that value of continuous current which produces the same average heating through the same resistance over a given time, and since the instantaneous heating of the alternating current varies from instant to instant, we may write

$$I^2rT=\int_0^T i^2r\,dt, \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \ (6)$$

where I is the effective value of the a.c. current, or, is the c.c. current t
which it is numerically equal,
T is a time interval,
r is a resistance,
and i is the instantaneous value of the current.

As the sine wave is generally plotted in terms of radians, instead of seconds, to make the values of the abscissæ repeating, the variable measured along the *X*-axis is really angle, instead of time. So, instead of considering the differential interval of time as dt, we take it as $d(\omega t)$. As the curve repeats itself after ωt has been through 2π radians, and further, as the squared curve repeats itself after the angle ωt has been through π radians, it is not necessary to integrate Eq. (6) over an interval greater than π radians. As a matter of fact, owing to the symmetry of the squared curve, about a vertical line erected at the point on the *X*-axis where $\omega t=\pi$ 2, it is really necessary only to integrate over the distance π 2; however, we shall integrate over one complete cycle of the squared curve.

We substitute for i its value, $I_m \sin \omega t$, and for dt its equivalent $d(\omega t)$, and for the time interval, T, a value of ωt equal to π; Eq. (6) then becomes

$$I^2 = \frac{I_m^2}{\pi} \int_0^\pi \sin^2 \omega t \; d(\omega t).$$

Substituting for $\sin^2 \omega t$, its value $\dfrac{1-\cos 2\omega t}{2}$,

$$I^2 = \frac{I_m^2}{\pi} \int_0^\pi \left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t),$$

$$I^2 = \frac{I_m^2}{\pi} \left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_0^\pi.$$

Since the second term yields zero value at both limits, this reduces to

$$I^2 = \frac{I_m^2}{\pi} \left[\frac{\pi}{2}\right] = \frac{I_m^2}{2}.$$

Taking the square root of both sides, we find

$$I = \frac{I_m}{\sqrt{2}} = \frac{I_m}{1.414} = 0.707 I_m. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

14. Effective Value of an Alternating Voltage.—What has been said in regard to current waves holds good also for voltage waves; therefore, we have, when $e = E_m \sin \omega t$

$$E = \frac{E_m}{\sqrt{2}} = 0.707 E_m, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

where e = the instantaneous value of the voltage,
E_m = the maximum value of the voltage,
E = the effective value of the voltage.

15. Average Value of a Sine Wave.—Since half of the instantaneous values of a sine wave of current or voltage are positive and the other half are negative, and since both positive and negative values follow the same sequence, the average of a complete cycle of values must be zero. It is sometimes useful, however, to know the average value of a sine wave over one alternation, which value evidently is not zero. This average value may easily be obtained by measuring the area between the sine curve and the axis and then dividing this area by the base of the curve; it may also be obtained by adding the sines for a large number of angles from 0° to 90°, differing from each other by small, equal increments, and dividing the sum obtained by the number of values taken. If this is done as indicated in Table I, beginning with 1°, it will be found that the sum of the 45 values is 28.64789, the average of which is 0.636. This same value would have been obtained by averaging over 180°, because of the symmetry of the curve. Thus the ratio of average value of one alternation to the maximum value of a sine wave of current or voltage is 0.636, or,

$$I_{av}=0.636I_{max}=\frac{2}{\pi}I_m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The exact determination of the ratio of the average value of a sine wave to its maximum value must be done by calculus.

We have then,

$$I_{av}=\frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t=\frac{I_m}{\pi}\Big[-\cos \omega t\Big]_0^{\pi}=I_m\cdot\frac{2}{\pi}=0.636I_m. \quad . \quad . \quad (10)$$

16. Effective Value of a Complex Wave.—The effective value of a complex or distorted wave, made up of a fundamental and its upper harmonics, is found in exactly the same way as was indicated for a sine wave. (It is to be pointed out again that, in general, only the odd harmonics occur in the ordinary electrical circuit.) We may either find the average of a large number of squared instantaneous values and extract the square root of this average, or we may plot a curve having ordinates equal to the squares of the ordinates of the given wave, and find the average value of this squared wave by measuring the area of its loops by a planimeter and dividing by the base; the effective value is then the square root of this average.

The effective value of a complex current wave, supposedly having only odd harmonics, is given by the expression

$$I=\sqrt{I_1^2+I_3^2+I_5^2+\ldots} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where I, I_1, I_3, I_5, etc., are the effective values of the complex wave, the fundamental, the third harmonic, the fifth harmonic, etc., respectively.

The effective value of a complex voltage wave is given by a similar expression,

$$E=\sqrt{E_1^2+E_3^2+E_5^2+\ldots} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The proof of this relation requires the use of the calculus. Consider the complex voltage waves of Figs. 6, 7, and 11, which are all made up of a fundamental and a third harmonic. The complex wave may be expressed as an equation

$$e=E'_m \sin \omega t+E'''_m \sin (3\omega t+\theta),$$

where E'_m and E'''_m are the maximum values of the fundamental and third harmonic respectively, and θ is the angle on the X-axis between corresponding zero values of the two waves. In Fig. 6, $\theta=0°$, in Fig. 7, $\theta=180°$, and in Fig. 11, $\theta=270°$, θ being given in degrees corresponding to the third harmonic curve.

The effective value, squared, of the complex waves is then

$$E^2=\frac{1}{\pi}\int_0^\pi\{E'_m \sin \omega t+E'''_m \sin (3\omega t+\theta)\}^2 d(\omega t).$$

Let

$$E'_m=A_1 \quad \text{and} \quad E'''_m=A_3.$$

Then

$$E^2=\frac{1}{\pi}\int_0^\pi A_1{}^2 \sin^2 \omega t\, d(\omega t)+\frac{1}{\pi}\int_0^\pi A_3{}^2 \sin^2 (3\omega t+\theta)d(\omega t)$$

$$+\frac{1}{\pi}\int_0^\pi 2A_1A_3 \sin \omega t \sin (3\omega t+\theta)d(\omega t).$$

The values of the first two integrals are, from section 13, evidently $\frac{A_1{}^2}{2}$ and $\frac{A_3{}^2}{2}$, respectively. The third integral can be obtained by expanding the term $\sin (3\omega t+\theta)$.

The third integral

$$=\frac{2A_1A_3}{\pi}\int_0^\pi \sin \omega t \sin (3\omega t+\theta)d(\omega t)$$

$$=\frac{2A_1A_3}{\pi}\int_0^\pi \sin \omega t(\sin 3\omega t \cos \theta+\cos 3\omega t \sin \theta)d(\omega t)$$

$$=K\int_0^\pi (\sin 3\omega t \sin \omega t)d(\omega t)+K_1\int_0^\pi (\sin \omega t \cos 3\omega t)d(\omega t),$$

where K and K_1 are the products of the constants in the terms of the previous equation.

Now it is shown in any standard calculus that integrals of the form $\int(\sin \theta \sin n\theta)d\theta$, or $\int(\sin \theta \cos n\theta)d\theta$, ($n$ being an integer), are equal to zero, providing the integration is extended over one cycle of the lower frequency curve. That is, if we integrate either of the above expressions over one cycle of the fundamental current or voltage, the integration yields zero.

Hence we conclude that the effective value of the complex voltage, consisting of fundamental and third harmonic (the latter in any phase whatever with respect to the fundamental), is given by the relation

$$E^2=\frac{A_1{}^2}{2}+\frac{A_3{}^2}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

or,

$$E=\sqrt{E_1{}^2+E_3{}^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where E_1 and E_3 are the effective values of the components of the complex wave.

In case the wave is made up of more than two components the same relation holds good; *the effective value of a complex wave is the square root of the sum of the squares of the effective values of all the components.*

This effective value is what an alternating-current meter would read. If two alternators of different frequencies are connected in series, one frequency being 25 cycles and the other 75 cycles, and each alternator is adjusted to give 100 volts, a voltmeter across both in series will be actuated by the complex wave made up of 100 volts fundamental and 100 volts third harmonic. It will read $\sqrt{100^2+100^2}$ or, 141 volts, and not 200 volts, as might be expected.

It is to be noted that any wave made up of a fundamental and a third harmonic of equal amplitudes will have the same effective value, regardless of the value of θ, the angle on the X-axis between corresponding zero values. In other words, the three complex waves of Figs. 6, 7, and 11, have the same effective value.

The effective value of a complex wave of current is similarly

$$I=\sqrt{I_1^2+I_3^2+I_5^2+\ldots} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where I_1, I_3, I_5, etc., are the effective values of the fundamental, third, fifth, etc., harmonics, respectively.

17. Form Factor.—The form factor of any wave is defined as the ratio of its effective to its average value. For a sine wave, the effective and average values of which are 0.707 and 0.636 respectively, we have

$$\text{Form factor}=\frac{\text{Effective value}}{\text{Average value}}=\frac{E_m}{\sqrt{2}}\div E_m\times\frac{2}{\pi}=\frac{0.707}{0.636}=1.11,$$

or,

$$\text{Effective value} = \text{average value}\times\text{form factor}.$$

The form factor is an indication of the type of wave; for a wave more peaked than a sine wave, the form factor is greater than 1.11, and for a flat-topped wave it is less than this value. Two extreme cases are given in Fig. 15. It is to be noted that, for the flat-topped wave, which is practically that generated by a conductor in a continuous-current machine, the average and effective values are equal.

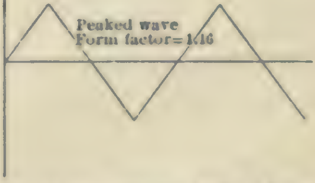


Fig. 15.—Showing how the form factors of two extremely distorted waves differ from that of a sine wave.

18. Rate of Change of a Current or Voltage of Sine-wave Form.—It has been previously shown, in the study of continuous-current circuits, that if the current in an inductive circuit changes, an e.m.f. of self-induction is set up, this voltage tending to prevent the change of current. The magnitude of this induced voltage was shown *to depend upon the rate of change of current.* Now, as an alternating current is changing at every instant, and as practically every circuit has more or less self-induction, it is evident that in alternating-current circuits the counter electromotive force of self-induction must play an important part in determining how much current flows. It therefore becomes important to study the exact rate of change of current in an alternating-current circuit; in fact, in the solution of the average alternating-current problem, the magnitude of the

TABLE II

Angle $2\pi ft$ Measured in Deg.	Sine $2\pi ft$	Change in Current in $\frac{1}{180f}$ of a Sec.	Change per Sec.	Change Divided by $2\pi f = 6.283f$	Mean Angle $2\pi ft$ Measured in Deg.	Cosine $2\pi ft$
-1 $+1$	$-.01745$ $+.01745$	$+.03490f$	$6.283f$	1.000	0	1.000
4 6	.06975 .10452	$+.03477f$	$6.259f$	.996	5	.996
14 16	.24192 .27563	$+.03371f$	$6.069f$	.966	15	.966
29 31	.48480 .51503	$+.03023f$	$5.441f$	.866	30	.866
44 46	.69465 .71933	$+.02468f$	$4.443f$	.707	45	.707
59 61	.85716 .87461	$+.01745f$	$3.141f$	.500	60	.500
74 76	.96126 .97029	$+.00903f$	$1.626f$	.259	75	.259
84 86	.99452 .99756	$+.00304f$	$.547f$	.087	85	.087
89 91	$+.99984$ $+.99984$	0	0	0	90	0
119 121	$+.87461$ $+.85716$	$-.01745f$	$-3.141f$	$-.500$	120	$-.500$

rate of change of the current is more important than is the magnitude of the current itself. As the rate of change of a complex wave is itself very complex, and as complex current waves are associated with inductive circuits rather infrequently, we shall investigate the rate of change of current for simple sine form only.

Consider a sine wave of current the maximum value of which is one ampere, and the frequency of which is f cycles per second; its instantaneous values will then be numerically equal to the value of the sine at any instant. When the angle ωt changes from $-1°$ to $+1°$, the sine curve of current changes from $-.01745$ to $+.01745$ ampere, a total change of $+.03490$ ampere. The time of one complete cycle is $1/f$ seconds, so that a 2-degree change in the angle requires $2/360$ of $1/f$ second, or $1/180f$ second. The current is therefore changing at a rate of $\frac{+.03490}{\frac{1}{180}f} = +.03490 \times 180f = 6.283f$ amperes per second. Now, if the angle is allowed to change from 4° to 6°, the current changes from $+.06975$ to $+.10452$ ampere, a total change of $+.03477$ ampere in $1/180f$ second, or, at a rate of $6.249f$ amperes per second.

In Table II this calculation is indicated for a number of 2-degree changes in the current, and it may be seen that if the rate of change of the current in amperes per second is divided by $2\pi f = 6.283f$, the quotient is in all cases equal to the cosine of the average angle over which the current change was calculated. When the current is increasing algebraically, the rate of change is positive; but when it is decreasing algebraically, the rate of change is negative. Thus, when the current changes from its 119° to its 121° value, it is decreasing in value algebraically and its rate of change is therefore negative, even though the current itself is positive. The rate of change of a sine wave is negative between the 90° and 270° points of the cycle and is positive from zero to 90° and from 270° to 360°.

We reach the conclusion, therefore, that the maximum rate of change of a current of sine form, the maximum value of which is one ampere, and the frequency of which is f cycles per second, is $2\pi f$ amperes per second, and that this maximum rate of change takes place when the current itself is passing through zero.

If the current assumed had not had unity maximum value, but any maximum value whatever, I_m, the instantaneous values and the values of change in amperes per second would have been multiplied by the factor I_m, and it would have been necessary to divide the change in amperes per second by $2\pi f I_m$ to get the cosine. We may say, then, that for any sine wave of current with maximum value I_m, the frequency of which is f cycles per second, the rate of change of current at any instant t, is equal to $2\pi f I_m \cos 2\pi f t$, which evidently is a cosine wave. The maximum value of this rate of change occurs when $\cos 2\pi ft = 1$ and is evidently equal to $2\pi fI_m$.

In Fig. 16 the values obtained by the calculation indicated in Table II are plotted, the full-line curve being a sine wave of maximum value of one ampere and of frequency of one cycle per second. The broken curve shows the rate of change of the current in amperes per second, its scale for convenience being taken as 1/4 of that of the sine wave of current. If the frequency of the sine wave is f cycles per second, the rate of change of the current is obtained by multiplying the ordinates by f.

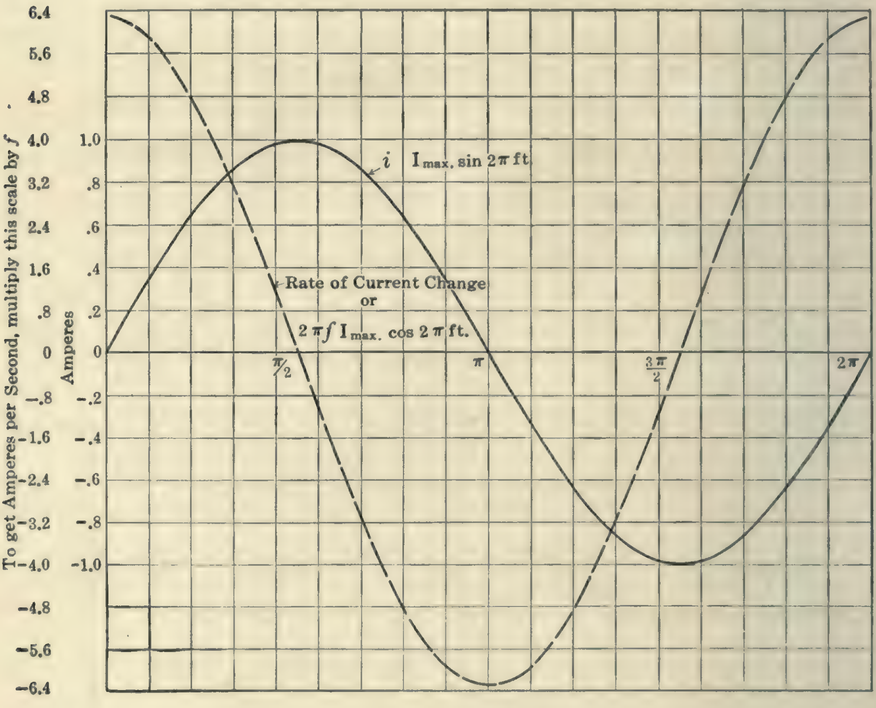


FIG. 16.—The rate of change of a sine wave of current is a cosine wave of the same frequency as the sine wave, and amplitude $2\pi f$ times as great.

The expression for the maximum rate of change of a sine wave of current already determined may be obtained in still another manner. Consider a sine wave of current of frequency f and of maximum value I_m, as was shown in Fig. 12. During a cycle the current changes four times from zero to maximum value, or *vice versa*, or once each quarter cycle. The time of a quarter cycle is $1/4f$ second; so that if the current changes from 0 to I_m amperes or from I_m to 0 amperes in one quarter cycle, its average rate of change is $\frac{I_m}{\frac{1}{4}f}=4fI_m$ amperes per second. We may write, then, that the average rate of change of current

$$\text{Av.}\left(\frac{I}{t}\right)=4fI_m.$$

We have already shown that the rate of change of a sine wave of current is itself a cosine wave and, as a sine wave and a cosine wave follow the same sequence of values, it follows that the relations between average, effective, and maximum values of a sine wave hold equally well for a cosine wave. We have, then,

$$\text{Maximum }\left(\frac{I}{t}\right)=\frac{\text{Av.}\left(\frac{I}{t}\right)}{0.636}=\frac{\text{Av.}\left(\frac{I}{t}\right)}{2\ \pi}=\frac{4fI_m}{2\ \pi}=2\pi fI_m.$$

To determine by the calculus an expression for the rate of change of a sine wave of current, it is only necessary to differentiate the expression $i=I_m \sin 2ft$ with respect to time.
Thus,

$$\frac{di}{dt}=\frac{d}{dt}(I_m \sin 2\pi ft)=2\pi fI_m \cos 2\pi ft, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which indicates that the rate-of-change curve is a cosine wave of maximum value equal to $2\pi fI_m$.

19. Law of Reactions in an Alternating-current Circuit.—If a brick is moved along a smooth board, a frictional reaction is developed, which acts to stop the motion of the brick; if we neglect temporarily the force required to accelerate the brick, we may say that the impressed force, acting to make the brick move, is always equal and opposite to this frictional reaction. In some cases the frictional reaction may be independent of the speed, as is nearly the case with the sliding brick; in others, as, for example, in the case of a stick being pushed through a viscous fluid like oil or molasses, the frictional reaction increases rapidly with increasing velocity. It is the same change in the frictional resistance which keeps the speed of a boat low; the force required to overcome the frictional resistance of the water varies nearly with the square of the speed.

If the brick is made to slide up an inclined board, the impressed force has to overcome two reactions of the system, the frictional reaction just mentioned and the force of gravity which is tending to make the brick slide down the board. In this case, therefore, we write:

Impressed force = frictional reaction + force due to a component of gravitational force.

The impressed force may be divided into two components, that required to overcome friction and that required to overcome gravitational force.

These two components of the impressed force will be equal to the respective reactions they serve to overcome, but are opposite to them in direction. The same ideas hold good in electric circuits.

In the case of continuous-current circuits containing only resistance, the law of action and reaction is known as Ohm's law, $E=IR$, which states that the impressed force is balanced by a resistance reaction, and we see that the resistance reaction is directly proportional to the current so that this electric circuit somewhat resembles the second analogue given above.

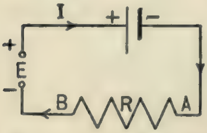


FIG. 17.—A battery being charged through a resistance.

It is well, at this point, to distinguish between a reaction and the component of the impressed force which overcomes the reaction. These are always equal, but are opposite in direction. Consider a simple circuit, as in Fig. 17, in which a battery of terminal voltage, e, and a resistance, R, are placed in series across a source of potential difference, E, the voltage of the battery being in the direction opposite to that of the source, with respect to the current.

If the voltage E is greater than that of the battery, current will flow as indicated, and a resistance reaction will be developed in the resistance, the direction of which is such as to stop the current; the reaction, through the resistance, must act in a direction from B to A. The "drop" across the resistance, or the component of E which acts to overcome the resistance reaction, acts through the resistance from A to B. Similarly, the voltage of the battery is a reaction with respect to E, and there must be a component of E which overcomes the battery voltage, e.

If we express the conditions of the circuit in the form

$$E-e-IR=0,$$

or,

$$E+(-e)+(-IR)=0,$$

the terms $(-e)$ and $(-IR)$ are the reactions of the circuit, but if we consider the terms e and IR, these are evidently the components of E which overcome the reactions of the circuit.

The law that action and reaction are equal is a general one, applying to all systems, mechanical or electrical. In a system in which various reactions are taking place, we may then state the law by saying that *at any instant the algebraic sum of all the reactions in the circuit must be exactly equal to the impressed force.*

If, then, a varying force is impressed on a system, the system must produce reactions of whatever kind it is capable, such that their algebraic sum at any instant is equal to the value of the impressed force at the same instant. If the *impressed force is varying, the sum of the individual reactions will also be varying, in like degree.*

There are two other kinds of reactions, besides the resistance reaction, and counter voltage of a battery, etc., which occur in alternating-current circuits; these will be taken up later.

20. Circuit Containing Resistance Only.—If a sine wave of voltage, $e=E_m \sin 2\pi ft$, is impressed upon a circuit containing only resistance, R, as in Fig. 18*a*, the circuit must develop a reaction which at any instant is equal and opposite to the value of the impressed voltage at that instant. Thus, in Fig. 18*b*, the full-line curve represents the voltage $e=E_m \sin 2\pi ft$; the curve of reaction must then be equal and opposite to it at every instant. It is shown in broken line; its equation is

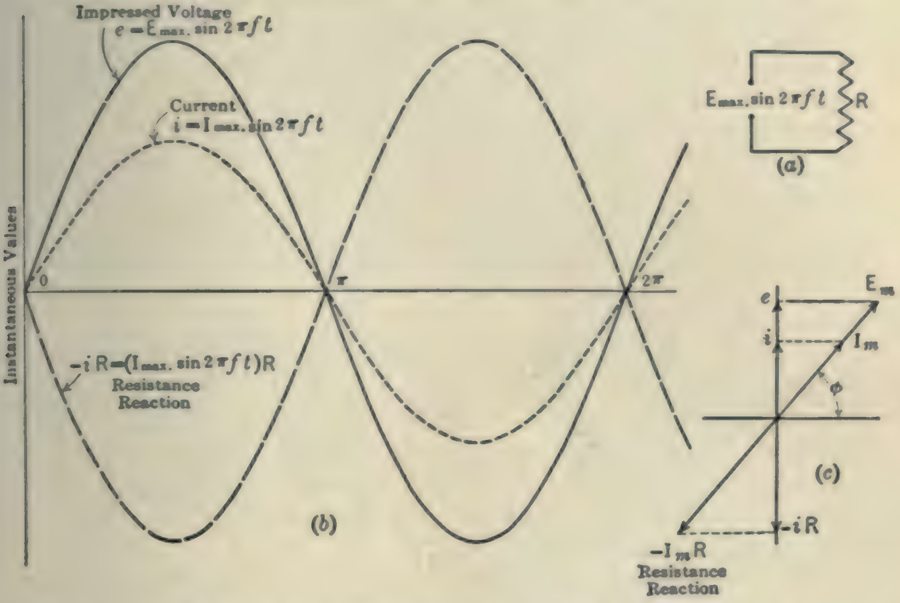


FIG. 18.—Curve diagram and vector diagram of current and voltages in a resistive circuit.

$$\text{Reaction} = -E_m \sin 2\pi ft. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

A circuit containing only resistance is capable of developing only resistance reaction whenever a voltage is impressed; the resulting current will therefore be of such magnitude that, at any instant,

$$e=IR,$$

where i is the instantaneous value of the current. To keep the equation

[illegible]

true at all parts of the cycle, it follows that the current must be of the form, $i = I_m \sin 2\pi ft$, so that

$$E_m \sin 2\pi ft = (I_m \sin 2\pi ft)R. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Since R is a constant, current and voltage must be proportional at every instant, and the current will be of sine form, as indicated by the dotted curve of Fig. 18*b*.

In this case, current and voltage, having proportionate positive or negative values at any instant and passing through zero together, are said to be *in phase*, this term being used to indicate that the time or angular difference between corresponding values is zero. The waves of impressed voltage and reaction are said to be 180° out of phase, since the impressed-voltage wave reaches its positive maximum value 180° before (or after) the reaction arrives at its positive maximum. Similarly, the impressed-voltage wave passes through zero with increasing values, 180° before the reaction passes through the same value, etc. We also speak of the impressed-voltage wave as leading (or lagging) the reaction wave by 180°, or of the reaction wave as lagging (or leading) 180° behind the impressed-voltage wave.

The same ideas that the curves of Fig. 18*b* are intended to convey may be indicated as a vector diagram, as in Fig. 18*c*. The rotating vector E_m represents the maximum value of the impressed voltage; the rotating vector I_m, drawn in phase with E_m, represents the maximum value of the current, and the rotating vector $-I_mR$ represents the maximum value of the resistance reaction. The last-named vector is, of course, drawn equal and opposite to the impressed voltage.

The instant indicated in Fig. 18*c* is that at which the angle $2\pi ft$ is equal to θ degrees, after the vector E_m passes through zero with increasing values. Instantaneous values of all three vector quantities are indicated by the vertical projections of the vectors.

When current and voltage reach their maximum values, we have

$$E_m = I_mR, \text{ or that } R = E_m/I_m. \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The relation between the effective value and the maximum value of a sine wave of current or voltage has been shown to be

$$E = 0.707E_m \text{ and } I = 0.707I_m.$$

If, therefore, we divide the voltage and current of Eq. (19) by 0.707, we have

$$E = IR \text{ and } R = E/I, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where E and I are the effective values of the voltage and current, respectively.

The last equations, which, however, apply to circuits containing only resistance, are in agreement with the ideas involved in the definition of effective values of current and voltage.

21. Power in a Circuit Containing only Resistance.—Under steady conditions, the power in a continuous-current circuit is always given by the product of amperes flowing and the voltage impressed. *At any one instant*, the alternating-current circuit is the same as a continuous-current circuit; therefore, the instantaneous power in an a.c. circuit may be obtained by multiplying the instantaneous value of the voltage by the instantaneous value of the current.

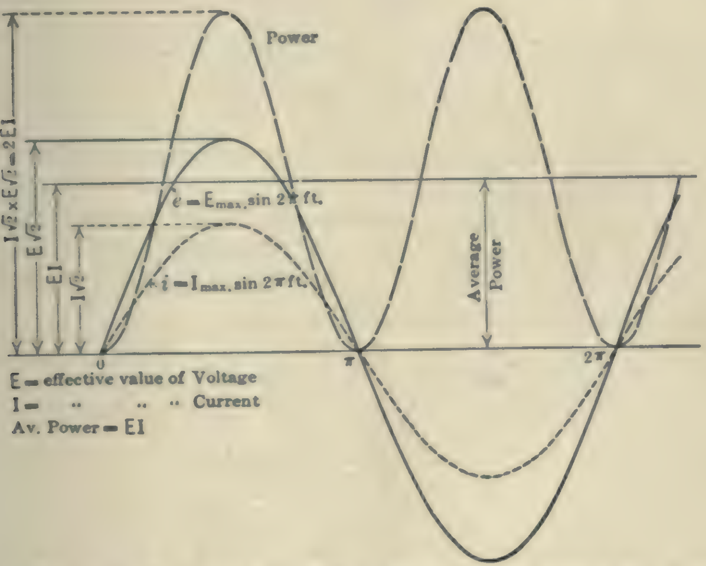


FIG. 19.—The curve of power in a resistive circuit is of sine wave form, everywhere above the axis.

We have seen that, in a circuit containing only resistance, the current is in phase with the voltage. If the instantaneous values of two such waves are multiplied and plotted, as in Fig. 19, a wave of instantaneous power results; this is seen to be a sine wave of double frequency. Since the current and voltage are in phase, both have either positive or negative values at any instant, and their product, the power, is always positive. The instantaneous power for the circuit assumed is given off as heat, and this is given off independent of the direction of current flow.

The maximum values of the current and voltage waves being I_m and E_m, respectively, it follows that the maximum value of the power wave is E_mI_m;

and since the latter is also a sine wave similar to the power wave of Fig. 14, we may write

$$\text{Average power}=\frac{E_mI_m}{2}=\frac{E_m}{\sqrt{2}}\,\frac{I_m}{\sqrt{2}}. \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Now $\frac{I_m}{\sqrt{2}}$ and $\frac{E_m}{\sqrt{2}}$ are, by definition, the effective values of the current and voltage, respectively; therefore, the *average power in a circuit containing only resistance* is equal to the product of the effective value of the current and the effective value of the voltage, i.e.,

$$\text{Power}=EI. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

If the value of E from Eq. (20) be substituted, we have

$$\text{Power}=EI=IR\cdot I=I^2R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

which is in agreement with our assumptions in determining the effective value of a current of sine-wave form.

To determine by the calculus an expression for the average power when a voltage of the form $E_m \sin \omega t$ is impressed on a circuit of resistance only, and a current of the order $I_m \sin \omega t$ results, the current being in phase with the voltage, we need only evaluate the expression

$$\begin{aligned}\text{Average power}&=\frac{1}{\pi}\int_0^{\pi} E_m \sin \omega t\cdot I_m \sin \omega t\, d(\omega t) \quad . \quad . \quad (24)\\ &=\frac{E_mI_m}{\pi}\int_0^{\pi}\sin^2 \omega t\, d(\omega t)\\ &=\frac{E_mI_m}{\pi}\left[\frac{\omega t}{2}-\frac{\sin 2\omega t}{4}\right]_0^{\pi}\\ &=\frac{E_mI_m}{\pi}\left[\frac{\pi}{2}\right]\\ &=\frac{E_mI_m}{2}=\frac{E_m}{\sqrt{2}}\cdot\frac{I_m}{\sqrt{2}}=EI, \quad . \quad . \quad . \quad . \quad . \quad (25)\end{aligned}$$

where E and I are the effective values of the voltage and current, respectively.

22. Continuous-current Crcuit Containing Inductance.—From Volume I we have seen that when the current flowing in a continuous-current circuit having inductance, varied with time, the magnetic field set up by the current varied accordingly with the time. As the current increased, the field, and therefore the flux interlinkages, increased; and when the current decreased, the field and the number of flux interlinkages decreased. Further, we saw that as the flux interlinkages of a system were changed, a counter voltage of self-induction was generated, the direction of which was always such as to oppose the change in the interlinkages taking place; the counter e.m.f. of self-induction always acted to oppose the change taking place in the current.

The action is the same as the force of inertia developed by an accelerating body; this force always acts in such a way as to oppose the acceleration, or rate of change of the velocity, of the body. The force of inertia comes into play only when the body starts to accelerate, and disappears as the velocity becomes uniform.

In the electrical circuit, the effect produced by a changing current is in accord with Lenz' law. With an increasing current, the increase in flux interlinkages immediately brings into existence an opposing, or counter, e.m.f., which tends to retard the increase in current, thereby opposing the increase in flux interlinkages. Conversely, when the current in a circuit dies down, the number of interlinkages decreases, and a voltage of self-induction is at once induced, which, being in the same direction as the impressed force, tends to retard the decrease in current, thereby again opposing the change taking place in the flux interlinkages.

It was further shown in Volume I that when the current in a circuit changed at a uniform rate of I/t amperes per second, and the permeability of the magnetic circuit was considered constant, the flux interlinkages would change at a uniform rate of $\frac{N\phi}{t}$ interlinkages per second, and the average value of the c.e.m.f. of self-induction would be

$$\text{Average } e = \frac{N\phi}{t10^8} \text{ volts. } \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The expression for the value of flux set up in a magnetic circuit is

$$\phi = \frac{0.4\pi N I \mu A}{l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

for a magnetic channel of definite dimensions only

where N is the number of turns of wire,
I is the current in amperes,
μ is the permeability of the magnetic circuit,
A is the area of the magnetic circuit,
l is the length of the magnetic circuit.

If the value of ϕ in the last expression is substituted in Eq. (26), we have

$$\text{Average } e = \frac{0.4\pi N^2 \mu A}{l} \cdot \frac{I}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The part $\frac{0.4\pi N^2 \mu A}{l}$ is a constant and is called the coefficient of self-induction, or merely the self-inductance of the circuit, and is generally represented by the symbol L. Thus

$$L=\frac{0.4\pi N^2\mu A}{l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and we have

$$\text{Average } e=L\frac{I}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

23. Alternating-current Circuit Containing only Inductance.—A circuit containing only inductance is, of course, a hypothetical one; the simplest inductive circuit consists of a coil of wire, and must therefore contain some resistance. It is, however, possible to neglect the resistance in a theoretical discussion, and thus the effect of inductance alone may be considered.

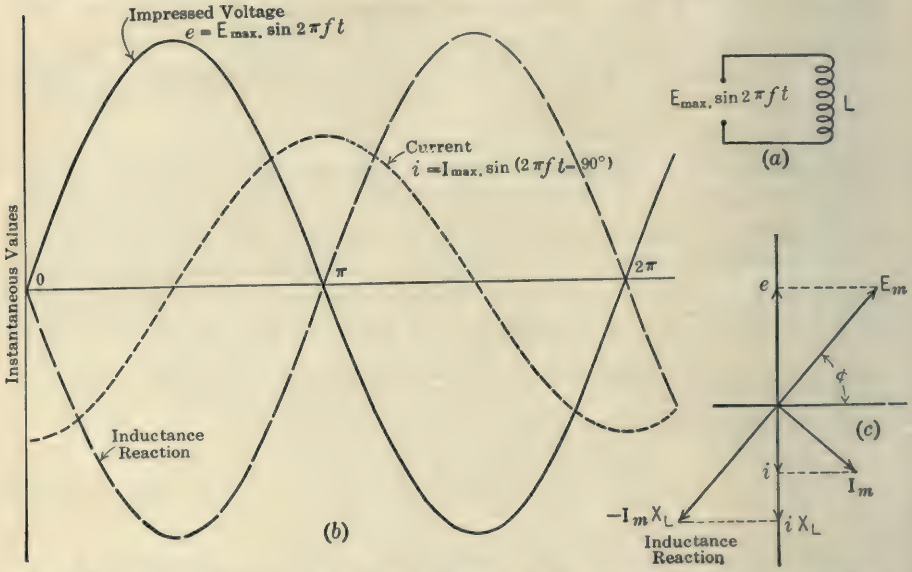


FIG. 20.—Curve diagram and vector diagram of a theoretical circuit containing inductance only; the current lags 90° behind the impressed force.

We have said that whenever a voltage is impressed on a circuit, the circuit must develop reactions, the algebraic sum of which must at any instant be equal and opposite to the impressed force. In Fig. 20*a* is indicated a circuit containing only inductance, across which a sine voltage, $e=E_m \sin 2\pi ft$, is impressed. If the full-line curve of Fig. 20*b* represents the impressed voltage, then the broken-line curve represents the total reaction developed by the circuit. In other words, as soon as the form and magnitude of the impressed voltage are fixed, the form, magnitude *and position, or time phase*, of the total reaction are also fixed; the total reaction must always be 180° from the impressed voltage.

It is evident that, in a circuit containing only inductance, there can be no other reaction than an inductance reaction; and since the impressed voltage has sine form, it follows that the inductance reaction must, in order to be equal and opposite to the impressed voltage at every instant, be also of sine form. *It is therefore necessary that the current take such position and assume such form and magnitude as to develop a reaction of sine form and of definite magnitude. In other words, the current must adapt itself to suit the condition that the reaction must balance the impressed force.*

Now, from the definition of the coefficient of self-induction we know that, if a small change in current, Δi, takes place in a small time, Δt, the induced voltage is given by the expression

$$e = -L\frac{\Delta i}{\Delta t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where e is the counter e.m.f. of self-induction, which, in the alternating-current circuit, is called the inductance reaction. From this definition of L it follows that when $\frac{\Delta i}{\Delta t}$ is positive, the inductance reaction is negative, and when $\frac{\Delta i}{\Delta t}$ is negative, the inductance reaction is positive.

Further, as we have already shown, if the impressed e.m.f. of the circuit of Fig. 20*a* is a sine wave, the form of the inductance reaction must be a sine wave, and, as the form of the inductance reaction is determined entirely by the form of $\frac{\Delta i}{\Delta t}$, it follows that $\frac{\Delta i}{\Delta t}$ must be a sine wave with respect to time. But if the rate of change of a current is to be a sine wave, it follows that the current itself must be a cosine wave.

We arrive, then, at the conclusion that, if a sine wave of voltage is impressed on a circuit containing inductance only, the current which flows must be a cosine wave, that is, another sine wave displaced 90° with respect to the voltage wave. Now the question to be determined is whether the wave of current is displaced 90° ahead or 90° behind the impressed voltage.

From consideration of Eq. (31), we see that when the inductance reaction is positive, $\frac{\Delta i}{\Delta t}$, the rate of change of current must be negative, and when the inductance reaction is negative, the rate of current change must be positive.

In Fig. 20*b*, the inductance reaction is shown in its proper phase with respect to the impressed voltage. As the reaction is negative from 0 to π, it follows that the rate of current change must be positive during this interval. This condition can only be satisfied by having the current at the negative maximum value at time, $t=0$, as shown by the dotted curve of Fig. 20*b*. It is to be noted particularly that, *although the current itself is shown negative from* 0 *to* $\pi/2$, *the rate of change of current during this same interval is positive.*

It will be seen that, when the current is plotted as determined in the above paragraph, and as shown in Fig. 20*b*, the condition that the rate of change of current must be negative when the inductance reaction is positive, is also satisfied; this condition holds from π to 2π. It is further seen that plotting the current in this phase also satisfies the condition that the maximum value of the inductance reaction occurs when the current is changing at its maximum rate, which occurs when the current is going through its zero values.

To write the equation of inductance reaction, we remember that it is equal to the coefficient of self-induction, taken with a negative sign, multiplied by the rate of change of current. But the current shown in Fig. 20*b* is a negative cosine wave, negative because it has the value (-1) at time, $t=0$. Now, as the rate of change of a cosine wave is a negative sine wave, the rate of change of the current shown in Fig. 20*b* is a positive sine wave, represented by the equation $\frac{\Delta i}{\Delta t}=2\pi f I_m \sin 2\pi f t$, if the current has been represented by the equation $i=-I_m \cos 2\pi f t$.

The expression for the inductance reaction is, therefore, for the case shown in Fig. 20,

$$\text{Inductance reaction} = -2\pi f L I_m \sin 2\pi f t. \quad . \quad . \quad . \quad (32)$$

If the current had been taken as a sine wave (as might have been done by taking the impressed e.m.f. as a cosine wave), the equation for current would be $i=I_m \sin 2\pi f t$, and we should then have

$$\text{Inductance reaction} = -2\pi f L I_m \cos 2\pi f t. \quad . \quad . \quad . \quad (33)$$

It will be noticed, for either case, that *the inductance reaction lags behind the current by 90°;* that is, the current leads the inductance reaction by 90°. Now, as the impressed force is just opposite to the inductance reaction (if, as assumed here, the circuit has no resistance), it follows that in such a circuit *the current lags behind the impressed voltage by 90°.*

Now, as the impressed force and inductance reaction are always equal to each other and opposite in direction, we may write the equality between them for any convenient time in the cycle. Selecting the time when the impressed voltage has its maximum value E_m, at which instant the inductance reaction has its maximum value, $-2\pi fLI_m$, and bearing in mind that the sum of the impressed force and reaction must be zero, we write

$$E_m - 2\pi fLI_m = 0,$$

or

$$E_m = 2\pi fLI_m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

The quantity $-2\pi fLI_m$ is the reaction itself; the quantity $2\pi fLI_m$ is the component of the impressed force that overcomes the reaction.

Multiplying both sides of the last equation by 0.707, we have

$$E = 2\pi fLI, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

which gives us the relation in effective values of both voltage and current.

Equation (34) may be derived in another manner. Consider that the current, taken by a circuit possessing a self-inductance L, is a sine wave of frequency f, and of maximum value I_m. During one cycle the current changes four times from zero to maximum value, or *vice versa*, or once each quarter cycle. The time of one quarter cycle is $1/4f$ second; therefore, if the current changes from zero to I_m amperes, or from I_m to zero amperes in one-quarter cycle, its average rate of change is $\frac{I_m}{1/4f} = 4fI_m$ amperes per second. The average voltage of self-induction will then be

$$\text{Average}\left(L\frac{\Delta i}{\Delta t}\right) = 4fLI_m,$$

which in turn must equal the average value of the impressed voltage.

Thus

$$\text{Average } E = 4fLI_m.$$

Since

$$E_m = \frac{\text{average } E}{2/\pi},$$

we have

$$E_m = \frac{4fLI_m}{2/\pi} = 2\pi fLI_m.$$

By use of calculus, this expression for the reaction due to self-induction is easily obtained. By the definition of the coefficient of self-induction, the back e.m.f. of self-induction, i.e., the inductance reaction, is given by

$$\text{Inductance reaction} = -L\frac{\Delta i}{\Delta t},$$

which in terms of differentials is

$$\text{Inductance reaction} = -L\frac{di}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

If we now assume the current a sine wave, $i = I_m \sin 2\pi ft$, we have

$$\text{Inductance reaction} = -L\frac{di}{dt} = -L(2\pi fI_m \cos 2\pi ft)$$

$$= -2\pi fLI_m \cos 2\pi ft.$$

This relation is shown in Fig. 21; and as the impressed force must be just equal and opposite to the inductance reaction, this derivation also shows that the current in a circuit, containing inductance only, lags 90° behind the impressed force.

24. Inductive Reactance.—The quantity $2\pi fL$ in Eq. (32) is a constant for any given frequency, provided the permeability of the flux path is constant; it is called the *inductive reactance* of the alternating-current circuit, and is designated by the symbol X_L. We have, then,

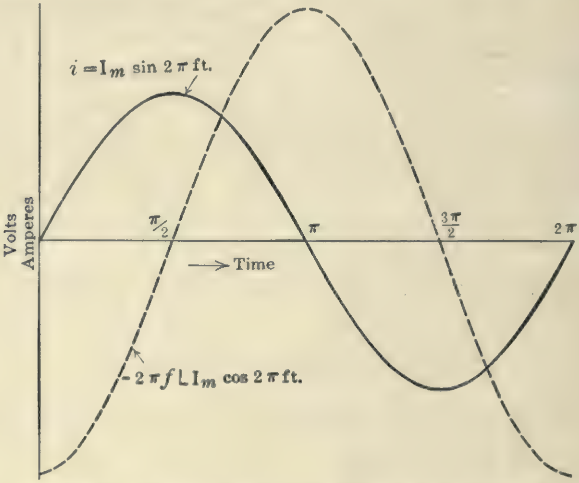


FIG. 21.—Showing how the inductance reaction and current are related.

$$\text{Maximum inductance reaction} = -I_mX_L,$$

and

$$E_m = I_mX_L \quad \text{and} \quad E = IX_L,$$

from which

$$X_L = \frac{E_m}{I_m} \quad \text{and} \quad X_L = \frac{E}{I}. \quad \ldots\ldots\ldots \quad (37)$$

This last equation is similar to the expression $R = \frac{E}{I}$, and since R is expressed in ohms, it is to be expected that X_L is also expressed in ohms.

In Fig. 20c, the vector diagram for an inductive circuit is shown; the inductance reaction is drawn 180° from the impressed voltage and the current 90° behind the impressed voltage.

The inductance reaction of any circuit varies directly with the frequency. A circuit possessing a self-inductance of 0.5 henry, has, when connected to a 25-cycle voltage, a reactance of $2\pi \cdot 25 \cdot 0.5 = 78.5$ ohms. When connected to a 60-cycle voltage the reactance of the same circuit is $2\pi \cdot 60 \cdot 0.5 = 188$ ohms.

25. Power in a Circuit Containing only Inductance.—The curve of instantaneous power, for a circuit containing only inductance, is constructed by plotting the product of the instantaneous values of current and voltage. In Fig. 22, the full-line curve represents the impressed voltage, the dotted curve the current (lagging 90° behind the voltage), and the broken-line curve the instantaneous power; the latter is seen to be a curve of double frequency with equal positive and negative loops. Consideration of the values of voltage and current will show that the power must have an instantaneous value of zero four times per cycle, or whenever either voltage or current is zero. Between 90° and 180° and between 270° and 360°, the current and voltage act in the same direction; during these intervals the power is positive. Between 0° and 90° and between 180° and 270°, current and voltage have opposite directions and the power is therefore negative.

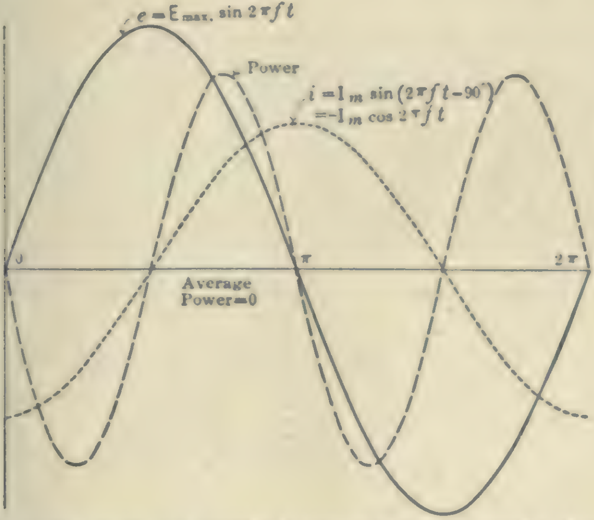


FIG. 22.—The power curve in a purely inductive circuit is equally above and below the axis, indicating that the average value of power in such a circuit is zero.

Whenever current is increasing in a circuit containing inductance, a magnetic field is being established. A magnetic field represents the storage of an amount of energy; therefore, as the current increases, the circuit is storing energy and taking it from the source. Power taken by a circuit from its source of energy is always considered as positive power. During the period that the current is decreasing, the magnetic field is collapsing, and the magnetic energy stored in the magnetic field is being given up or restored to the source from which it originally came. Whenever a circuit restores energy to its source, the power is considered negative.

The circuit having been considered as having no resistance, no power is being lost as heat, and we see that the circuit must restore as much energy during a quarter cycle as it absorbed during the previous quarter cycle. This is indicated in our power curve by the equal positive and negative loops.

We may write, then, for a circuit containing *only* inductance,

$$EI = \text{zero power.}$$

To determine by the calculus the average power in a circuit containing only inductance, it is necessary to integrate the product $E_m \sin \omega t(-I_m \cos \omega t)$ over a complete cycle. As a complete cycle of power occurs during the time of one-half cycle for either voltage or current, we need to integrate over only the time from zero to π. Thus,

$$\begin{aligned}\text{Average power} &= \frac{1}{\pi}\int_0^\pi \{E_m \sin \omega t(-I_m \cos \omega t)\}\,d(\omega t)\\ &= \frac{-E_mI_m}{\pi}\int_0^\pi \sin \omega t \cos \omega t\; d(\omega t)\\ &= \frac{-E_mI_m}{\pi}\int_0^\pi \left(\frac{\sin 2\omega t}{2}\right) d(\omega t)\\ &= \frac{-E_mI_m}{\pi}\left[-\frac{\cos 2\omega t}{4}\right]_0^\pi = 0.\end{aligned}$$

26. Volt-amperes. Power Factor.—In any alternating-current circuit the product of volts and amperes is stated in terms of a unit called the *volt-ampere.* We have seen that, in the case of a circuit containing only resistance, the volt-amperes are equal to the watts, whereas, in a circuit containing only inductance, the power was zero.

It will be shown later that, in any circuit,

$$\text{Power} = EI \cos \phi,$$

where ϕ is the angle between voltage and current; $\cos \phi$ is called the *power factor.* We have then

$$\cos \phi = \frac{\text{Power in watts}}{\text{Volt-amperes}} = \frac{P}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

In a circuit containing only resistance, the watts are equal to the volt-amperes: the power factor has its greatest possible value, or one. The current and voltage being in phase, the angle between them is zero and the cosine for a zero angle is one. In a circuit containing only inductance, the angle between current and voltage is 90° and the cosine of 90° is zero. Therefore the power is zero.

27. Condensers.—As shown in elementary physics, an electrical condenser consists of two conducting plates, separated by some insulating dielectric, such as air, oil, or paraffined paper. In a perfect condenser there is no leakage of current from one plate to the other through this dielectric; the insulation resistance is supposed to be infinity.

As it was known, in the early days of electrical experiment, that electricity could run into and out of such a combination of conductors and dielectric, and yet that electric current could not flow through the dielectric, it was supposed that the electricity condensed in some way in the dielectric; hence the name, "condenser."

Condensers are used to a large extent in alternating-current circuits; the telephone companies have millions of them in their networks of wires, and in recent years power companies have begun to use large condensers in their power houses to correct certain troubles arising from inductive loads. Of course, in radio communication, condensers are used a great deal, their capacitance (or capacity, as it was formerly called) being low and variable in amount.

The unit of capacitance is the *farad*; a condenser has a capacitance of one farad when one coulomb of electricity will charge it to a potential difference of one volt. The charging of a condenser takes place somewhat in this fashion: Consider a condenser, made of the two conducting plates *A* and *B*, insulated from one another, and connected in a circuit as shown in Fig. 23, the switch, *S*, being open. The two plates *A* and *B* are uncharged; that is, each has the proper number of free electrons to exactly neutralize the charge in the positive nuclei. When the switch is closed, the two plates must assume the potential of the respective poles of the battery, *E*, to which they are connected; that is, *A* must become positively charged with respect to *B*, by an amount equal to the voltage of the battery. From the electron viewpoint, this can only mean that some electrons leave *A* and a corresponding number collect on *B*. When the switch, *S*, is closed, therefore, there is a slight shift in all the electrons in the circuit, in a counter-clockwise direction. They do not move very far; the

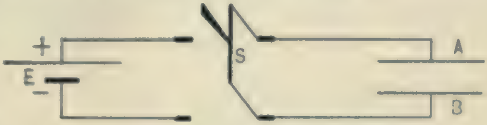


FIG. 23.—A condenser, to be charged by battery *E* when the switch is closed.

electrons that leave plate *A* do not flow around the circuit to plate *B*, as is frequently stated in text-books.

Let us suppose that the wire used in making the connections has a cross-section of 1 sq. mm.; there will be, per cm. length of this wire, about 10^{22} free electrons, the coordinated motion of which constitutes current in the wire. As one coulomb is equal to 6.28×10^{18} electrons, it is seen that to put one coulomb of electricity in the condenser, the electrons in the wire constituting the circuit would have to move less than 0.001 cm. What happens, then, if such a condenser as that shown in Fig. 23 is charged with one coulomb of electricity (an amount much greater than is ever stored in an actual condenser)? The charging process consists of a slight shift of all the electrons in the circuit, in counter-clockwise direction for charging the condenser as it would be in Fig. 23, and clockwise if the battery should be reversed.

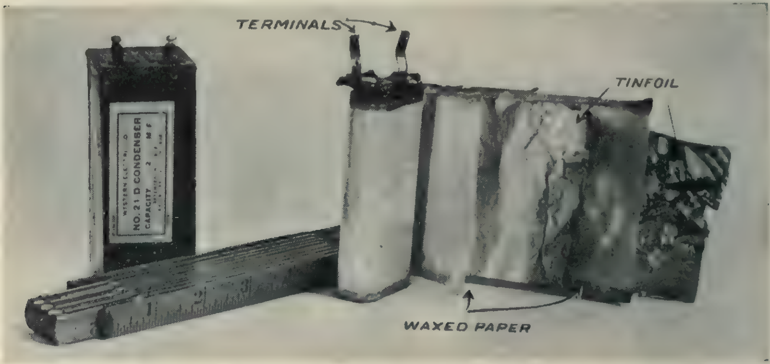


Fig. 24.—The ordinary condenser used in telephone circuits is made up of very thin metallic sheets (aluminum foil or tin foil) separated by thin wax-impregnated paper.

The unit of capacitance is the farad; a condenser having this amount of capacitance would require one coulomb of electricity to charge it to a potential difference of one volt. We might also say that such a condenser shows a difference of potential of one volt, if 6.28×10^{18} electrons are taken from one plate and this same number added to the other. This unit of capacitance is much too large for practical purposes; the ordinary unit is the *microfarad*, or one-millionth of a farad; condensers of one microfarad, or a fraction thereof, are used extensively in telephone wiring schemes. In radio, the variable condensers used for tuning in different signals are generally measured in micro-microfarads, this unit being one millionth of one millionth of a farad. The ordinary tuning condenser has a maximum capacitance of 1000 micro-microfarads, or less.

The ordinary telephone condenser, used in telephone circuits, is made of two long, narrow sheets of tinfoil or aluminum foil, separated by thin paraffined paper, all wrapped tightly together and squeezed into a small metal container, the two tinfoil (aluminum foil) sheets being connected to thin copper strips which project through two holes in the case. The roll of tinfoil and paper is immersed in hot wax to drive out all the air, and is subjected to high pressure to squeeze out most of the wax before being fitted into a thin metal box. Such a condenser, as well as one which has been taken out of its containing box and partly unrolled to show the tinfoil sheets, is shown in Fig. 24. The paraffined paper in such a condenser is about one-thousandth of an inch in thickness; yet so well is it impregnated with paraffin that the insulation resistance is generally a good many million ohms.

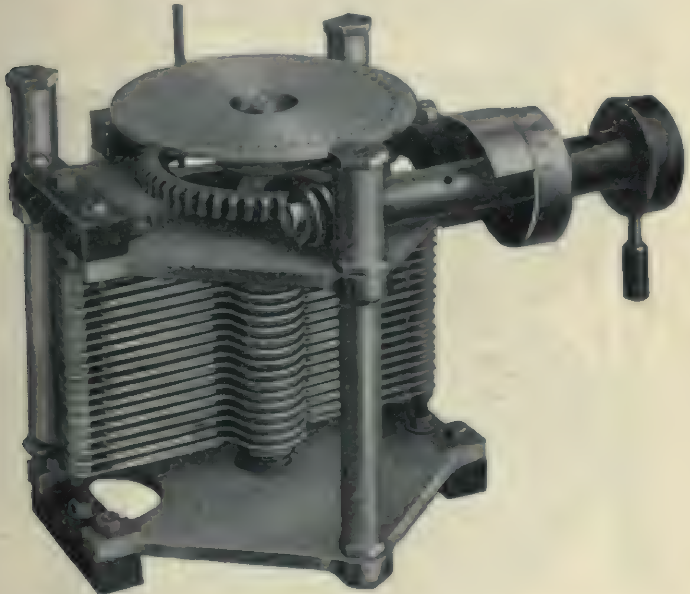


FIG. 25.—Variable condensers, using air for dielectric, are extensively used in radio circuits. Shown here is a high-grade condenser, having a slow-motion on the moving plates; its maximum capacity is 0.0015 microfarad.

In Fig. 25, is shown a variable condenser for a radio circuit, each of the plates of the condenser in this case being a group of semicircular plates which fit into one another more or less as the axis is rotated. The dielectric in this case is air; the condenser shown in the photograph has a maximum capacitance of 0.0015 microfarad.

In Fig. 26 is shown a group of condensers intended for use in an alter-

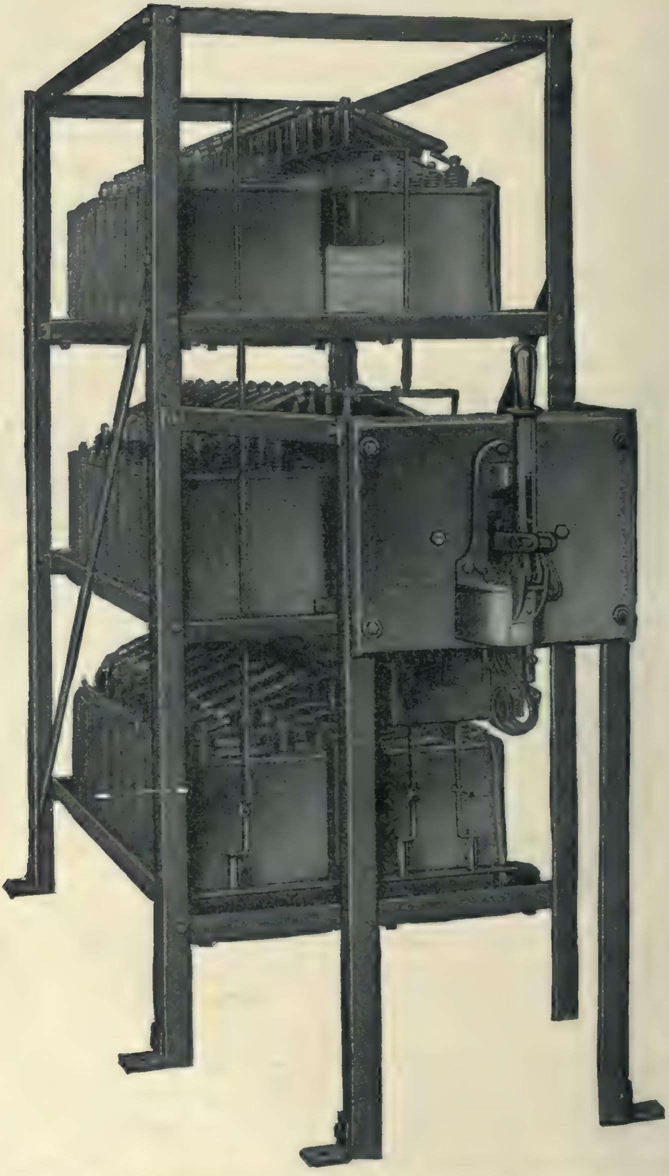


FIG. 26.—Condensers of high voltage and current capacity are being used more and more extensively in power plants to compensate for the inductive loads usually met in practice. This bank of condensers is for a three-phase line; each is in series with a fuse which will blow if the condenser breaks down while in service.

nating-current power plant; they consist of thin metal plates immersed in tanks filled with good insulating oil. Such condensers are generally designed for operation on a 2200-volt circuit, being sufficiently well insulated by proper separation of the plates and oil as the dielectric so that it will stand safely about 10,000 volts.

28. Alternating-current Circuit Containing only Capacitance.—We have seen that when a continuous voltage, E, is impressed on a condenser, the latter must have a quantity of electricity, Q, in coulombs, on its plates, such that $E=Q/C$, where C is the capacitance of the condenser in farads.

If the voltage impressed on the condenser is increased by a small amount ΔE, the capacitance of the condenser remaining constant, the quantity of electricity stored in the condenser must increase by an amount ΔQ, such that,

$$E+\Delta E=\frac{Q+\Delta Q}{C},$$

or,

$$Q+\Delta Q=C(E+\Delta E).$$

Since $Q=E/C$, it follows that

$$\Delta Q=C(\Delta E). \qquad (39)$$

The passage of a quantity of electricity, over the wires leading to the condenser, is a current; and since the passage of one coulomb of electricity in one second constitutes one ampere, we may say that the current flowing, when the voltage is increased by an amount ΔE in a short time interval Δt second, is

$$\frac{\Delta Q}{\Delta t}=i=C\frac{\Delta E}{\Delta t}. \qquad (40)$$

Thus, with an increasing impressed voltage, the greater the rate of change of voltage, the more quickly the condenser must charge, or absorb coulombs of electricity, and therefore the greater the current flowing into it. With a decreasing voltage, the condenser decreases its charge, and the greater the rate of change in the voltage, the more quickly the coulombs of electricity will leave the condenser, and the greater the current.

We deduce, therefore, from Eq. (40), that the current flowing in a condenser is equal to the capacitance of the condenser multiplied by the rate of change of the voltage. This equation also indicates that, when the voltage is increasing numerically and its rate of change is positive, voltage and current have the same direction. When the voltage is positive, and decreasing numerically, its rate of change is negative and the current and voltage have opposite signs.

When a sine wave of voltage is impressed on a condenser, as in Fig. 27*a*, the condenser must develop a reaction voltage, which is equal and opposite to the impressed force at any instant; this is indicated by the full-line and broken-line curves of Fig. 27*b*. As the impressed voltage begins increasing positively from zero, it does so at its greatest rate; this rate of increase gradually decreases, reaching zero when the voltage reaches its maximum or 90° value. The current, being proportional to the rate of change of the voltage, must therefore also be positive, and have its maximum value when the voltage is passing through its zero value; when the voltage is at its 90° value and not changing, the current will have fallen to zero.

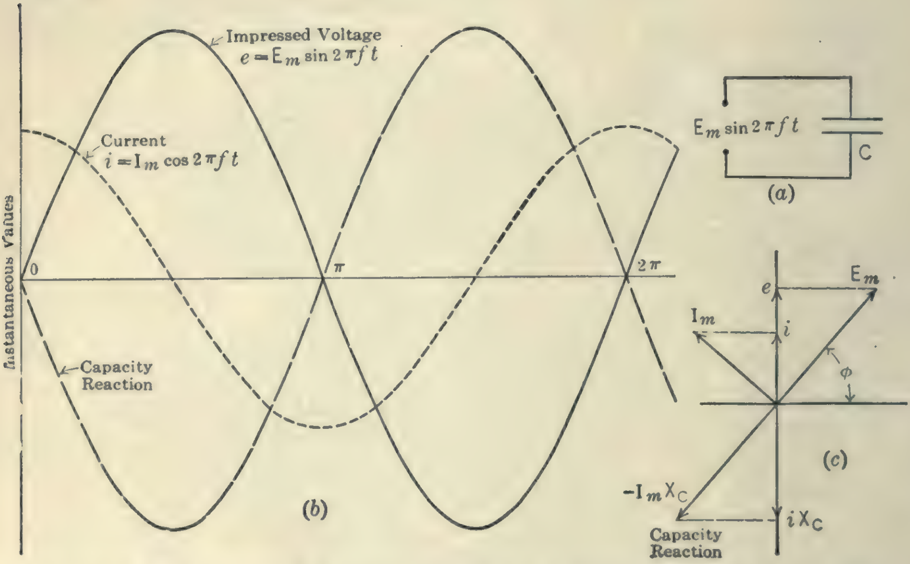


FIG. 27.—Curve diagram and vector diagram for a condensive circuit of no losses; the current leads the impressed voltage by 90°.

When the voltage starts to decrease, the reaction voltage must also decrease, and in order that the reaction voltage may decrease, the condenser must give up some of its charge of electricity. Hence a current will flow in the direction opposite to that of the impressed voltage. Between 90° and 180°, while the voltage decreases from its maximum positive value to zero, the current increases, with negative value, from zero to its maximum negative value. We see then that the current must lead the impressed voltage by 90°.

This deduction would have followed directly from Eq. (40), in which the quantity $\frac{\Delta E}{\Delta t}$ represents the rate of change of the voltage. From the proof given in section 18, that the rate of change of a current, $i = I_m \sin 2\pi f t$, is equal to $2\pi f I_m \cos 2\pi ft$, we may say directly that the rate of change of a voltage, $e = E_m \sin 2\pi ft$, is equal to $2\pi f E_m \cos 2\pi ft$.

Therefore, as the current into a condenser is equal to its capacitance multiplied by the rate of change of the impressed voltage, the current flowing into a condenser of capacitance C, may be written

$$i = 2\pi f C E_m \cos 2\pi ft, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

which evidently leads the impressed voltage, $E_m \sin 2\pi ft$, by 90°.

To derive the above expression by the calculus, we have

$$i = C\frac{dE}{dt} = C\frac{d}{dt}(E_m \sin 2\pi ft) = 2\pi f C E_m \cos 2\pi ft.$$

From Eq. (41), it follows that the maximum value of the current is

$$I_m = 2\pi f C E_m, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

or that

$$E_m = \frac{I_m}{2\pi f C} = I_m \cdot \frac{1}{2\pi f C}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

From our original assumption, the reaction voltage developed by the circuit is

$$-E_m \sin 2\pi ft,$$

so that the maximum value of the reaction voltage is equal to E_m, and from Eq. (43) this is equal to

$$-I_m \cdot \frac{1}{2\pi f C},$$

so that the full equation for the capacitance reaction is $-\frac{I_m}{2\pi f C} \sin 2\pi ft$.

Since the algebraic sum of the impressed force and the reaction must be zero, we have

$$E_m + \left(-I_m \cdot \frac{1}{2\pi f C}\right) = 0,$$

or,

$$E_m = I_m \cdot \frac{1}{2\pi f C} = \frac{I_m}{2\pi f C}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

which checks Eq. (43).

The quantity $I_m \cdot \frac{1}{2\pi f C}$, is therefore the component of the impressed force that overcomes the reaction of the condenser.

Equation (44) may also be derived in a different manner. Consider the sine curves of current and voltage in Fig. 27*b*. When the voltage is at its maximum value, E_m, at the end of the first quarter cycle, the charge on the plates of the condenser is also of maximum value, Q_m, such that

$Q_m = E_mC$. This charge was put on the plates during the same quarter cycle by the current, which decreased from its maximum value to zero. Since current is, by definition, the quantity of electricity conveyed per second, quantity of electricity is also the product of the average current and time. The quantity of electricity, therefore, conveyed during a quarter cycle, is

$$Q_m = \text{Average } I \cdot \frac{1}{4f},$$

since the time of one quarter cycle is $1/4f$ second.

We have seen that

$$E_mC = Q_m = \frac{\text{Av. I}}{4f},$$

$$E_m = \frac{\text{Av. } I}{4fC}.$$

Since, for a sine wave

$$I_m = \frac{\text{Av. } I}{2/\pi} \quad \text{or,} \quad \text{Av. } I = I_m \cdot \frac{2}{\pi},$$

we have

$$E_m = \frac{I_m}{2\pi fC}.$$

29. Capacitive Reactance.—The quantity $\frac{1}{2\pi fC}$ is a constant for any given frequency, provided the capacitance in the circuit is constant; it is called the *capacitive reactance*, or *anti-inductive reactance*, and is designated by the symbol X_c. We have seen that

$$\text{Maximum capacitive reaction} = I_mX_c,$$

and that

$$E_m = I_mX_c \quad \text{and} \quad E = IX_c,$$

from which

$$X_c = \frac{E_m}{I_m} \quad \text{and} \quad X_c = \frac{E}{I}. \quad \quad (45)$$

The last expression is similar to the equations $X_L = \frac{E}{I}$ and $R = \frac{E}{I}$; it is therefore to be expected that X_c will also be expressed in ohms.

In Fig. 27*c*, the vector diagram for a capacitive, or anti-inductive, circuit is shown; the capacitive reaction is drawn 180° from the impressed voltage, and the current 90° ahead of the impressed voltage.

The capacitive reactance of any circuit varies inversely as the frequency. A circuit possessing a capacitance of 100 microfarads, or 100×10^{-6} farads, when connected to a 25-cycle voltage, has a reactance of $\frac{1}{2\pi \cdot 25 \cdot 100 \cdot 10^{-6}} =$

63.66 ohms. When connected to a 60-cycle voltage, the reactance of the same circuit is $\frac{10^6}{2\pi \cdot 60 \cdot 100} = 26.53$ ohms.

30. Power in a Circuit Containing only Capacitance.—The curve of instantaneous power, for a circuit containing only capacitance, is constructed by plotting the product of instantaneous values of current and voltage. In Fig. 28 the full-line curve represents the impressed voltage, the dotted curve the current leading the impressed voltage by 90°, and the broken-line curve the instantaneous power. The latter is seen to be a curve of double frequency with equal positive and negative loops; therefore the average power is zero. This was to be expected; as the circuit is assumed to have no resistance, no power is lost as heat, and the condenser restores as much energy during one quarter cycle as it absorbed during the previous quarter cycle.

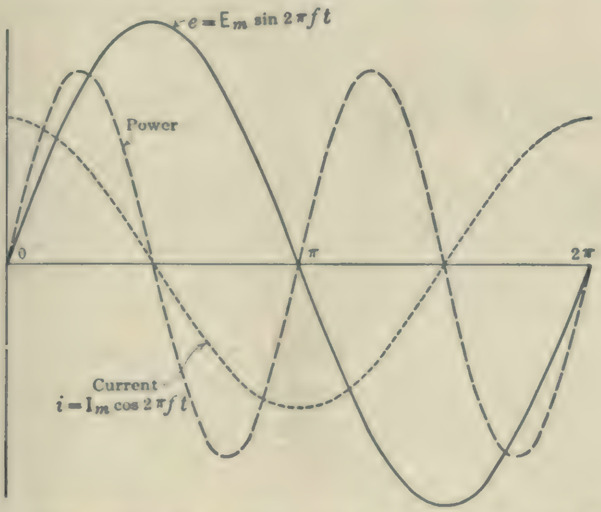


FIG. 28.—The power curve for a purely condensive circuit is equally above and below the axis, indicating that the average value of power in such a circuit is zero.

We may write then, for a circuit containing *only* capacitance,

$$EI = \text{zero power.}$$

In the expression

$$\text{Power} = EI \cos \phi,$$

where ϕ is the angle between voltage and current, $\cos \phi$ is equal to zero, when applied to a circuit containing only capacitance, since current and voltage differ in phase by 90°; the power is therefore zero, as stated before.

To determine by the calculus the average power in a circuit containing only capacitance, it is necessary to integrate the product $E_m \sin \omega t \cdot I_m \cos \omega t$, over a complete cycle of the power curve. Thus,

$$\begin{aligned}\text{Average power} &= \frac{1}{\pi}\int_0^\pi (E_m \sin \omega t \cdot I_m \cos \omega t)d(\omega t)\\ &= \frac{E_m I_m}{\pi}\int_0^\pi (\sin \omega t \cdot \cos \omega t)d(\omega t)\\ &= \frac{E_m I_m}{\pi}\int_0^\pi \frac{\sin 2\omega t}{2}d(\omega t)\\ &= \frac{E_m I_m}{\pi}\left[-\frac{\cos 2\omega t}{2}\right]_0^\pi = 0.\end{aligned}$$

PROBLEMS

1. What is the instantaneous value of an alternating voltage (sine form), having a maximum value of 150 volts, 35° after passing its zero value, going positive? 150°? 480°?

2. Draw the voltage wave (on cross-section paper) represented by the equation

$$e = 100 \sin 2\pi\, 60t + 40 \sin 2\pi\, 180t + 25 \sin 2\pi\, 300t.$$

Construct the curve carefully, for one alternation of the 60-cycle value. In a similar manner construct the curve

$$e = 100 \sin 2\pi\, 60t + 11 \sin 2\pi\, 300t + 9 \sin 540t.$$

3. Assuming 10^{24} molecules of copper per cubic centimeter, and one free electron per molecule, what is the average to-and-fro motion of an electron in a No. 10 wire carrying a current of 50 amperes, 60 cycles? Of the electrons in a radio set, made up of No. 24 wire, carrying a current of 50 microamperes at one million cycles per second?

4. What is the maximum voltage of a 60-cycle, 110-volt circuit. A spark gap set to break down at 20,000 volts will require what effective voltage to break down?

5. What is the effective value of the two voltages given in Prob. 2? What is the maximum voltage of each of these waves?

6. What is the form factor of each of the waves of Prob. 2?

7. What is the maximum rate of change of a 100-ampere, 60-cycle current? Of a 25-ampere, 500,000-cycle current?

8. A condenser of 150 microfarads offers how many volts reaction to further charge after 0.03 coulomb of electricity has flowed into it? Answer same question for a radio condenser of 0.005 microfarad after a current of 3 microamperes has flowed into it for 0.0015 second.

9. A resistance of 15.5 ohms is connected to a 110-volt, 60-cycle circuit. How much power is used? What would be the answer if the current were of 25-cycle frequency?

10. In a circuit of negligible resistance and 0.26 henry inductance, how much current will flow if it is connected to a 110-volt, 60-cycle line? A 220-volt, 25-cycle line? A 10-volt circuit of 2000 cycles frequency?

11. What is the reactance of a coil of 0.062 henry connected to a 60-cycle supply? What would be the reactance if that coil were used in a telephone circuit of 800 cycles frequency?

12. What would be the maximum reacting force if 0.25 ampere was flowing in the coil of Prob. 11?

13. A 20-microfarad condenser is connected to a 60-cycle line, the maximum voltage of which is 300 volts. What charging current flows?

14. An antenna of 0.0026 microfarad capacity is connected to a high frequency alternator generating 150 volts at 200,000 cycles. How much charging current flows into the antenna?

15. What is the reactance of the condensers of Probs. 13 and 14?

16. A long transmission line has a capacity of 2.3 microfarads. How much charging current must be supplied by a 150,000-volt, 25-cycle alternator connected to this line?

17. Useful voice frequencies lie between 100 and 2000 cycles per second. What range of inductive reactances do the voice current encounter as they pass through a coil of 65 millihenries inductance?

18. If the voltage wave of the second part of Prob. 2 is impressed on a condenser of 20 microfarads, how much current will each of the component voltages cause to flow? Will the charging current be more or less distorted than the voltage impressed?

19. What will be the maximum value of the current in Prob. 18? What will be its form factor?

20. If the voltage of Prob. 18 is impressed on a coil of negligible resistance and 0.372 henry, how much current will each of the component voltages cause to flow? Will the resulting current be more or less like a sine wave than the impressed voltage?

CHAPTER II

THE USE OF VECTORS IN ALTERNATING-CURRENT CIRCUITS

31. Scalars and Vectors.—Quantities generally may be divided into *scalars* and *vectors*.

A scalar quantity is one completely specified by *magnitude* and "*sense*" (i.e., whether positive or negative). Examples of scalar quantities are bushels, time, work, power, temperature, mass, continuous currents and voltages, etc. Scalar quantities are added algebraically.

A vector quantity is one that requires for its complete specification both *magnitude* (or intensity) and *direction*. A vector quantity may be represented by a line, equal in length to the magnitude of the quantity, and parallel to the direction of the quantity. Such a line is called a vector. Examples of vector quantities are forces, velocities, etc.; furthermore, we continually use rotating vectors to represent alternating currents and voltages, and so must consider vector operation before trying to solve alternating-current problems.

32. Vector Addition and Subtraction.—If two forces, F_1 and F_2, act on a body, their directions are usually defined with respect to two arbitrarily chosen lines or axes of reference which lie in the same plane as the forces. These axes of reference are practically always taken as vertical and horizontal lines; thus the two forces are defined as in Fig. 29, F_1 making an angle α with the horizontal and F_2 making an angle β with the horizontal. If more than two forces, not lying in the same plane, are to be combined, they must be defined with respect to three axes of reference, generally at right angles to each other. As the vectors used in representing alternating-current circuits always lie in the same plane, we shall consider only the addition and subtraction of vectors lying in the same plane.

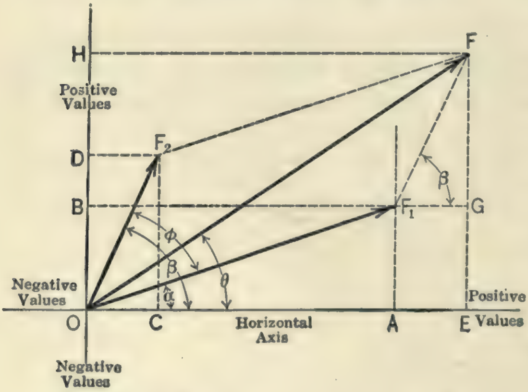


FIG. 29.—Vector addition by polar diagram.

The two force vectors in Fig. 29 may be replaced by a single vector, called the resultant, which will have the same effect on the body as the two component forces acting in conjunction, at the same point of application. This resultant is obtained by completing the parallelogram OF_1FF_2; the diagonal OF is the resultant, as may readily be proved if the component forces F_1 and F_2 are each resolved into their respective horizontal and vertical components.

Horizontal component of $F_1 = OF_1 \cos \alpha = OA$;
Horizontal component of $F_2 = OF_2 \cos \beta = OC = F_1G = AE$;
Horizontal component of $F \ = OF \ \cos \theta = OE = OA + AE$;
Vertical component of $F_1 = OF_1 \sin \alpha = AF_1 = OB$;
Vertical component of $F_2 = OF_2 \sin \beta = CF_2 = GF = BH$;
Vertical component of $F \ = OF \ \sin \theta = OH = OB + BH$;

$$OF = \sqrt{\overline{OE}^2 + \overline{OH}^2} \quad \text{and} \quad \tan \theta = \frac{OH}{OE}.$$

The horizontal component of the resultant is the algebraic sum of the horizontal components of the different vectors, and the vertical component of the resultant is the algebraic sum of the vertical components of the different vectors.

The same graphical operation might have been carried out somewhat differently. If the vector F_1 were defined as making an angle α with the horizontal, and F_2 were defined as leading F_1 by ϕ degrees, the position of the two vectors would be fixed as before in Fig. 29. If F_1 is properly laid off and then F_2 is drawn making an angle ϕ with OF_1 produced, the angle ϕ being laid off in counter-clockwise direction (since F_2 leads F_1), the construction would appear as in Fig. 30. It will be seen that the tail of the second vector, F_2, has been placed at the head of the first vector, F_1. The vector OF is then the resultant, joining the tail of the first vector with the head of the last.

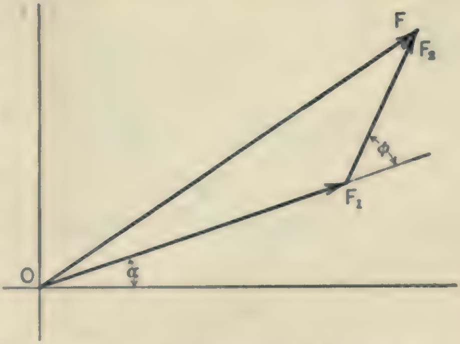


FIG. 30.—Vector addition by topographic diagram.

A diagram like that of Fig. 29, in which all the component vectors are drawn as emanating from a common origin, and the resultant is obtained by the completion of parallelograms, we shall call a polar diagram. It is the one most often used for representing alternating-current circuits. The second method, in which the tail of each successive vector is laid off at the head of the preceding one and the resultant is obtained by completing the polygon, is at times more convenient to use. We shall call it a topographic, or polygon, vector diagram.

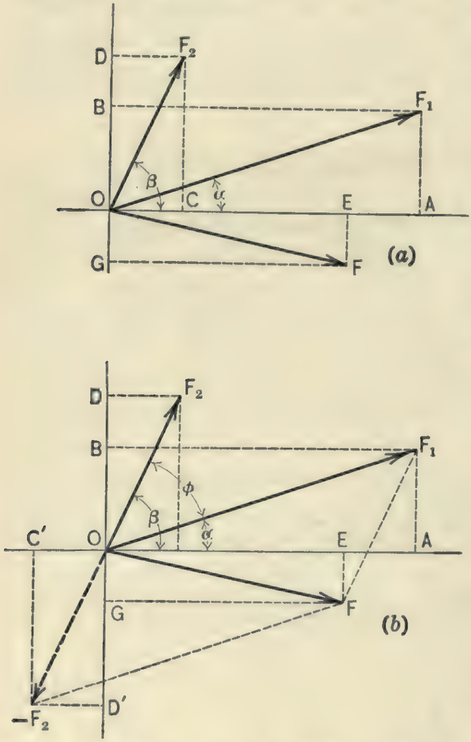


Fig. 31.—Showing how to subtract vector F_2 from vector F_1 by polar diagram.

If the difference between two vectors, as F_1 and F_2 in Fig. 31, is desired the process is the same. Suppose that we wished to subtract vector F_2 from F_1. We might then subtract the horizontal component of F_2 from that of F_1, and the vertical component of F_2 from that of F_1. That is,

Horizontal component of resultant $= OF_1 \cos \alpha - OF_2 \cos \beta$
$= OA - OC = OA - EA = OE;$

Vertical component of resultant
$= OF_1 \sin \alpha - OF_2 \sin \beta$
$= OB - OD = -BD = OG.$

Now, "minus the horizontal component of F_2," is the same as "plus the horizontal component of $-F_2$," where $-F_2$ is simply vector F_2 reversed; and "minus the vertical component of F_2" is the same as "plus the vertical component of $-F_2$," as is easily seen from Fig. 31*b*. Therefore, to subtract F_2 from F_1 we need

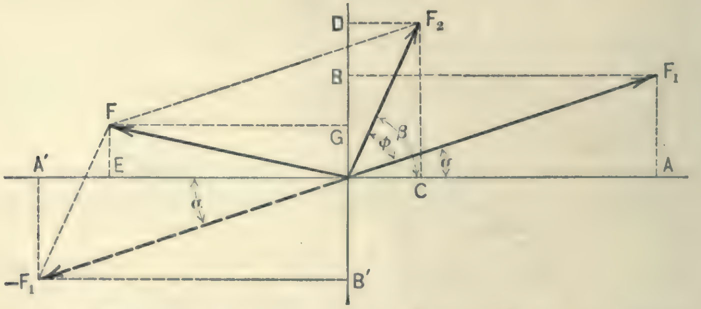


Fig. 32.—Showing how to subtract vector F_1 from F_2 by polar diagram.

only reverse F_2, and add $-F_2$ to F_1. The process is the same as in algebra; to subtract we simply change the sign of the quantity to be subtracted, and add.

In Fig. 32 is shown the result of subtracting F_1 from F_2. The process is the same, but it should be noted that, whereas the resultant is the same in magnitude, it lies 180° from the resultant in Fig. 31 where F_2 was subtracted from F_1.

In the polygon, or topographic, diagram the process is similar. To subtract vector F_2, leading F_1 by ϕ degrees, we lay off F_2 reversed, as in Fig. 33, the direction of $-F_2$, extended backwards, making an angle ϕ with F_1 extended forwards. The resultant is obtained as before, by joining the tail of the first vector with the head of the last. In Fig. 34 vector F_1 is shown subtracted from F_2.

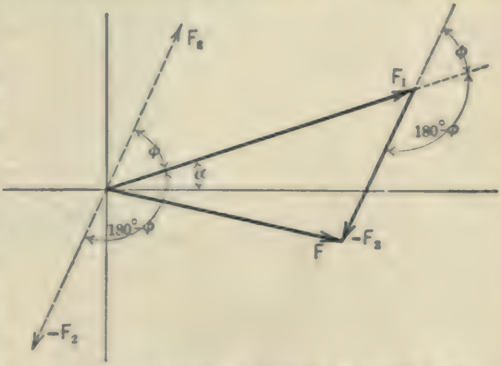


FIG. 33.—Subtracting vector F_2 from F_1 by topographic diagram.

In Fig. 35 is shown a polar diagram for finding the vector sum of three vectors, F_1, F_2, and F_3. The resultant of any two vectors may first be found by completing the parallelogram, and this resultant may be combined in the same way with the remaining vector. Thus, in Fig. 35, F_{1-2} is the resultant of vectors F_1 and F_2. By combining in the same way the resultant F_{1-2} with F_3, the resultant of all three vectors, F, is obtained.

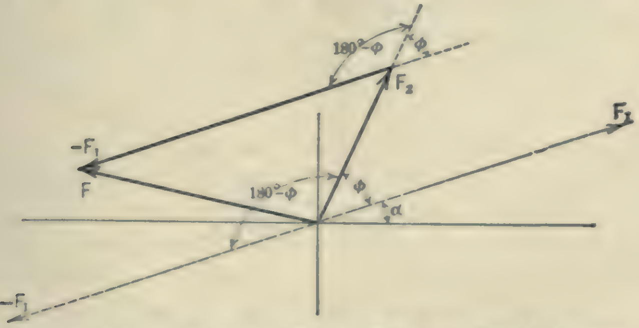


FIG. 34.—Subtracting vector F_1 from F_2 by topographic diagram.

The resultant may be obtained trigonometrically by combining horizontal and vertical components,

Horizontal component of $F_1 = OA$;
Horizontal component of $F_2 = OB$;
Horizontal component of $F_3 = OC$ (to the left);
Vertical component of $F_1 = OD$ (down);
Vertical component of $F_2 = OE$;
Vertical component of $F_3 = OG$.

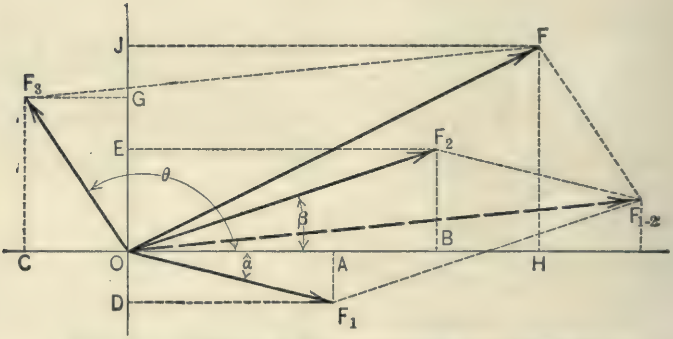


FIG. 35.—Addition of three vectors by polar diagram.

It must be noted that the horizontal component of F_3, reaching out to the left, is negative with respect to the other horizontal components, and that the vertical component of F_1, reaching below the origin, is to be considered negative. We have then

Horizontal component of $F = OH = OA + OB - OC$,
Vertical component of $F = OJ = OE + OG - OD$.

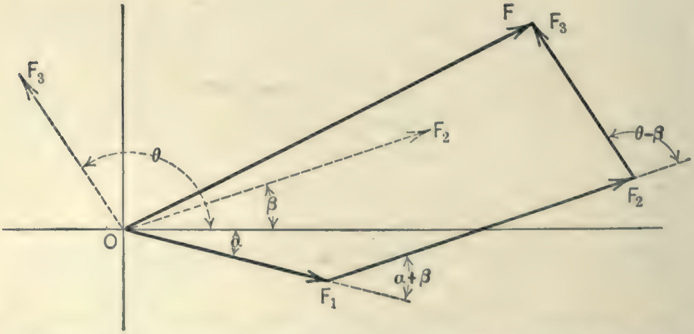


FIG. 36.—Addition, by topographic diagram, of same three vectors as used in Fig. 35.

In Fig. 36 is shown the vector summation of the same three vectors by the polygon or topographic method.

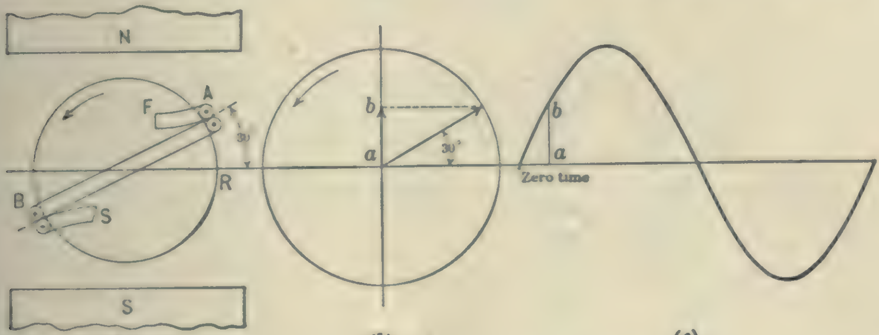
In order to indicate that the vector operation of quantities is intended in an equation, we shall use the symbols $\oplus$ and $\ominus$ to show vector addition and vector subtraction, respectively. The operation indicated in Fig. 29 would be indicated by the equation $F=F_1 \oplus F_2$; that of Fig. 31 by the equation $F=F_1 \ominus F_2$; and that of Fig. 32 by the equation $F=F_2 \ominus F_1$.

33. Notation of Voltage Vectors.—Consider a coil of two turns, wound on an armature and rotating in a bipolar field of uniform density, as in Fig. 37*a*. The coil, when rotating at uniform speed, will generate a sine wave of voltage of maximum value E_m (section 2), i.e., the instantaneous value of the voltage, e, may be represented by

$$e=E_m \sin \theta, \qquad (46)$$

where $\theta=2\pi ft$.

According to the conventions of trigonometry, the sine wave of voltage generated by the coil must be similar to that shown in Fig. 37*c*. Beginning with zero time, the instantaneous values of the voltage generated must be positive in value during the first 180° of rotation of the coil, i.e., for θ between 0° and 180°; and negative in value during the next 180° of rotation, i.e., for θ between 180° and 360°. This must be true for whatever direction of rotation is assumed for the coil, and whatever polarity is assumed for the field in which the coil is rotating.



(*a*) (*b*) (*c*)

Fig. 37.—If it is assumed that a coil generates a sine wave of voltage, starting from a certain reference axis, due regard must be had to the direction of voltage in the coil to be called positive.

If it is assumed that the coil is rotating in counter-clockwise direction, that time is reckoned from the instant when coil-side A is at R, and that the direction of the field is down, the upper pole being north and the lower pole south, it will be seen by the application of Fleming's right-hand rule, that in the inductors moving under the north pole, the voltage induced is in a direction out of the paper (designated by a dot, representing the head end of an arrow); in the inductors moving under the south pole, the voltage induced is in a direction into the paper (designated by a cross, representing the tail end of an arrow).

For the instantaneous position of the coil shown in Fig. 37*a*, the coil having moved 30° from its position at zero time, the voltage is therefore acting through the coil from *S* to *F*, and this value must be that indicated by the ordinate *ab* of the curve of Fig. 37*c*; i.e., this voltage is positive, as it evidently must be for a value of $\theta = 30°$, if the voltage is to be represented by Eq. (46). Thus, having assumed the direction of rotation and polarity of the field as in Fig. 37*a*, and reckoning time from the instant when coil-side *A* is at *R*, we are forced to consider instantaneous voltages acting through the coil from *S* to *F* as positive; it follows then that instantaneous voltages acting through the coil from *F* to *S*, must be considered negative.

In order to clearly designate the vector by which the voltage wave of Fig. 37*c* is generated, some convention must be adopted; we shall designate this particular voltage vector by $E_{m(SF)}$. The idea to be conveyed by the subscripts *SF* (as contrasted to the subscripts *FS*) is that $E_{m(SF)}$ is that vector the projections of which give the sine wave whose *positive instantaneous* values act through the coil in a direction from *S* to *F*. To correspond to the instantaneous position the coil has in Fig. 37*a*, the vector $E_{m(SF)}$ must be drawn as in Fig. 37*b*; in order that its vertical projection, *ab*, shall be equal to the ordinate *ab* of the curve, it must also have moved through an angle of 30° from its position at zero time.

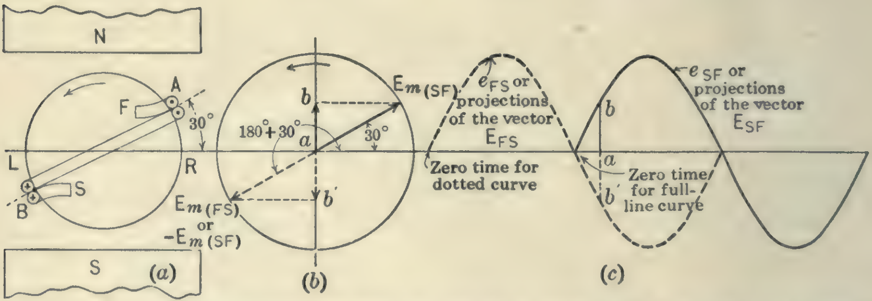


FIG. 38.—From the significance of voltage notation to be used in this text the voltage E_{FS} must be considered as leading the voltage E_{SF} by 180°, that is one voltage is the opposite of the other.

In view of what has been stated above, let us see what would be signified by the vector $E_{m(FS)}$. To be consistent with the convention adopted above, we must mean that voltage vector, the projections of which give a sine wave of voltage, the positive instantaneous values of which act through the coil in a direction from *F* to *S*. Now for the position of the coil shown in Fig. 38*a*, the instantaneous value of the voltage generated is acting from *S* to *F*, or in a direction opposite to that now assumed as positive for this new vector. This instantaneous negative voltage, as generated in Fig. 38*a*, which was positive for the vector $E_{m(SF)}$, is now to be considered negative for the new vector $E_{m(FS)}$, and is represented in Fig. 38*c* by the negative ordinate *ab'* of the broken-line curve. It follows, from what was said in the second paragraph of this section, that time must have been reckoned from the instant that coil-side *B* coincided with *R*, the coil having moved through an angle (180°+30°) to reach the position shown in Fig. 38*a*. The voltage vector $E_{m(FS)}$ must also have moved through an angle of (180°+30°) from its position at zero time, as in Fig. 38*b*. Its vertical projection is *ab'*, corresponding to the negative ordinate *ab'* of the broken-line curve of Fig. 38*c*.

It is evident from Fig. 38 that the voltage vector $E_{m(FS)}$ must lie 180° from the voltage vector $E_{m(SF)}$, or that the voltage vector $E_{m(FS)}$ is the same as the voltage vector $-E_{m(SF)}$. In accordance with the convention adopted for designating the voltage vectors $E_{m(SF)}$ and $E_{m(FS)}$, we may designate the full-line voltage wave as e_{SF} and the broken-line voltage wave as e_{FS}, meaning thereby that the positive instantaneous values of voltage act through the coil from *S* to *F* and *F* to *S* respectively.

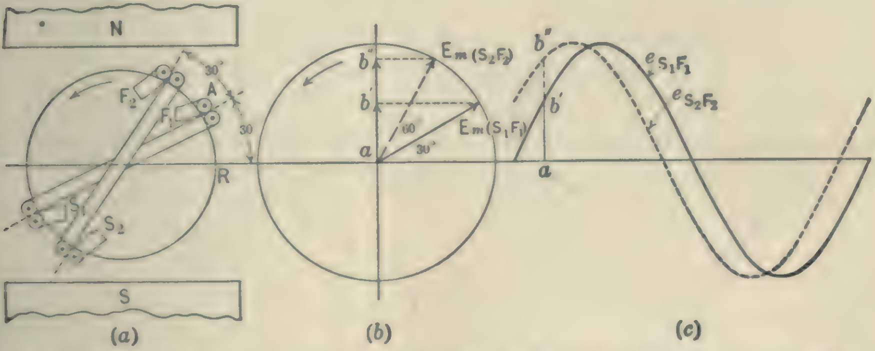


FIG. 39.—In such an arrangement of coils as this, it is evident that the voltage $E_{S_2F_2}$ must lead the voltage $E_{S_1F_1}$ by 30°.

34. Voltage of a Group of Coils.—Consider that a second coil is placed upon the armature of Fig. 37, coil No. 2 being wound in the same direction as was coil No. 1, but placed on the armature 30° ahead of it. Coil No. 2 is indicated in Fig. 39 as S_2F_2.

Now as coil No. 2 is identical with coil No. 1 (except for its angular position on the armature core), it must generate exactly the same kind of a voltage wave as does coil No. 1, that is, a sine wave. However, as coil No. 2 occupies a position on the armature 30° ahead of that occupied by coil No. 1, its cycle of voltage values must occur 30° ahead of the corresponding cycle of values for coil No. 1. The voltage of coil S_2F_2 is thus properly shown by the dashed curve of Fig. 39*c*. If we reckon time as zero when coil S_1F_1 is generating no voltage, its coil-side *A* being at *R* of

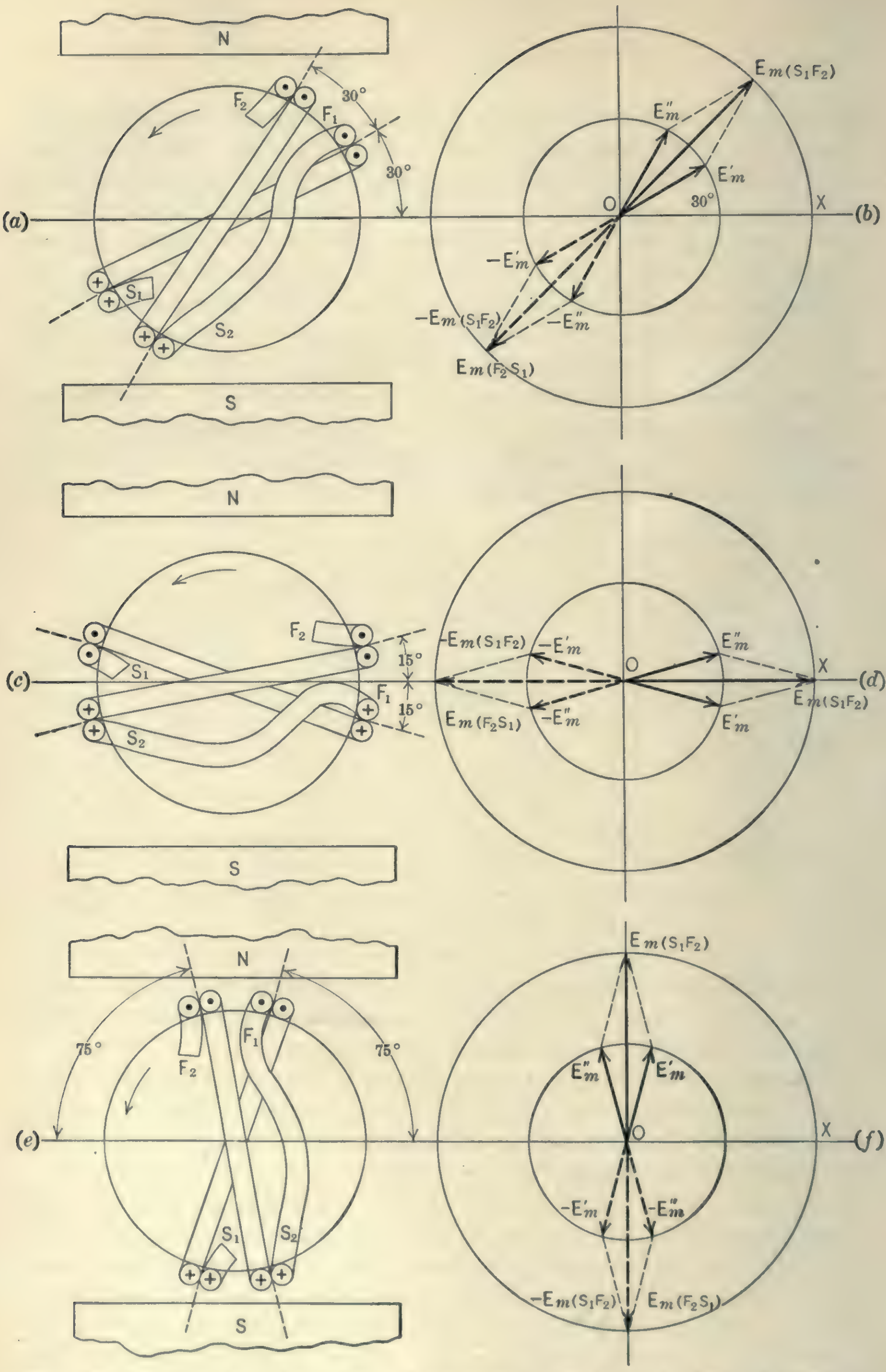


FIG. 40.—Showing how the voltage of a group of coils is obtained for all positions of the armature, by properly adding the respective vectors of each coil.

Fig. 39a, then evidently at time $t=0$, coil No. 2 is already generating considerable voltage because its sides are already cutting flux.

As we have shown that sine waves may be represented by rotating vectors, it follows that the voltage of coil No. 2 may be represented by a vector, and that this vector must be in a position 30° ahead of the vector representing the voltage of coil No. 1. This relative phase of the two vectors (shown in Fig. 39b), is evidently required if their projections on the vertical axis are to represent the voltage waves given in this figure.

We shall now consider what voltage will be generated if the two coils are connected in series. In making the connection of the two coils, four possibilities exist; we may connect F_1 to S_2, F_2 to S_1, F_1 to F_2, or S_1 to S_2. However, if we want the voltage of the two coils to add up (this is the reason several coils are always connected in series in armature windings), the natural way to connect them would be to connect the end of coil No. 1 (F_1) to the beginning of coil No. 2 (S_2). This is just what would be done if two similar centrifugal water pumps were to be connected in series to get a greater pressure than one alone could produce.

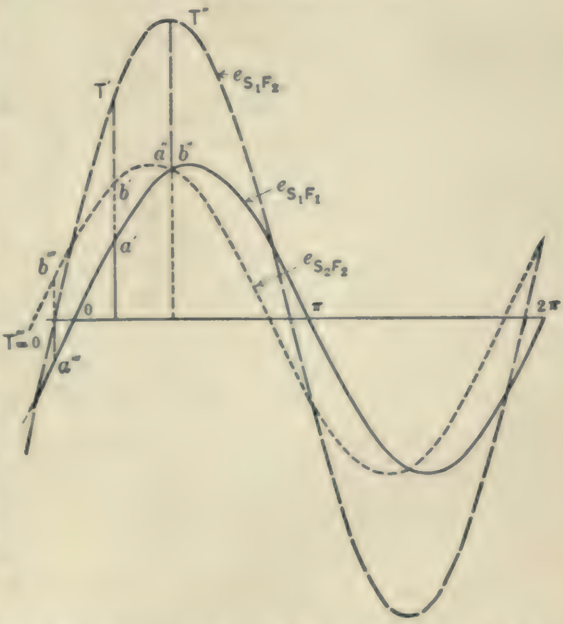


FIG. 41.—If two coils, each of them generating a sine wave of voltage, are connected in series, the voltage of the combination will also be a sine wave, somewhat less in magnitude than the arithmetical sum of the two voltages.

The end of pump No. 1 (its outlet) would be connected to the beginning of pump No. 2 (its intake); the pressure generated by the pair (difference in pressure of intake of No. 1 and outlet of No. 2) would be the *sum* of the pressures due to each pump.

If, however, the outlet of pump No. 1 were connected to the outlet of No. 2, the difference in pressure between the two available water openings (intake of No. 1 and intake of No. 2), would be zero, that is, the *difference* of the pressures due to the two pumps.

It will be seen, then, that the voltage of the two coils, connected as shown in Fig. 40 (F_1 to S_2), will be obtained by adding at every instant the voltages generated by each. This is shown in Fig. 41, the full curve and the dotted curve being the voltages of coils S_1F_1 and S_2F_2 respectively, that is $e_{S_1F_1}$ and $e_{S_2F_2}$. The sum of these voltages is shown by the dashed curve, which is itself a sine curve of nearly twice the amplitude of each of the individual voltages. Furthermore, the phase of this combined voltage is not the same as that of either coil, but is intermediate between the two.

Instead of adding the voltage waves of the two coils, we could just as well have obtained the voltage of the combination by vector addition of the voltage vectors, as is indicated in Fig. 40. In part *a* of this figure, the two coils, connected in series, are shown after coil No. 1 has rotated 30° from its zero position. The vector OE'_m, to represent its voltage at this instant, is therefore properly shown 30° (counter-clockwise rotation) from the axis of reference OX. The voltage of coil No. 2 is then shown in this same figure by vector OE''_m, 30° ahead of OE'_m. The sum of these two vectors is shown at OE_m and this is the vector which represents the voltage of the two coils in series.

The position of this resultant vector is evidently ahead of OE'_m by 15°, as it bisects the parallelogram $OE''_mE_mE'_m$. In magnitude it is equal to twice the voltage of one coil multiplied by cos 15°.

If, therefore, the voltage of coil No. 1 is given by

$$e' = E'_m \sin \omega t,$$

that of coil No. 2 is given by

$$e'' = E''_m \sin (\omega t + 30°),$$

and that of the two coils connected in series, (as $E'_m = E''_m$), is given by

$$e = E'_m (2 \times \cos 15°) \sin (\omega t + 15°)$$

$$= 1.932\, E'_m \sin (\omega t + 15°). \quad . \quad . \quad . \quad . \quad . \quad (47)$$

As a check on the accuracy of this expression for the voltage of both coils in series, let us assume that each coil generates a maximum voltage of ten volts, and consider the position of the coils as shown in Fig. 40*a*.

Here the voltage of coil No. 1 is

$$e' = 10 \sin 30° = 5.00 \text{ volts},$$

the voltage of coil No. 2 (30° ahead of coil No. 1) is

$$e'' = 10 \sin (30° + 30°) = 10 \sin 60° = 8.66 \text{ volts},$$

and the sum of these two voltages must be the voltage of both coils in series, for the instant shown; the voltage of both coils at this instant is

$$8.66 + 5.00 = 13.66 \text{ volts}.$$

According to Eq. (47) the voltage of both coils is

$$e = 1.932 \times 10 \times \sin\,(30° + 15°)$$

$$= 1.932 \times 10 \times \sin 45° = 13.66 \text{ volts.}$$

For position *c* in Fig. 40,

$$e' = 10 \sin\,(-15°) = -2.59 \text{ volts,}$$

$$e'' = 10 \sin 15° \qquad = 2.59 \text{ volts,}$$

so that the voltage of both coils in series should be zero.

According to Eq. (47),

$$e = 1.932 \times 10 \sin\,(-15° + 15°) = 0,$$

as shown.

For position *e* in Fig. 40,

$$e' = 10 \sin 75° = 9.66 \text{ volts,}$$

$$e'' = 10 \sin 105° = 9.66 \text{ volts,}$$

so that both coils in series must be generating at this instant 19.32 volts.

According to Eq. (47)

$$e = 1.932 \times 10 \sin\,(75° + 15°)$$

$$= 19.32 \text{ volts.}$$

If we had desired to get the voltage of the two coils (connected in series as above), from F_2 to S_1, by vector construction as previously shown, then evidently we would have had to consider the sum of the vectors $E_{m(F_2S_2)}$ and $E_{m(F_1S_1)}$. These will be the negatives of the vectors previously used, E'_m and E''_m, as is shown in Fig. 40 by the dotted vector construction. In each case it is apparent that $E_{m(F_2S_1)}$ is the equal and opposite of $E_{m(S_1F_2)}$, or that $E_{m(F_2S_1)}$ is the same as $-E_{m(S_1F_2)}$. The curves of these reversed voltages are shown in Fig. 42.

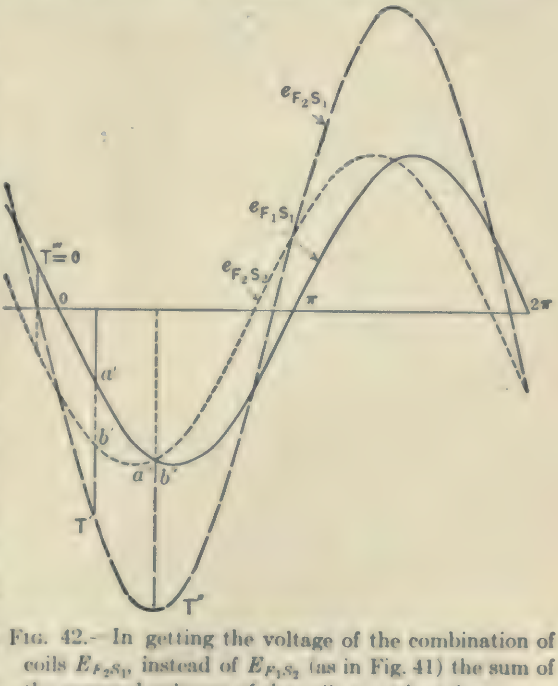


FIG. 42.—In getting the voltage of the combination of coils $E_{F_2S_1}$, instead of $E_{F_1S_2}$ (as in Fig. 41) the sum of the reversed voltages of the coils must be taken.

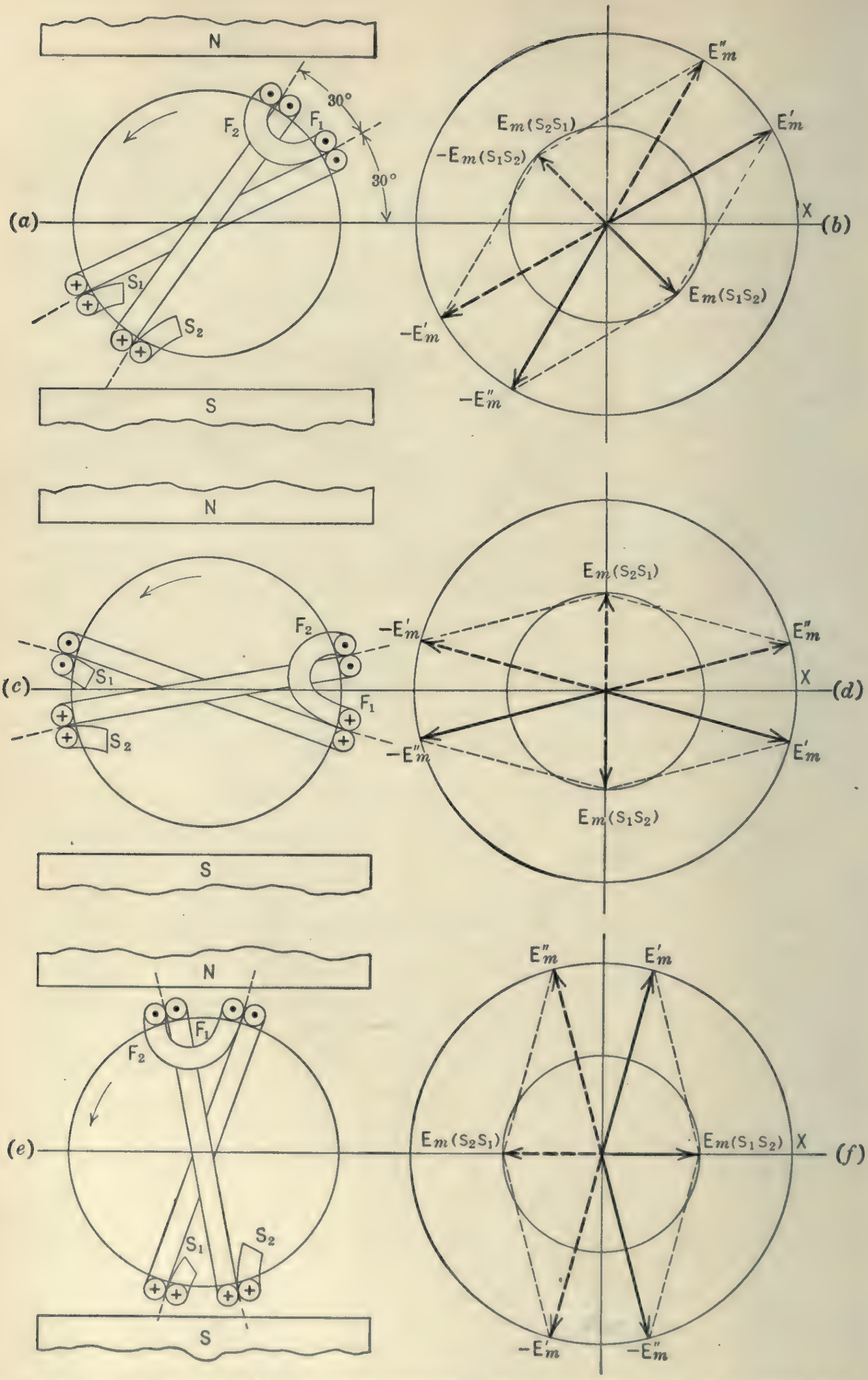


Fig. 43.—If the coils of Fig. 40 are connected in the reversed sense the voltage of the group is obtained by taking a reversed vector for the coil which has been reversed, in this case, coil 2.

Now, suppose that instead of connecting the coils F_1 to S_2, we had connected F_2 to F_1 as in Fig. 43, and so obtained S_1 and S_2 for the two terminals of the coil group. The voltage $e_{S_1S_2}$ would then be obtained at any instant by adding the voltage generated in coil No. 1 to that generated in coil No. 2 in the reverse sense; that is, we should take $e_{S_1F_1}$ and add it to the voltage $e_{F_2S_2}$. But, as the latter is evidently the negative of $e_{S_2F_2}$, the voltage $e_{F_2S_2}$ at any instant is as shown by the dot-and-dash line of Fig. 44. The sum of this curve and the curve for $e_{S_1F_1}$ gives the dashed curve for $e_{S_1S_2}$; the vector construction for the voltage of the two coils so connected is shown in Fig. 43. It is evident that such a connection

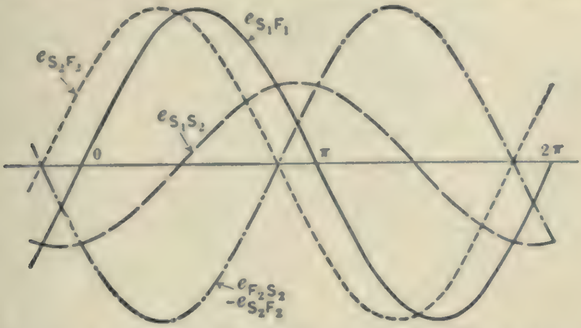


FIG. 44.—In case one of the coils is connected in the reversed fashion the voltage of the group is much less than it is with the normal connection.

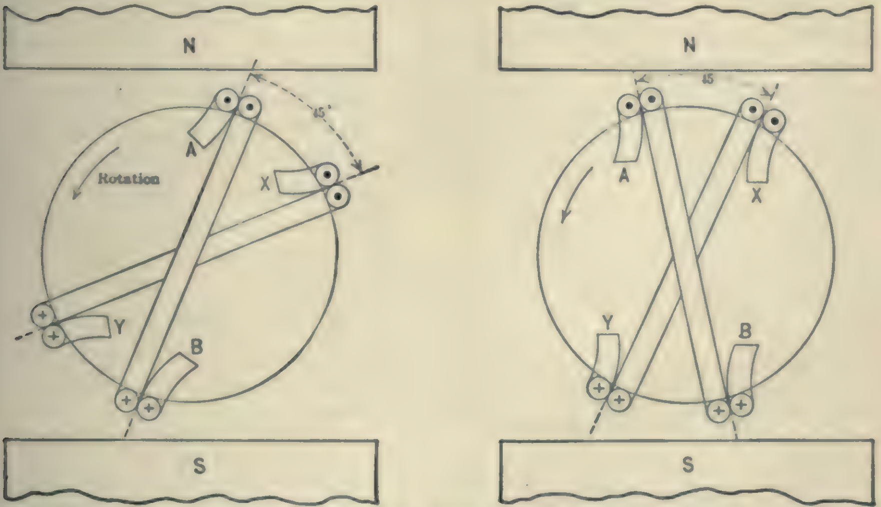


FIG. 45.—The coil A–B is evidently 45° ahead of the coil X–Y, but the coil B–A is 135° behind the coil X–Y.

of coils would never be used, because the function of a generator is to produce voltage, and this scheme of connection gives actually less voltage for the group than for one coil alone.

The other two possible connections between coils, that is, joining F_2 to S_1, or S_1 to S_2, would give results identical with those obtained above.

35. Relative Voltage Phases Fixed by Coil Positions.—To find the resultant voltage across the ends of a number of coils in series, when the various voltages generated are not in phase, it is necessary to know the phase relationship of the various voltages, and the order of connection of the coils in which the voltages are generated. If we have a view of the coils as actually wound on the armature, the phase relationship can readily be determined. Thus, in Fig. 45, if it is assumed that a voltage from B to A is positive, and a voltage from Y to X also positive, then voltage BA will lead the voltage YX by 45°. This follows because the coil BA goes through its sequence of positions 45° earlier than coil YX. But if the voltage BA is again assumed as positive, and the voltage Y to X is assumed as negative (that is, X to Y as positive), then voltage BA will lag $180° - 45°$ behind the voltage XY, or, voltage XY will lead the voltage BA by 135°. This follows because the coil BA goes through its sequence of positions 135° later than does the coil XY. Thus, as soon as we have assumed the positive direction of the voltages, their phase relationship follows from the position of the coils on the armature.

36. Notation of Coil Diagrams.—It is, however, not necessary to picture the coils as actually wound on the armature. We may represent the coils simply, as in Fig. 46*a*, and indicate their phase relationship by a vector diagram as in Fig. 46*b*; the latter figure indicates clearly that the voltage through the first coil, from B to A, leads the voltage through the second coil, from Y to X, by 45°.

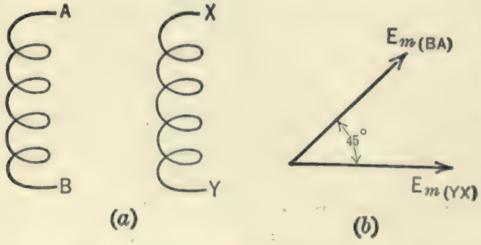
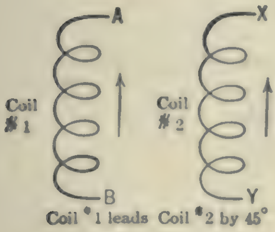


FIG. 46.—One way of denoting the relative positions of coils in an armature; the coil B-A is 45° ahead of the coil Y-X, as shown by diagram (*b*).

The same ideas may be conveyed, as in Fig. 47, by indicating with arrows the direction assumed as positive for the instantaneous voltages of each coil, and stating that coil No. 1 leads coil No. 2 by 45°. It follows then that the voltage BA will lead the voltage YX by 45°.

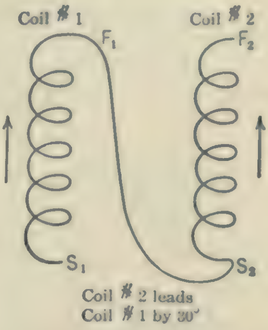
It is to be emphasized that the idea to be conveyed by the arrows in Fig. 47 is the direction of positive voltage (instantaneous value) through each coil; *they are not intended to indicate the direction of the instantaneous voltages in the different coils at the same instant.* The arrows therefore indicate only the direction of positive values whenever they may occur; the phase relationship, which indicates the time interval between the maximum positive values of the different voltages, must be indicated in some other way.

Referring again to the coils of Fig. 39, we may indicate the fact that the voltage of coil No. 2 from S_2 to F_2 leads the voltage of coil No. 1 from S_1 to F_1 by 30°, and that S_2 and F_1 are joined, as in Fig. 48. From this figure we are at once enabled to draw a vector diagram as in Fig. 49*a*, to indicate the phase relationship of the two voltages. One of the vectors may be drawn wherever we please; the other must then be drawn 30° ahead or 30° behind, depending on which vector we first selected.



Coil # 1 leads Coil # 2 by 45°

FIG. 47.



Coil # 2 leads Coil # 1 by 30°

FIG. 48.

FIG. 47. Another way of indicating the relative phase positions of two coils; this means that the voltage E_{BA} goes through its sequence of values 45° ahead of the voltage E_{YX}.

FIG. 48.—Such a diagram at once permits the vector calculation of the voltage of the group of coils.

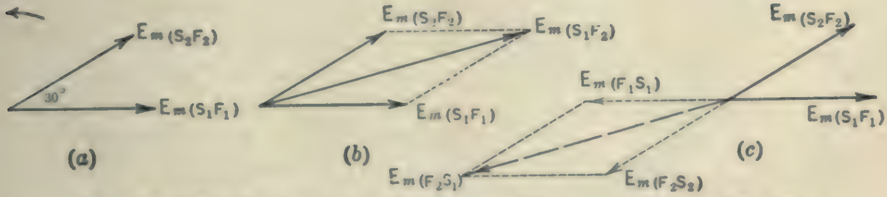


FIG. 49.—Showing how to get the voltage of the group of coils of Fig. 48.

If we desire the voltage S_1 to F_2, we find that we must combine vectors $E_{m(S_1F_1)}$ and $E_{m(S_2F_2)}$; the voltage S_1 to F_2 is the vector sum of the voltages S_1F_1 and S_2F_2. Or we may write that

$$E_{m(S_1F_2)} = E_{m(S_1F_1)} \oplus E_{m(S_2F_2)}.$$

This operation is carried out in Fig. 49*b*.

To find the voltage F_2S_1 (Fig. 39), in proceeding through the coils, we pass through both coils in a direction opposite to the arrow of each coil, which indicates that we must add the voltages $E_{m(F_2S_2)}$ (or $-E_{m(S_2F_2)}$), and $E_{m(F_1S_1)}$ (or $-E_{m(S_1F_1)}$). Thus the voltage F_2S_1 is the vector sum of the voltages $-E_{m(S_2F_2)}$ and $-E_{m(S_1F_1)}$. Or we may write

$$E_{m(F_2S_1)} = \ominus E_{m(S_2F_2)} \ominus E_{m(S_1F_1)},$$

$$E_{m(F_2S_1)} = -E_{m(S_2F_2)} \oplus (-E_{m(S_1F_1)}).$$

This operation is carried out in Fig. 49*c*.

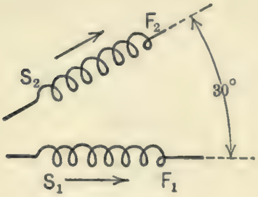


FIG. 50.

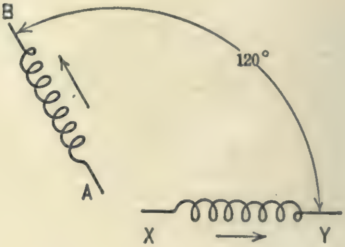


FIG. 51.

FIG. 50.—Such a diagram indicates that the coil S_2F_2 is 30° ahead of S_1F_1.
FIG. 51.—This diagram indicates that E_{AB} leads E_{XY} by 120°.

It is often convenient to indicate the phase relationship between the voltages generated by coils on an armature by the relative position of the coils in the diagram. Thus in Fig. 50, the coils are drawn making an angle of 30° with each other. The direction of the arrows then indicates that voltage S_2F_2 leads voltage S_1F_1 by 30°, thus giving sufficient information to enable us to draw the vector diagram of Fig. 49*a*. Similarly, Fig. 51 conveys the information that voltage AB leads voltage XY by 120°.

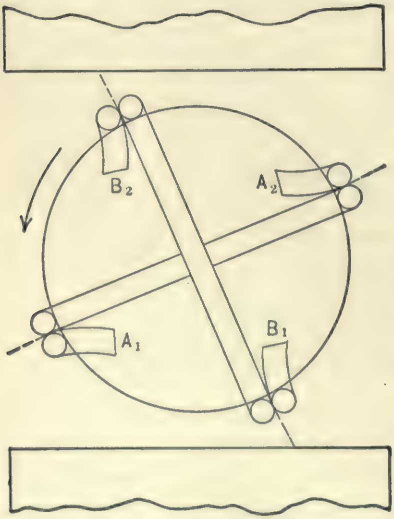


FIG. 52.—In a two-phase winding two coils, or group of coils, 90° apart are used.

37. Examples of Practical Coil Connections. Two-phase Winding.—Consider an armature with two identical coils placed 90° from each other as in Fig. 52. If it is assumed that a voltage A_1A_2 is positive, and a voltage B_1B_2 is positive, it follows that voltage B_1B_2 leads voltage A_1A_2 by 90°. Had it been assumed that voltages A_1A_2 and B_2B_1 were positive, then voltage A_1A_2 would lead B_2B_1 by 90°. The same information is conveyed by Fig. 53, the two parts of the figure corresponding respectively to the two possible assumptions made above. From the information given in Fig. 53, the corresponding vector diagrams of Fig. 54 may at once be drawn.

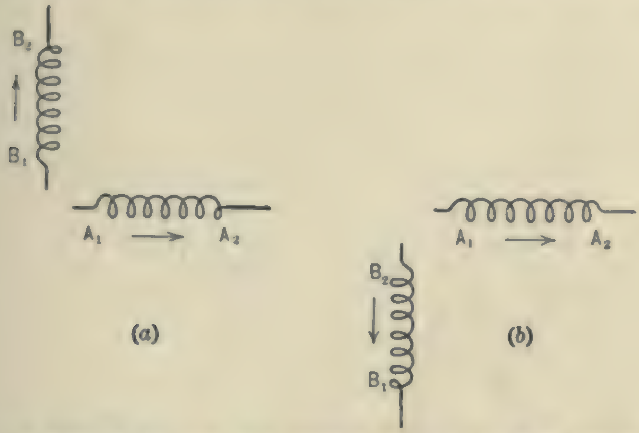


FIG. 53.—The voltage $E_{B_1B_2}$ of the diagram in Fig. 52 leads $E_{A_1A_2}$ by 90°, but if we consider the voltage $E_{B_2B_1}$, this lags 90° behind $E_{A_1A_2}$.

If the two coils are put in series by joining ends A_1 and B_1 as in Fig. 55, and the voltage B_2A_2 is desired, it is necessary to add vectorially the voltages B_2B_1 and A_1A_2; this follows for either of the assumptions represented in Fig. 53. The proper vector construction is indicated in Fig. 56. In case we desire to obtain the voltage A_2B_2, the voltage A_2A_1 must be added to voltage B_1B_2.

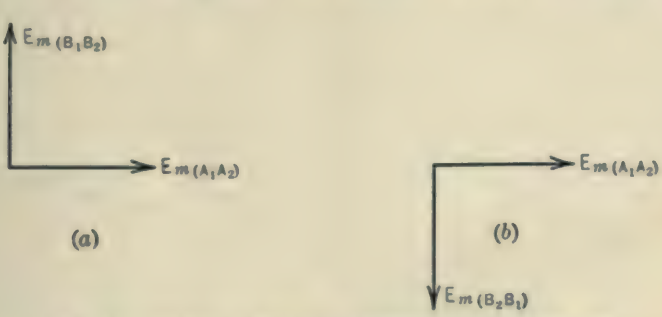


FIG. 54.—The vector diagrams corresponding to Fig. 53.

If there are two actual coils on an armature, and they are connected in series in a certain definite fashion, the resultant voltage across the free ends must have a definite magnitude and phase. Hence, if our method of obtaining this resultant voltage is accurate, it must yield the same definite voltage, no matter what assumptions we make as to the positive and negative directions of voltages through the two coils. Thus, in Fig. 55 we have assumed in one case the voltage acting through coil B as positive from B_1 to B_2, and, as shown in Fig. 56*a*, the voltage B_2A_2 is 45° behind A_1A_2. In Fig. 55*b* the coils are shown connected in the same way as before, but the voltage through coil B is assumed as positive from B_2 to B_1, that is, opposite to what it was for the previous case. The resultant voltage B_2A_2 is shown in Fig. 56*b* to be just the same voltage as in Fig. 56*a*, again lagging 45° behind the reference voltage A_1A_2.

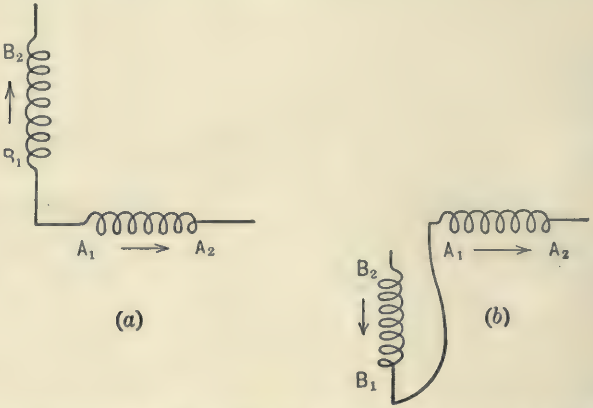


FIG. 55.—The two coils of Fig. 52 may be connected in either of the ways indicated here.

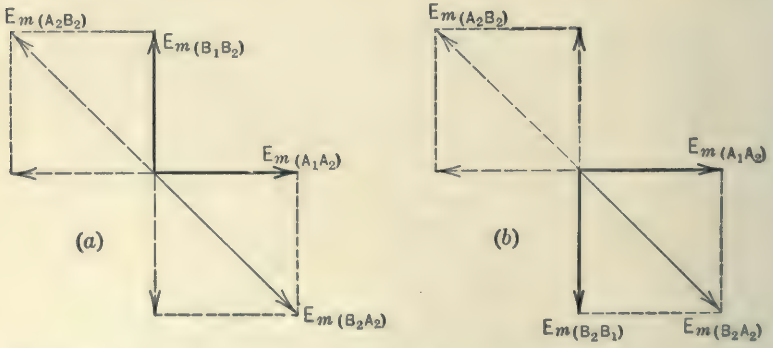


FIG. 56.—The vector diagrams of voltage for the two schemes of connection shown in Fig. 55.

If the two coils are put in series by joining ends A_1 and B_2 as in Fig. 57, the voltage B_1A_2 will be the vector sum of the voltages B_1B_2 and A_1A_2 as is indicated in Fig. 58. Similarly, the voltage A_2B_1 will be the vector sum of the voltages A_2A_1 and B_2B_1.

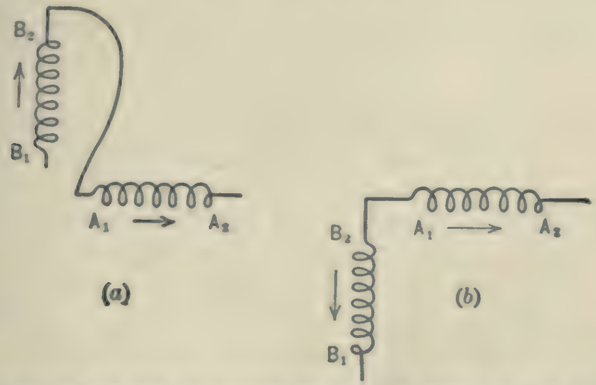


Fig. 57.—Indicating how another connection than that shown in Fig. 55 may be used.

The other two possible connections of the coils, that is, joining A_2 to B_2, or B_1 to A_2, will give the same results as those considered above.

Two important conclusions are to be drawn from the above consideration. Whenever two identical coils, placed 90° apart on an armature, are joined in any order whatever, the resultant voltage across the open ends is the same in magnitude. If the maximum value of the voltage of each coil is E_m, the maximum value of the combination is

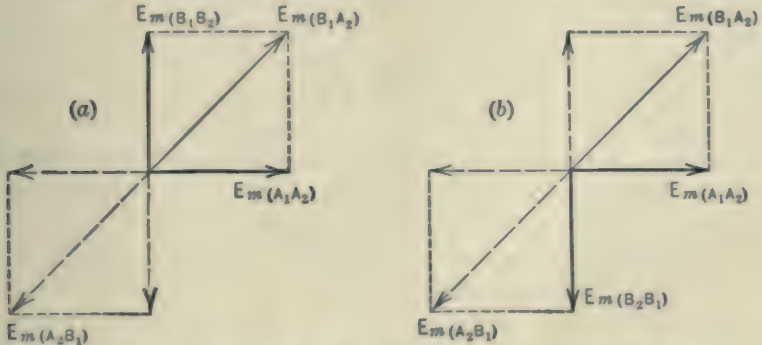


Fig. 58.—The vector diagrams for Fig. 57; the group voltage is now 90° from what it was in Fig. 56.

$$\frac{E_m}{\cos 45^\circ}=\frac{E_m}{0.707}=1.414E_m,$$

as may be seen from Figs. 56 and 58. If the connection of one coil with respect to the other is reversed, the resultant voltage has the same magnitude but shifts its phase by 90°. Thus, in Fig. 55, end B_1 was joined to end A_1, and the resultant group voltage was found (Fig. 56) to lie either in the second or the fourth quadrant. When end B_2 was connected to end A_1, as in Fig. 57, the resultant group voltage was found either in the first or the third quadrant (Fig. 58).

An alternator which has the coils on its armature so arranged as to give two equal voltages 90° apart, is called a *quarter-phase*, or a *two-phase* alternator. Obviously the two-phase alternator represented in Fig. 52 would generate but a few volts; in order to generate voltages as high as 15,000 volts, which may be required in commercial machines, two-phase alternators must have many coils, each coil consisting of many turns, on their armatures. The group of coils which generates one of the voltages is spoken of as one of the *phases* of the machine, and the group of coils which generates the other voltage, 90° away in phase from the first, would be called the other phase of the alternator. In practice the two phases of a two-phase alternator are nearly always electrically separate within the machine, the four ends of the two phases coming out through the frame as in Fig. 59. The terminals of the coils as they come from the machine are usually so marked that ends of different phases may be distinguished; thus in Fig. 59 terminals AA are ends of the one phase, and terminals BB are the ends of the other phase.

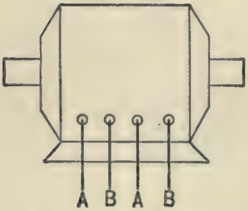


Fig. 59.—A two-phase machine is fitted with leads as shown, no polarities being indicated, so if the coils are connected together either of the conditions of the preceding four figures may be encountered.

In actually carrying out connections of the two phases of an alternator as in Fig. 59, unless definite information is available as to the order in which the coils have been wound and placed on the armature, we shall not know, with a given connection, which of the cases indicated in Figs. 55 and 56 we have. The conclusions mentioned on page 67 will, however, always be true.

38. Reversing Direction of Rotation of Armature.—It is to be noted that, if the direction of rotation of the armature of Fig. 52 be reversed, the voltage A_1A_2 will lead voltage B_1B_2 by 90°, whereas with the counter-clockwise rotation as supposed in Fig. 52, the voltage B_1B_2 leads voltage A_1A_2 by 90°. Thus, reversing the direction of rotation reverses the *phase rotation*, or order in which the phases generate their positive values.

With ends A_1 and B_1 joined and clockwise rotation of the armature of Fig. 52, conditions are as indicated in Fig. 60, in which counter-clockwise rotation of the vectors is assumed, *as is the convention for all vector diagrams, no matter what the direction of armature rotation.* The voltage

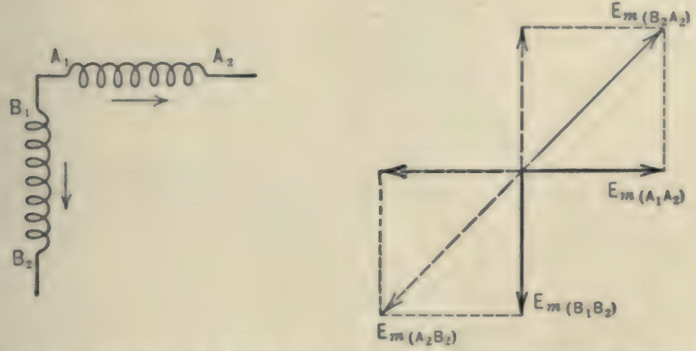


FIG. 60.—If the direction of rotation of a two-phase machine is reversed, the same change in vector relations occurs as when the connection of one coil to the other is reversed, rotation being the same.

B_2A_2 is then the vector sum of the voltages B_2B_1 and A_1A_2, and is found to be in the first quadrant. This voltage B_2A_2 leads the voltage A_1A_2 by 45°, whereas with counterclockwise rotation of the armature this same voltage lagged behind the voltage A_1A_2 (Fig. 56). This same shift in phase of 90° occurred from Fig. 55 to 57, in which the order of connection of coil B to coil A was reversed. Thus *reversing the direction of rotation of the armature of an alternator produces similar results to reversing the connections of one phase with respect to the other*, as may be seen by comparing Figs. 56 and 58 with Fig. 60.

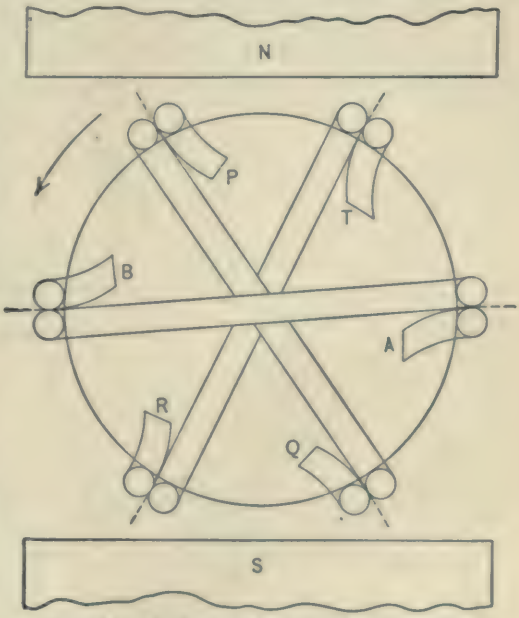


FIG. 61.—A three-phase armature winding.

39. Three-phase Coil Connections. Y-connection.—Consider an armature upon which three coils have been placed, as in Fig. 61. If we assume that voltages through the coils from B to A, from P to Q and from T to R

are to be considered as positive, we see that coil BA will lead coil PQ by 60°, and the latter in turn will lead coil TR by 60°. The voltage waves e_{BA}, e_{PQ} and e_{TR}, generated by the coils, will bear the same relations to each other, as represented in Fig. 62b, being produced by the rotating vectors $E_{m(BA)}$, $E_{m(PQ)}$, and $E_{m(TR)}$, respectively, of Fig. 62a. We may say then that the coils BA, PQ, and TR have been wound upon the armature 60° apart.

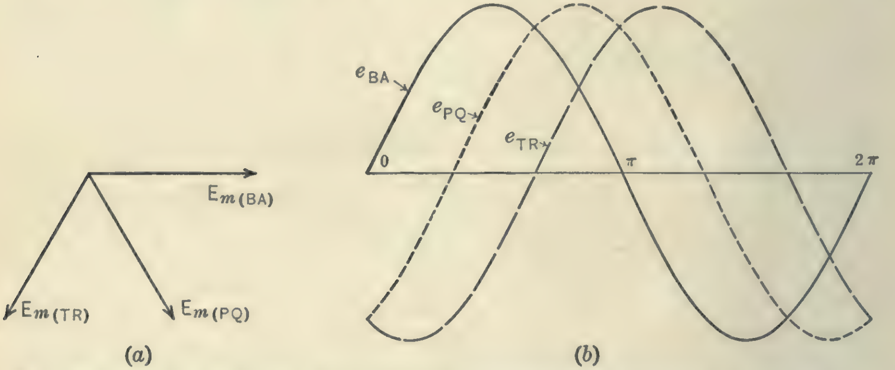


FIG. 62.—Taking the coils in the sequence indicated by the subscripts of diagram (a) the three coils are 60° apart.

However, if we assume that voltages B to A, and T to R are positive as before, but now consider that voltage Q to P is positive, we see that coil BA leads coil TR by 120°, and that coil TR leads coil QP by 120°. The voltage waves, e_{BA}, e_{TR}, and e_{QP} are then 120° apart and may be represented as in Fig. 63b, being produced by the rotating vectors $E_{m(BA)}$, $E_{m(TR)}$, and $E_{m(QP)}$, respectively, of Fig. 63a.

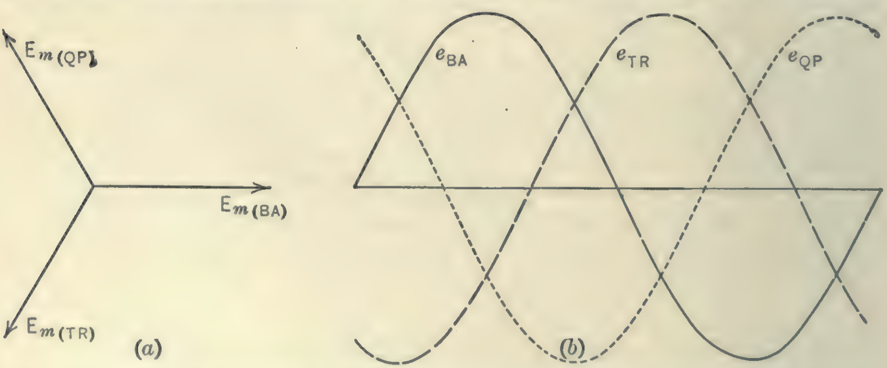


FIG. 63.—By considering coil Q-P instead of coil P-Q, the 60° voltage diagram of Fig. 62 is charged to a 120° diagram as indicated here. This is the proper phase relation for a three-phase alternator.

A three-phase alternator is one that generates three equal voltages 120° *apart.* It is evident from the above that in winding a simple three-phase bipolar alternator with but three coils, one coil would be wound in a certain order, the armature then rotated 120° and the second coil wound in exactly the same order as the first coil. The armature would then be rotated 120° further and the third coil wound in the same order as were the first and second. With all three coils wound in the same order, the starting ends of the three coils would be 120° apart mechanically, and the finish ends as well. The starting ends of the three phases might then be designated S_1, S_2, and S_3, and their finish ends as F_1, F_2, and F_3, as shown in either of the methods used in Figs. 64 and 65;

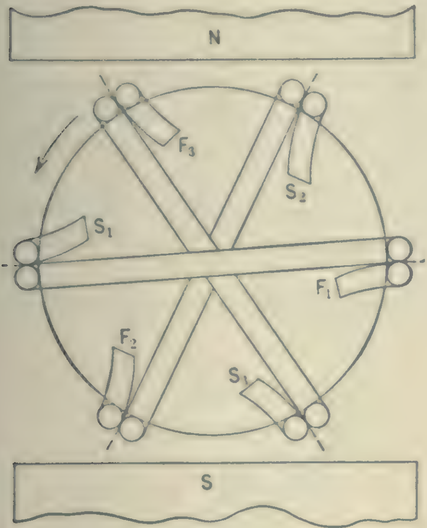


FIG. 64.

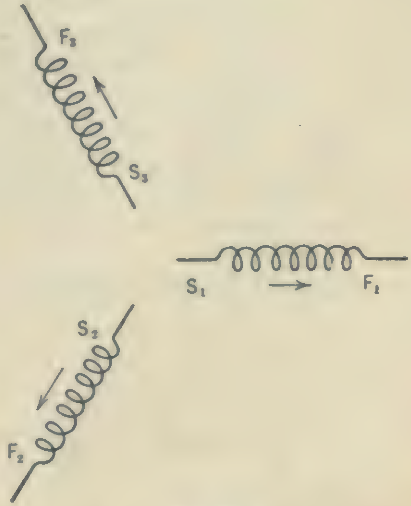


FIG. 65.

FIG. 64.—Indicating the proper sequence of winding the coils on a three-phase machine, *S* denoting the start of a coil.

FIG. 65.—A common method of indicating three coils 120° apart in phase.

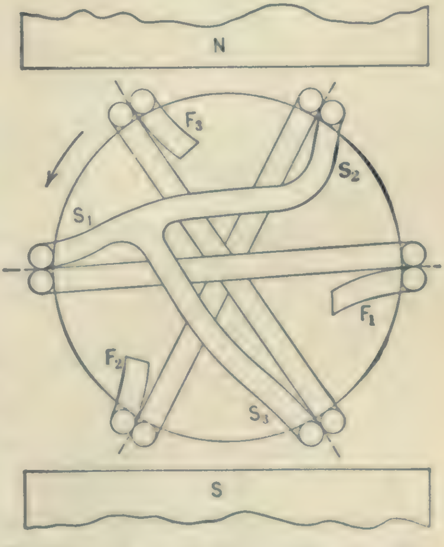
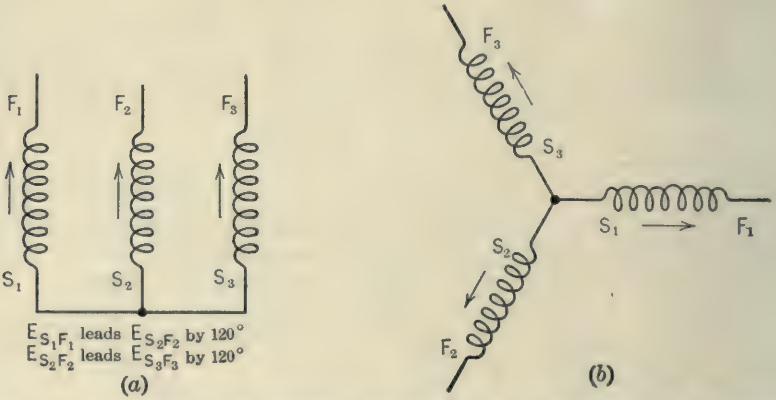


FIG. 66.—One way of inter-connecting the coils of a three-phase machine.

in these diagrams phase No. 1 leads phase No. 2 by 120°, and phase No. 2 leads phase No. 3 by 120°.

In actual machines, the three coils, placed 120° apart on the armature as in Fig. 64, are generally interconnected, and this may be done in either one of two schemes both of which will give a symmetrical arrangement. In one such scheme, the three starting ends of the coils (S_1, S_2, S_3), are joined together and the three finish ends used as terminals of the machine, or, the three finish ends (F_1, F_2, F_3) are joined together and the three beginning ends used as the machine terminals. In Fig. 66, ends S_1, S_2 and S_3 have been joined; the same fact is indicated by each of the two parts of Fig. 67.

From the similarity of its coil arrangement (Fig. 67b) to the letter Y, this method of connecting three coils placed 120° apart on an armature is called the *three-phase* Y-(*or wye*) *connection;* it is also known as the *star connection.*



F_1
F_2
F_3
S_1
S_2
S_3
$E_{S_1F_1}$ leads $E_{S_2F_2}$ by 120°
$E_{S_2F_2}$ leads $E_{S_3F_3}$ by 120°
F_3
S_3
S_1
F_1
S_2
F_2

(a) (b)

FIG. 67.—Two methods of showing that the coils of a three-phase machine have been inter-connected as shown in Fig. 66.

The voltages across the open ends of the three coils connected as in Fig. 67, are then $E_{m(F_1F_3)}$, $E_{m(F_3F_2)}$, and $E_{m(F_2F_1)}$, and by inspection of Fig. 67 we may write

$$E_{m(F_1F_3)} = E_{m(F_1S_1)} \oplus E_{m(S_3F_3)},$$

$$E_{m(F_3F_2)} = E_{m(F_3S_3)} \oplus E_{m(S_2F_2)},$$

$$E_{m(F_2F_1)} = E_{m(F_2S_2)} \oplus E_{m(S_1F_1)},$$

In Fig. 68 these additions have been carried out and it will be seen that $E_{m(F_1F_3)}$ leads $E_{m(F_2F_1)}$ by 120°, and that $E_{m(F_2F_1)}$ in turn leads $E_{m(F_3F_2)}$ by 120°, and that all three voltages are equal. With its three coils thus symmetrically connected, the alternator delivers three equal voltages 120° apart, which is the condition that a three-phase alternator must fulfill.

In practice, a three-phase alternator generally has the connections between its phases made inside its frame. If it is Y-connected, leads connected to the open ends of its coils are brought out and the load for the alternator is connected to them. The voltages between the open ends of the coils are called the *line voltages* of the alternator; the voltages generated by the three individual phases, i.e., the voltages S_1F_1, S_2F_2, S_3F_3, are spoken of as the *phase voltages* of the alternator.

The numerical relation between the phase and line voltages of a three-phase, Y-connected alternator may be derived from Fig. 68. It is evident that vector $E_{m(F_3F_2)}$ bisects the angle between $E_{m(S_2F_2)}$ and $E_{m(F_3S_3)}$, so that $E_{m(F_3F_2)} = 2 \times OX$. We have then,

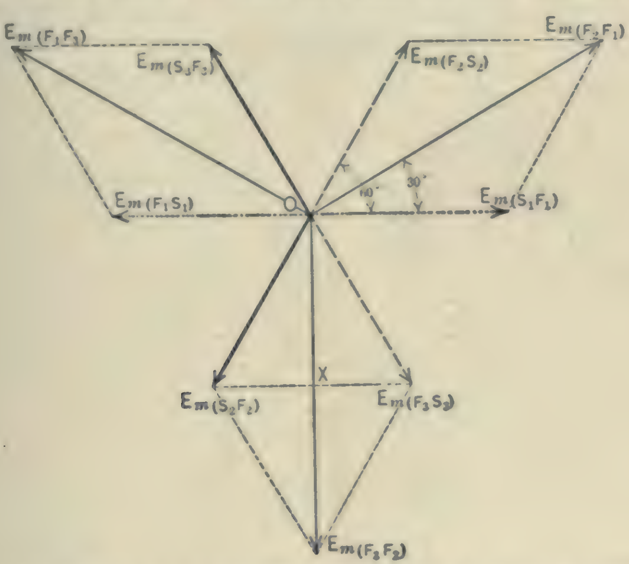


FIG. 68.—Vector diagram of the voltages between terminals of the winding of Fig. 67.

$$E_{m(F_3F_2)} = 2 \times OX = 2 \times E_{m(F_3S_3)} \times \cos 30°$$

$$= 2 \times E_{m(F_3S_3)} \times 0.866 = 2 \times E_{m(F_3S_3)} \times \tfrac{1}{2}\sqrt{3}$$

$$= 1.732\ E_{m(F_3S_3)}.$$

The maximum value of the line voltage is equal to the maximum value of the phase voltage multiplied by 1.732, provided, of course, that all three phases voltages are equal. We may also express the relation above as

$$E_m = 1.732 e_m,$$

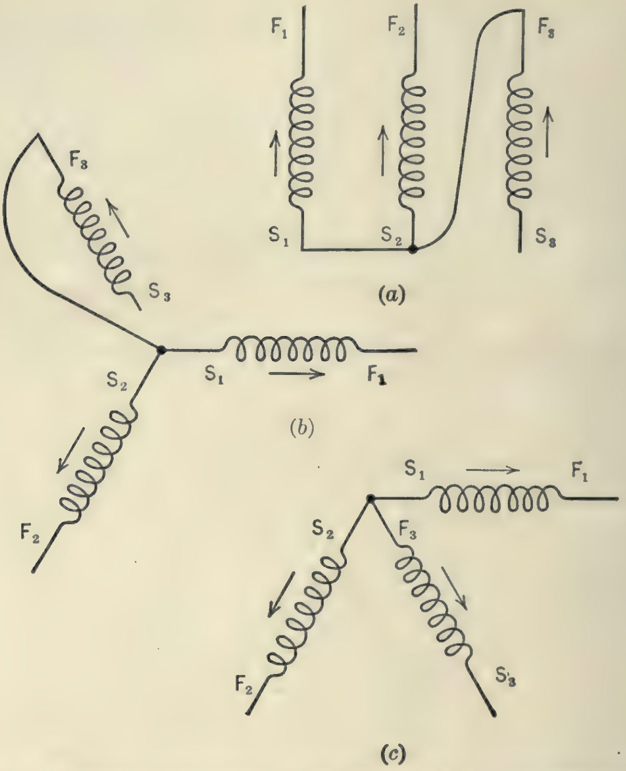


FIG. 69.—An incorrect connection of the three coils of a three-phase machine.

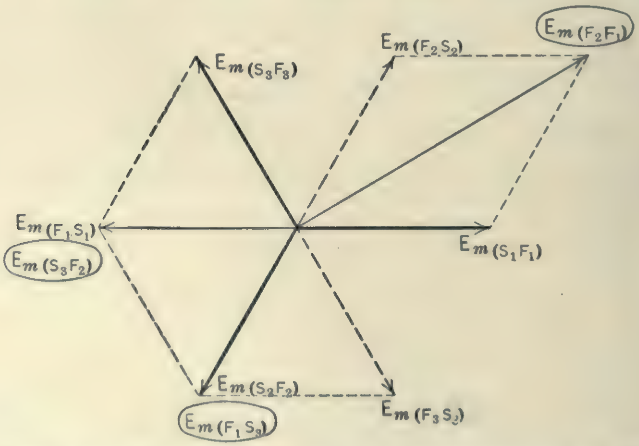


FIG. 70.—Showing the dissymmetrical voltage diagram resulting from the incorrect connection of Fig. 69.

where E_m = maximum value of line voltage,
e_m = maximum value of phase voltage.

Since the effective value of a sine wave is equal to 0.707 of the maximum value, we may write

$$E = 1.732e = \sqrt{3}e, \quad . \quad (48)$$

where E = effective value of line volts,
e = effective value of phase volts.

40. Incorrect Connections.—In connecting the coils of a three-phase alternator, it is possible to make an incorrect connection. Suppose that in connecting the three coils of Figs. 67, phase No. 3 was picked up in reverse order; i.e., instead of joining end S_3 to S_1 and S_2, we made a mistake and connected F_3 to S_1 and S_2, as in Fig. 69.

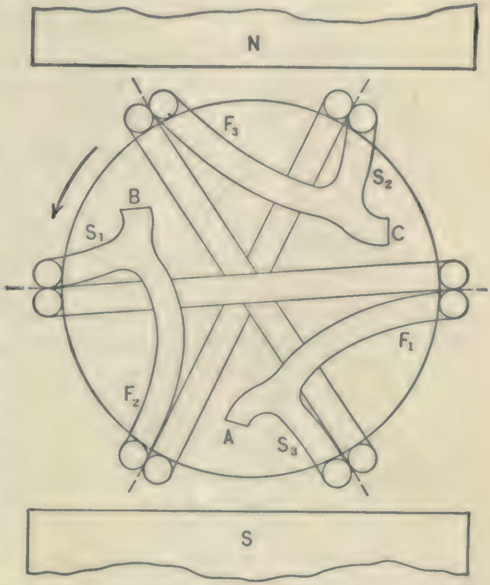


FIG. 71.—Another method of inter-connecting the coils of a three-phase machine; the terminals A, B, and C connect to the load.

The line voltages would then be

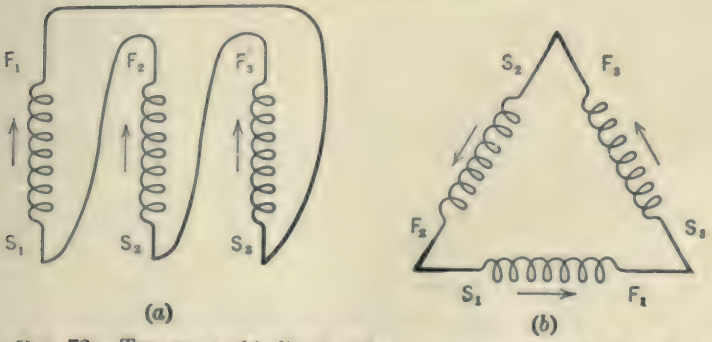
$$E_{m(F_1S_3)},\ E_{m(S_3F_2)} \text{ and } E_{m(F_2F_1)},$$

and we may write

$$E_{m(F_1S_3)} = E_{m(F_1S_1)} \oplus E_{m(F_3S_3)},$$

$$E_{m(S_3F_2)} = E_{m(S_3F_3)} \oplus E_{m(S_2F_2)},$$

$$E_{m(F_2F_1)} = E_{m(F_2S_2)} \oplus E_{m(S_1F_1)}.$$



(a) (b)

FIG. 72.—Two ways of indicating the connection scheme of Fig. 71.

These vector additions have been carried out in Fig. 70; it will be seen that the three line voltages are neither equal nor 120° apart. The machine with its coils connected incorrectly is therefore not a normal three-phase alternator. The improper connection of the phases as shown in Fig. 69, is sometimes called an unsymmetrical Y-connection.

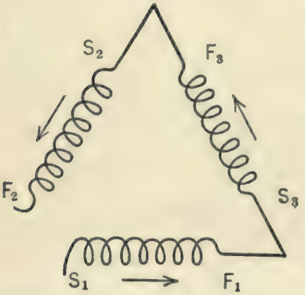


Fig. 73.—Before the delta connection is completed, test for voltage across the open ends S_1–F_2 should be made.

41. Delta-connection. — In the other scheme for connecting the three phases of a three-phase alternator, the three phases are placed in series as in Fig. 71, by joining ends S_1 to F_2, S_2 to F_3, and S_3 to F_1.

In Fig. 72 this connection is indicated diagrammatically. It will be seen that part *b* of this figure is in effect a polygon or topographic vector diagram, and from its similarity to the Greek letter Δ (delta), this method of connecting the three phases is known as the *delta-connection;* it is also sometimes known as a *mesh-connection.*

Inasmuch as an armature winding, when Δ-connected, closes on itself, it is evident that there must not be any resultant voltage within the closed circuit, i.e., the resultant of all the voltages throughout the circuit must be zero. If this is not the case, a large current will circulate within the delta, and damage the alternator within a very short time.

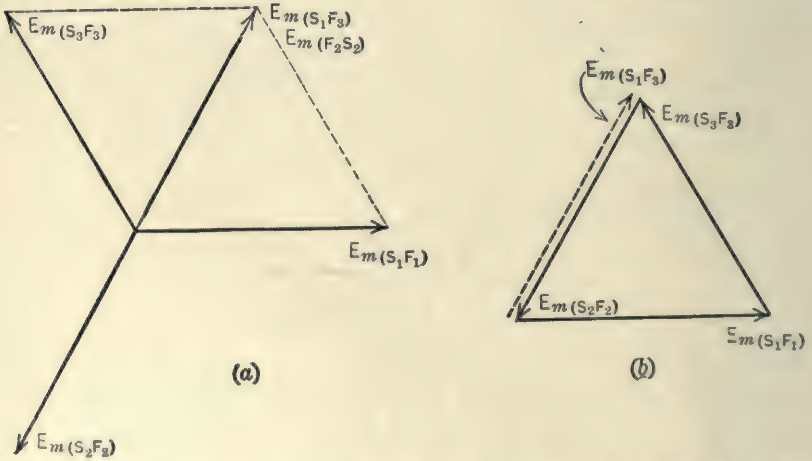


Fig. 74.—Showing why the voltage across the open ends S_1–F_2 of Fig. 73 is zero if the coils have been properly interconnected.

In making such a Δ-connection it is therefore advisable to leave one of the connections open until proper tests have been made. Consider then that the connection S_1F_2 has been left open, as in Fig. 73. If any resultant voltage exists within the delta, it will appear across the ends S_1 and F_2.

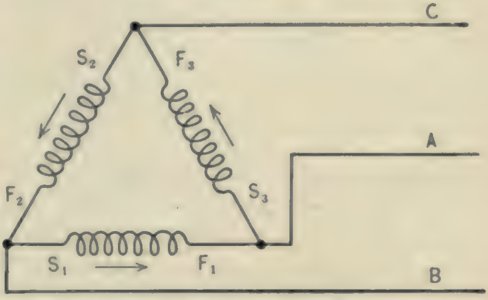


FIG. 75.—Showing how the delta-connected armature is connected to its load.

The voltage S_1F_2 will be the vector sum of voltages S_1F_1, S_3F_3, and S_2F_2. In carrying out this construction we first get the voltage of coils No. 1 and No. 3 in series, as shown in Fig. 74*a*; this is the voltage S_1F_3. We must now add to this resultant, the voltage of coil No. 2 (given in Fig. 74*a* as $E_{m(S_2F_2)}$), and evidently the sum of $E_{m(S_1F_3)}$ and $E_{m(S_2F_2)}$ is zero. The topographic diagram of Fig. 74*b* indicates at once that the resultant voltage within the delta is zero. If the resultant voltage across ends S_1 and F_2 is zero, they may be joined as in Fig. 72*b*, and no current will circulate within the delta.

If leads are brought out from the corners of the delta (marked A, B, and C), as in Figs. 71 and 75, there will be, across these three lines, three equal voltages 120° apart. It will be seen from Fig. 75 that the voltage impressed between lines A and B, that is, $E_{m(BA)}$, is that of phase No. 1, or $E_{m(S_1F_1)}$. The other two phases are, however, also acting between these two lines, so that we may say that $E_{m(BA)}$ is equal to the vector sum of $E_{m(F_2S_2)}$ and $E_{m(F_3S_3)}$. That is,

$$E_{m(BA)} = E_{m(S_1F_1)} = E_{m(F_2S_2)} \oplus E_{m(F_3S_3)}.$$

In a similar way it can be seen that

$$E_{m(CB)} = E_{m(S_2F_2)} = E_{m(F_3S_3)} \oplus E_{m(F_1S_1)},$$

and

$$E_{m(AC)} = E_{m(S_3F_3)} = E_{m(F_1S_1)} \oplus E_{m(F_2S_2)}.$$

These relations are further brought out by the construction of Fig. 76.

From Fig. 76 we see that, for the Δ-connection, phase and line voltages are numerically equal, i.e.,

$$E_m = e_m,$$

and

$$E = e, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Where E_m = maximum value of line voltage,
e_m = maximum value of phase voltage,
E = effective value of line voltage,
e = effective value of phase voltage.

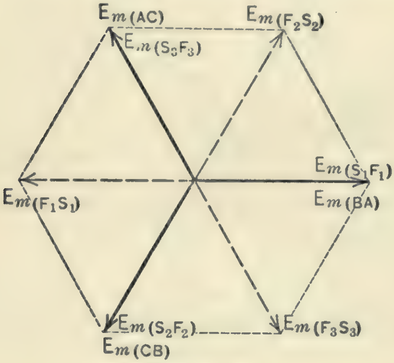


FIG. 76.

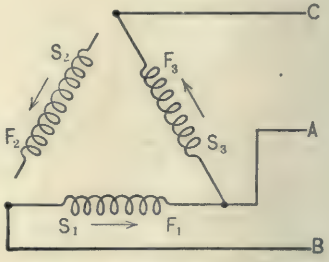


FIG. 77.

FIG. 76.—In a delta-connected armature the line voltages are the same as the phase voltages.

FIG. 77.—If only two phases are used, a so-called open-delta winding results.

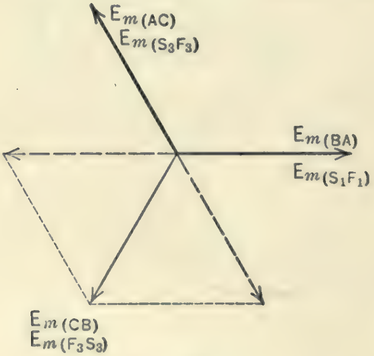


FIG. 78.

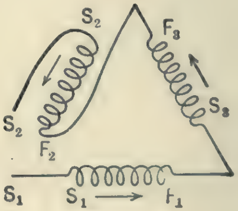


FIG. 79.

FIG. 78.—With the connection of Fig. 77, the line voltages are the same as though all three phases were used.

FIG. 79.—An incorrect delta-connection; the last coil has been connected to the others in reversed phase.

42. Open-delta, or V-connection.—If one of the phases of the Δ-connected alternator of Fig. 71 were disconnected, the machine would still be able to deliver three voltages 120° apart. Consider that only two phases, S_1F_1 and S_3F_3, are in use, ends S_1 and F_3 being still connected as in Fig. 77, and leads A, B, C, being brought out as before.

Voltages $E_{m(BA)}$ and $E_{m(AC)}$ would, as in the Δ-connection, be the same as voltages $E_{m(S_1F_1)}$ and $E_{m(S_3F_3)}$ respectively; voltage $E_{m(CB)}$, however, would be the vector sum of $E_{m(F_3S_3)}$ and $E_{m(F_1S_1)}$. This construction is carried out in Fig. 78 and indicates that the three line voltages are still equal and 120° apart.

From the similarity of its coil connection to the letter V, this connection is known as a V-connection; it is also known as an *open delta-connection*. It is never used in alternators, but is occasionally used in connecting up transformers. All that we have said regarding Δ- and Y-connections of the coils of alternators applies equally well to transformers, as we shall see later.

43. Incorrect Δ-connection.—If a mistake is made in connecting the three phases of an alternator in delta, and the error is not discovered before the machine is operated, the machine is likely to be damaged. Consider that in joining the phases we connected ends F_1 to S_3, F_3 to F_2, i.e., in connecting the phases in series, we joined phase No. 2 in reverse order, as in Fig. 79. The connection S_1S_2 is to be left open until proper voltage tests have been made.

The voltage S_1F_3 is, as before, the vector sum of the voltages S_1F_1 and S_3F_3 (Fig. 80). The voltage across the open ends, $E_{m(S_1S_2)}$, will then be the vector sum of voltages S_1F_3 and F_2S_2. These voltages are, however, equal and in phase; their vector sum, the voltage $E_{m(S_1S_2)}$, will therefore be the arithmetic sum of $E_{m(S_1F_3)}$ and $E_{m(F_2S_2)}$. Thus, if each phase of the alternator generated 100 volts, the voltage across the open ends, S_1 and S_2 would be 200 volts. If the ends S_1 and S_2 were to be joined, the resultant voltage of 200 volts would cause a current to circulate within the delta, which, because the impedances of the phases are small, would be so large that the machine winding would soon be burned out.

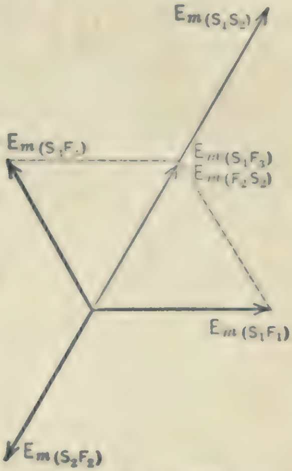


Fig. 80.—The voltage relations for the incorrect connection of Fig. 79.

It is therefore always advisable, before closing a Δ-connection, to read the voltage across the open ends with a voltmeter; if the connection is incorrect, the voltmeter will have a reading greater than the voltage per phase. If the connection has been correctly carried out, the voltmeter will read nearly zero.

PROBLEMS

1. A steamer makes a speed of 16 miles per hour heading northeast, while in an ocean current of 4½ miles per hour velocity in a direction due south. What is the actual velocity of the steamer? How fast is it moving north and how fast east?

2. Answer the same question if the steamer, heading in the same direction as before, reverses her propellers and gains a backward velocity of 6 miles per hour.

3. If, in Prob. 1, the captain is walking towards the left side of the bridge with a velocity of 3 miles per hour, what is his actual velocity? How fast is he moving north, and how fast east?

4. The coil of an armature generates a sine wave voltage of maximum value 16 volts. Three such coils 15 electrical degrees apart are connected in series. What will a voltmeter read, connected across the terminals of the group? The center one of the three coils is connected in backward; what will the voltmeter read?

5. There are two groups of coils on an armature, the groups being 90° apart. Each group of coils consists of 4 coils, 22½° apart. The maximum voltage generated by one coil is 18 volts. What will a voltmeter read if connected across the terminals of one group? What will be the maximum voltage of the winding, if the two groups are connected in series?

6. A two-pole, three-phase alternator has three coils per phase, the coils being 20° apart. Each coil generates a voltage of 10 volts (effective). What is the voltage of one phase? If the armature is Y-connected, what will be the line voltage?

7. In Prob. 6, phase No. 1 leads phase No. 2, which leads phase No. 3. The center coil of phase No. 1 has been connected in reversed. Answer the same questions as for Prob. 6.

8. If the machine of Prob. 6 is delta-connected, what will be the line voltage? If that of Prob. 7 is tested for voltage before making the last connection in the delta arrangement, what will the voltmeter read?

9. If the machine of Prob. 6 has had two groups of coils connected in series (preparing to make the delta connection), what might the voltmeter read across the ends of the two groups in series?

10. If, in Prob. 6, two of the phases have been connected to make the Y-connection, what might the voltmeter across the terminals of the winding read?

11. If, in making the delta connection for the machine of Prob. 6, the first two connections are made haphazard, what values of voltage might be read across the open points of the delta?

12. Voltage E_{1-2} is 90° away from voltage E_{3-4} and the two coils are connected in series. Where is the voltage E_{1-4} with respect to the voltage E_{1-2}?

13. There are 24 coils on the armature of a 4-pole, three-phase alternator. Each coil generates a maximum voltage of 100 volts. If the machine is Y-connected, what will be the line voltage?

CHAPTER III

THE SINGLE-PHASE CIRCUIT

44. Inductive Circuits.—In order that a circuit may possess self-inductance, or be inductive, it must be capable of setting up flux interlinkages; for the inductance to be appreciable, the circuit must be wound as a coil or be composed of loops of wire. We have seen that the self-inductance of a circuit may be expressed as in Eq. (29),

$$L=\frac{0.4\pi N^2\mu A}{l},$$

which indicates that the self-inductance of a circuit varies directly as the square of the number of turns, as the permeability of the medium, constituting the magnetic path, as the area of the magnetic path, and inversely as the length of the magnetic path

The unit of self-inductance is the *henry;* it may be defined in two ways. We may say that a circuit has unit self-inductance when a rate of change of one ampere per second induces a counter voltage of one volt. Or, we may say that a circuit possesses self-inductance of one henry if one ampere flowing through the circuit is capable of setting up 10^8 flux interlinkages.

It is often convenient to express the inductance of coils in subdivisions of the henry; the subdivisions used are the millihenry, or one-thousandth of a henry, and the microhenry, or one millionth part of a henry.

The fact that inductive circuits must be made of coils of wire at once indicates that "a circuit containing only inductance" is a hypothetical one. Every inductive circuit must evidently possess some resistance.

45. Examples of Inductive Circuits.—An example of the simplest form of inductance of commercial importance is the single-layer solenoid, a type of coil extensively used in radio receiving sets. Fig. 81 shows such a coil having an inductance of 1100 microhenries; the low value of inductance of the coil is primarily due to the few turns used, also, of course, to its comparatively small size and the absence of an iron core.

An air-core coil of 0.628 henry is shown in Fig. 82. It can be seen from the cut that there are many turns in this coil and that its diameter is much greater than that of Fig. 81. This coil has a resistance (by continuous-current measurement) of 10.5 ohms, so that its time constant is about 0.06 second. Seventy-five pounds of copper wire were required to make it.

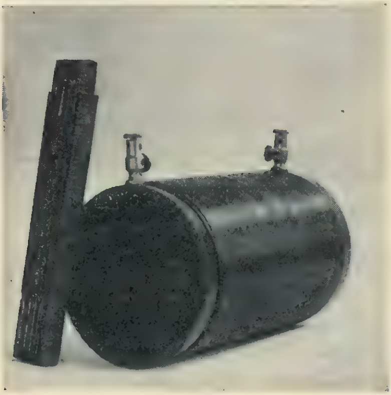


Fig. 81.

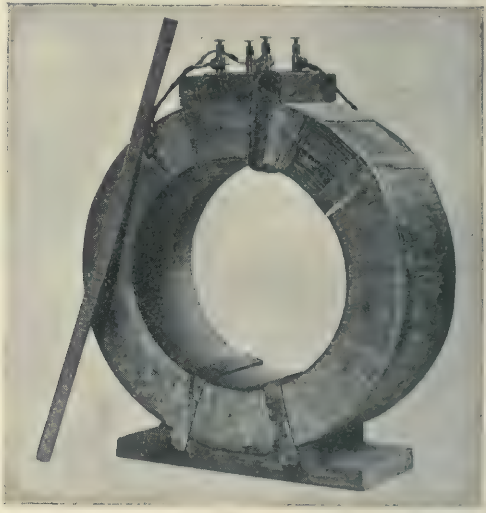


Fig. 82.

Fig. 81.—A single-layer solenoid, having 130 turns of No. 18 wire; it has about 1100 microhenries of inductance.

Fig. 82.—A large air-core coil having 30 layers with 50 turns per layer; it weighs 7 pounds, has a continuous-current resistance of about 10 ohms and inductance of 0.0628 henry

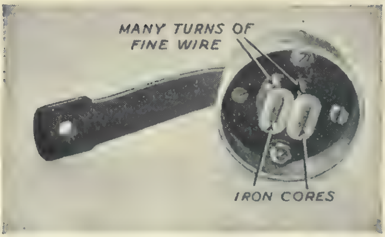


Fig. 83.

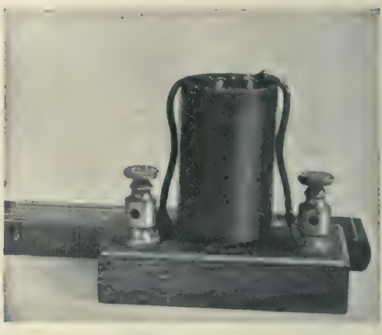


Fig. 84.

Fig. 83.—The magnets of a telephone receiver are wound with thousands of turns of No. 40 enameled wire; the inductance of a pair of magnets as shown here is about one henry.

Fig. 84.—By using many turns of fine wire and a magnetic circuit consisting entirely of iron the inductance may be made large; the coil has an inductance of 16 henries.

In Fig. 83 are shown the two small coils of a " watch-case " telephone receiver, such as is used in the ordinary head set. Although these coils are very small, their inductance is about 1 henry, more than that of the 75-pound coil of Fig. 82. The large inductance is due to the thousands of turns used, as well as to the fact that iron cores are used. The resistance of the wire used in these magnets is approximately 2000 ohms, so that the time constant of such an inductance is only 0.0005 second, about one thousandth that of the large coil.

The shunt-field circuit of large generators may have many henries of inductance, because of the many turns used, and because the magnetic path is almost completely through iron and this path is of large cross-section. In Fig. 84 is shown a small coil using a completely closed magnetic circuit; it is known as an ironclad coil. This is wound with many turns of No. 40 enameled wire and contains 16 henries of inductance.

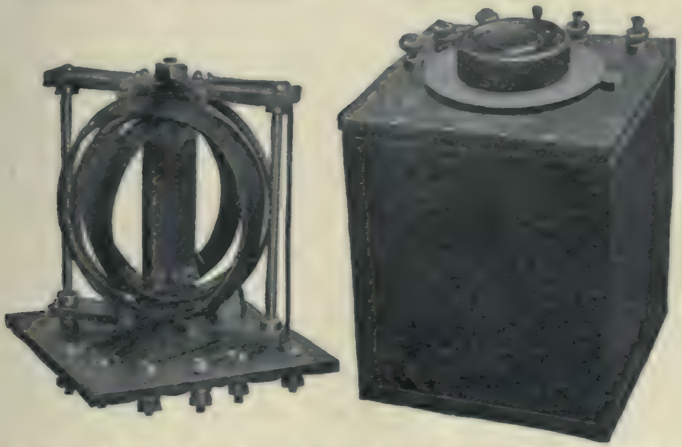


Fig. 85.—Here is shown a variable inductor for use primarily in a radio circuit. A rotary coil on a shaft, fitting inside a stationary coil, is in series with the stationary one. The inductance varies because of the variation of mutual induction.

Sometimes it is desired to have a continuously variable inductance; in Fig. 85 such a one is shown. It is made up of two parts, one rotatable inside the other, the two parts being electrically connected in series. The inductance of such a construction varies because of the change in the mutual induction as the inner coil is turned. The total variation in self-induction of such a *variometer* (as it is frequently called) is about ten to one.

It will be noticed that in all these inductances the resistance of the circuit is inseparably connected with the inductance; the two constants of the circuit, R and L, cannot be separated because, for a given weight of copper, the resistance increases as the inductance increases. For a given weight of copper, wound in a certain size coil, the ratio of resistance to inductance is independent of what size of wire is used. A No. 36 wire, for example, would give much more inductance than if the same weight of copper was used to wind the coil with No. 18 wire, but the ratio of resistance to inductance of the coil will be the same for the fine as for the coarse wire.

46. Circuit Containing Inductance and Resistance.—We have already seen how circuits containing resistance only, inductance only, or capacitance only, behave when an alternating voltage is impressed upon them. To consider the action of a circuit containing inductance and resistance, we shall consider that a pure resistance is placed in series with a pure, air-core inductance, as in Fig. 86. Such a circuit is an imaginary one, in that we cannot have a pure inductance; the resistance would actually be inseparably associated with the inductance.

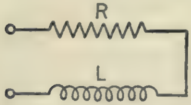


FIG. 86.—An imaginary circuit consisting of an inductance-less resistor in series with a resistance-less inductor.

When a voltage is impressed upon the series circuit of Fig. 86, a current will flow, and it is evident that the current must have the same value at any instant in every portion of the circuit; whatever the current value is in one part of the circuit at a given instant, it will be the same in any other portion. If this be not true, there will be an increasing accumulation of electrons at some points of the circuit, or *vice versa;* this is manifestly impossible.

Let us assume that the resulting current is of sine form, as shown by the dotted curve of Fig. 87; let us represent it by the equation $i = I_m \sin 2\pi ft$. We have seen (p. 25) that when such a current flows through a pure resistance, it will develop a resistance reaction

$$e_R = -iR = -I_mR \sin 2\pi ft.$$

This voltage wave is of sine form and must be opposite in phase to that of the current, or lie 180° behind the current, as is represented by the dashed curve in Fig. 87.

It has also been shown (p. 32) that when a current, $i = I_m \sin 2\pi ft$, flows through an inductance, it will develop an inductance reaction

$$e_L = -L\frac{di}{dt} = -2\pi fLI_m \cos 2\pi ft = -I_mX_L \cos 2\pi ft.$$

This inductance reaction, also of sine form, will lie 90° behind the current, as is represented by the dash-dot curve of Fig. 87.

There are thus two separate reactions produced by the current in the circuit of Fig. 86, a resistance reaction and an inductance reaction. From what has been said about reactions in an alternating-current circuit in Chapter I, it is evident that the total reaction offered by the circuit at any instant will be the algebraic sum of the instantaneous values of the two separate reactions, at the instant considered.

In constructing Fig. 87 we have assumed that the values of frequency, self-induction and resistance are such as to make the resistance equal to the reactance, that is, equal to $2\pi fL$. It follows that the maximum values of the resistance reaction and the inductance reaction are equal, as is indicated in Fig. 87. If the corresponding values of the two reactions are now added, as suggested in the previous paragraph, the wave of the total reaction of the circuit will be obtained; it is shown by the broken-line curve. It will be found that the wave of total reaction is also of sine form, and (for this case in which R and $2\pi fL$ have been assumed equal), lies 135° behind the current, or the current may be said to lead the reaction by 135°.

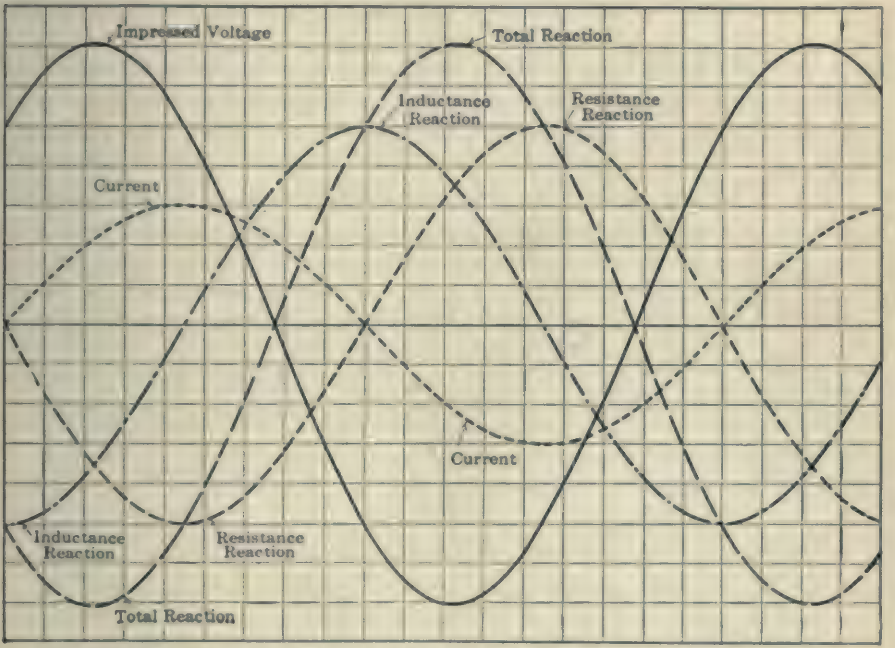


FIG. 87.—The forms and phases of the various reactions in an inductive circuit.

The wave of impressed voltage must lie 180° from, or be opposite to, the wave of total reaction, since these two voltages must be equal and opposite to each other at every instant; the wave of impressed voltage is shown in full line in Fig. 87. The total reaction being of sine form, it follows that the impressed voltage must also be of sine form; it may in this case be represented by the equation $e = E_m \sin\ (2\pi ft + 45°)$. As the current leads the total reaction by 135°, it must lag 45° behind the impressed voltage.

Examination of Fig. 87 will show that the relative position and maximum value of the total reaction will depend only upon the relative maximum values of the two separate reactions. As the maximum value of either reaction increases relatively to that of the other reaction, the maximum value of the total reaction will go more and more into phase with that reaction, the maximum value of which is increasing. In Fig. 87 it was assumed that the resistance and inductance reaction had equal maximum values, say 100 units. The maximum value of the total reaction must then occur at a point midway between the maximum values of like sign of the separate reactions. At this instant each separate reaction has an instantaneous value of 70.7 units, and the total reaction a value of 141.4 units. The effective value of both impressed voltage and total reaction would be $141.4 \times 0.707 = 100$ volts.

Had we started our analysis by assuming a sine wave of impressed voltage, $e = E_m \sin 2\pi ft$, we would at once have plotted a sine wave of total reaction, equal and opposite at every instant to the wave of impressed voltage. This wave of total reaction would be made up of two sine-wave components, a resistance reaction and an inductance reaction, 90° apart from each other. If the maximum values of the two component waves are assumed equal, one would have to lie 45° ahead of the total reaction wave, and the other 45° behind it. The current, of sine-wave form, would then lie opposite the wave of resistance reaction, or opposite that component lying 45° behind the total reaction, inasmuch as we have already seen that in a circuit containing inductance, the current must lag behind the impressed voltage by some angle between 0° and 90°. The current would be expressed by the equation, $i = I_m \sin (2\pi ft - 45°)$.

In this second method of analyzing this problem, we have assumed the two components of the total reaction to be each a sine wave. For two component reaction waves to yield a sine wave of total reaction, they must either both be sine waves or both suitably distorted waves; if one component is a sine wave and the other is distorted, they cannot yield a sine wave of total reaction. For both of them to be distorted it would be necessary for the current to have such a distorted form that it would produce a distorted wave of resistance reaction, and by its rate of change, a distorted wave of inductance reaction, these two being of such form that their sum, the total reaction, is a sine wave. This is impossible unless the resistance and inductance of the circuit change during the cycle in some very complicated manner.

47. Voltage Diagram for a Circuit Containing Resistance and Inductance.—The current being the same in all parts of a series circuit, it is most convenient to use it as the reference vector. We may then draw a vector, I_m, proportional to the maximum value of the current, as in Fig. 88; a vector equal to the maximum value of the resistance reaction, $-I_mR$, is then drawn 180° from the current, and one equal to the inductance reaction, $-2\pi fLI_m = -I_mX_L$, 90° behind the current. In Fig. 88 the vectors representing the resistance and inductance reactions are drawn equal, as was assumed for Fig. 87. The maximum value of the total reaction is then the vector sum of the two reactions; we shall call the total reaction $-I_mZ$. Equal and opposite to the total reaction is drawn the maximum value of the impressed voltage, E_m. It will be seen that the impressed voltage leads the current by 45°, as was the case in Fig. 87.

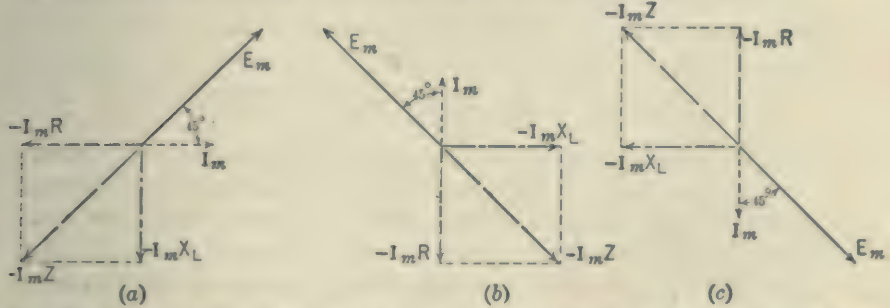


FIG. 88.—Vector diagram for an inductive circuit; the three positions for the rotating vectors represent different instants in one cycle.

In Fig. 88*a*, conditions corresponding to time $=0$ of Fig. 87 are represented. By taking the vertical projections of all the vectors, the instantaneous values of all quantities may be obtained. In Figs. 88*b* and *c*, conditions corresponding to times $\pi/2$ and $3\pi/2$, respectively, of Fig. 87 are indicated.

It will be seen from the construction of Fig. 88 that, as the impressed voltage is equal and opposite to the total reaction, the impressed voltage may be considered as made up of two components which overcome the two separate reactions. It is generally more convenient to draw these components of the total impressed voltage directly. Thus in Fig. 89 the component of the impressed voltage which overcomes the resistance reaction,

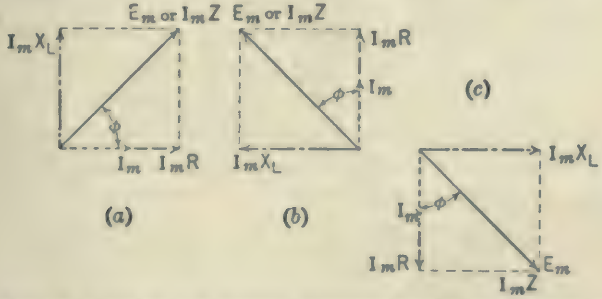


FIG. 89.—Instead of drawing the vectors representing the reactions of a circuit, it is more convenient to draw vectors showing the components of the impressed force (called "drops") which overcome the reactions.

$-I_mR$, has been drawn in phase with the current; it is marked I_mR and is known as the resistance drop. Similarly, the component of the impressed voltage which overcomes the inductance reaction is drawn 90° ahead of the current; it is marked I_mX and is known as the inductance drop, or inductive reactance drop.

The vector sum of the I_mR and I_mX drops is called the *impedance drop*, I_mZ; in the case assumed, there being no other reactions for the impressed voltage to overcome, the I_mZ drop is the impressed voltage. We have then the obvious relationship,

$$E_m = I_mZ = \sqrt{(I_mR)^2 + (I_mX)^2} = \sqrt{(I_mR)^2 + (2\pi f L I_m)^2}. \quad . \quad . \quad (50)$$

The three parts of Fig. 89 are drawn to correspond with the three instants of time chosen in Fig. 88; instantaneous values of the current, the impressed voltage, and its components are found by obtaining the vertical projections of the various vectors. In Fig. 92 the curves of IR, IX_L and impressed voltage are plotted in their correct phase position with respect to the current.

48. Voltage Diagram with Effective Values.—In problems dealing with a.c. circuits, the voltages and currents involved are always expressed in effective values; we are generally interested in knowing their effective values and their relative phases, or the angles they make with each other, but rarely are we concerned with their instantaneous values.

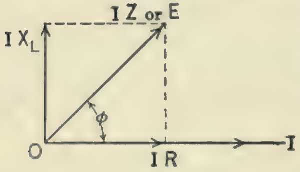


FIG. 90.—Instead of using maximum values of the various drops the effective values are generally employed.

Since the effective value of any sine wave is equal to 0.707 of its maximum value, if we multiply the vectors of Fig. 89 by 0.707, we will have a diagram as in Fig. 90, which expresses the quantities involved in effective values, and indicates their relative phase positions. The same result is obtained if all the terms of Eq. (50) are multiplied by 0.707, i.e.,

$$E = IZ = \sqrt{(IR)^2 + (IX)^2} = \sqrt{(IR)^2 + (2\pi f L I)^2}. \quad . \quad . \quad . \quad (51)$$

The effective voltages represented by IR, IX_L and IZ are what a voltmeter would read if applied to the hypothetical circuit of Fig. 86, across the resistance only, the inductance only, or the entire circuit, respectively, since all a.c. meters are calibrated to read in effective values. (It is to be remembered, as was mentioned before, that actually a coil cannot be wound without resistance, so that a drop, IX_L, cannot be measured. Actually the IZ drop of the inductance coil is the only quantity which can be measured by voltmeter.)

It is to be pointed out that vertical projections of the vectors of Fig. 90

will indicate instantaneous values of their respective voltages only if these projections are multiplied by 1.414 (or divided by 0.707). For this reason Fig. 90, when rotated, is intended to indicate only the relative magnitudes and the phase relationship of the various vectors, that is, the order in which the various vectors pass through corresponding values, as positive maxima, and the time interval between these corresponding values.

In Figs. 88 and 89 the vectors are all drawn to the same relative scales as the maximum values of the curves of Fig. 87; it is, of course, necessary that all the various voltage vectors in either Figs. 88 or 89 be drawn to the same scale. However, as the current vector is introduced only as a reference (all the voltage vectors having their phase relationships determined with respect to it), it may be drawn to any scale whatever, or only its position may be indicated, as in Fig. 90.

49. Impedance Diagram for a Circuit Containing Resistance and Inductance.—The relation between the various voltages in the vector diagram of Fig. 90 was given in Eq. (51) as

$$E=IZ=\sqrt{(IR)^2+(IX_L)^2}=\sqrt{(IR^2+(2\pi fLI)^2},$$

which equation may be rewritten as follows:

$$E=IZ=I\sqrt{R^2+X_L^2}=I\sqrt{R^2+(2\pi fL)^2}. \quad . \quad . \quad . \quad . \quad . \qquad (52)$$

If the last equation is divided through by the current, I, we have

$$\frac{E}{I}=Z=\sqrt{R^2+X_L^2}=\sqrt{R^2+(2\pi fL)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (53)$$

or

$$I=\frac{E}{Z}=\frac{E}{\sqrt{R^2+X_L^2}}=\frac{E}{\sqrt{R^2+(2\pi fL)^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (54)$$

Equation (52) indicates that, in a circuit containing resistance and inductance only, the impressed force is balanced by an impedance reaction, IZ, which is the product of the current and the *impedance*, Z. The impedance is dependent upon both resistance and reactance, and, as stated above, is given by the equation

$$Z=\sqrt{R^2+X_L^2}=\sqrt{R^2+(2\pi fL)^2}. \quad . \quad . \qquad (55)$$

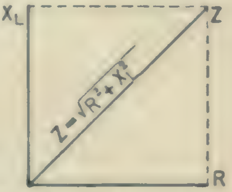


FIG. 91.—The impedance of a circuit may be obtained by properly combining the resistance and reactance as shown here.

Equation (55) indicates that to obtain the impedance, resistance and reactance must be combined at right angles to each other, the operation being indicated by the scalar diagram of Fig. 91. This figure may be said to have been obtained by dividing the IR, IX_L, and IZ drops of Fig. 90 by the current. It follows that in the impedance diagram of Fig. 91, the inductive reactance, X_L, should be drawn 90° ahead of the resistance to make this diagram consistent with the others we have used.

If the angle between current and voltage is designated as ϕ, it will be seen that from the voltage diagram (Fig. 90)

$$\left.\begin{aligned} \cos\phi &= \frac{IR}{IZ} = \frac{IR}{\sqrt{(IR)^2+(IX_L)^2}}, \\ \sin\phi &= \frac{IX_L}{IZ} = \frac{IX_L}{\sqrt{(IR)^2+(IX_L)^2}}, \\ \tan\phi &= \frac{IX_L}{IR} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (56)$$

and from the impedance diagram (Fig. 91)

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2+X_L{}^2}}, \quad \sin\phi = \frac{X_L}{Z} = \frac{X_L}{\sqrt{R^2+X_L{}^2}}, \quad \tan\phi = \frac{X_L}{R}. \quad (57)$$

50. Power in a Circuit Containing Resistance and Inductance.—We have seen in Chapter I that the product of a current and a voltage which is 90° away from the current, represents zero power, and that the product of a current and a voltage which is in phase with the current represents real power, measured in watts. Thus in Fig. 90 the product of the IX_L drop and the current, I, that is, $(IX_L)I$, cannot represent any real power. The product of the IR drop and the current, I, that is, $(IR)I$, or I^2R, must however indicate the actual power used in the circuit. It will be realized that in the circuit selected, one containing a pure inductance in series with resistance, no power can be dissipated in the inductance; whatever power is expended in the circuit can only be the I^2R loss in the resistance.

From Fig. 90 we have, since $IR = E\cos\phi$,

$$\text{Power} = P = (IR)\times I = EI\cos\phi, \quad . \quad . \quad . \quad . \quad . \quad (58)$$

where $\cos\phi$, the cosine of the angle between the current and the voltage, is called the *power factor;* it may also be defined as a number which, when multiplied by the *volt-amperes* (product of volts and amperes) of a circuit, gives the power in the circuit.

In Fig. 92 the curves of current, IR drop, IX_L drop, and impressed voltage, corresponding to the vector diagram of Fig. 89, are plotted. In Fig. 93 the corresponding volt-ampere curves are shown, being obtained by plotting instantaneous products of current and IR drop, IX_L drop, and impressed voltage, respectively. It will be seen that the volt-ampere

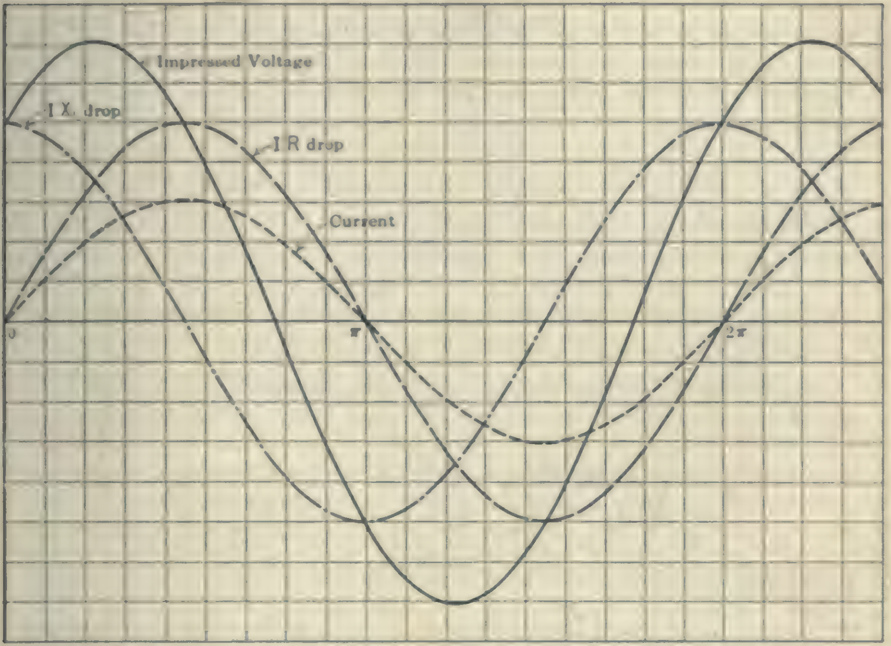


FIG. 92.—Curves of drops and impressed voltage in an inductive circuit.

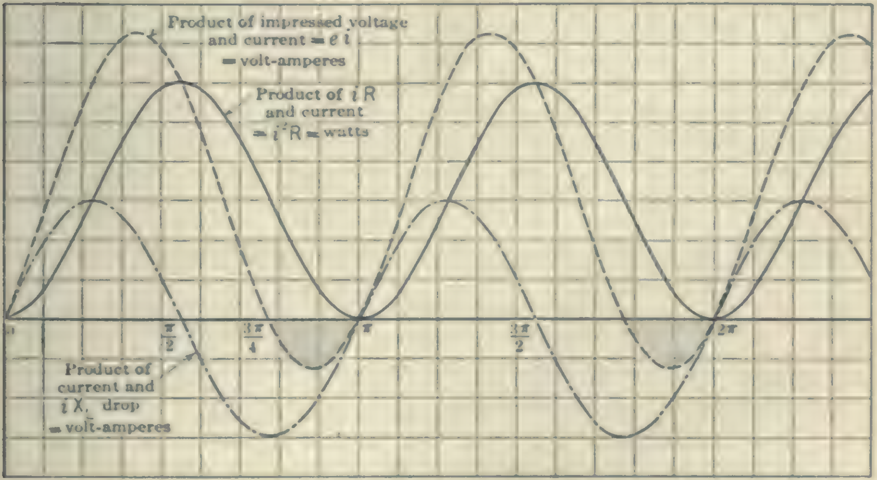


FIG. 93.—Showing curves of the volt-amperes of an inductive circuit; the active volt-amperes (or watts) are at no time negative whereas the reaction volt-amperes are as much negative as positive.

curve representing the product of instantaneous values of current and IX_L drop (dot-dash curve) has equal positive and negative loops, indicating that on the average the inductance dissipates no power. The curve representing the product of instantaneous values of current and IR drops (shown in full line), lies entirely above the axis of reference, indicating that all the power is positive, i.e., all of it is being dissipated. The volt-ampere curve representing the product of instantaneous values of current and impressed voltage (shown in broken line), has both positive and negative loops, the former being much larger than the negative ones. The difference in the areas of the positive and negative loops should represent the energy dissipated by the entire circuit, and should also be equal to the area under the full-line curve (product of current and IR).

If the areas of the loops of the curves of Fig. 93 were measured by planimeter over one-half cycle, i.e., from 0 to π, we should find that

Area under $i(iR)$ curve, (i^2R) $= 1.12$ sq. in.
Area of each loop of $i(iX_L)$ curve $= 0.36$ sq. in.
Area of positive loop of ei curve $= 1.20$ sq. in.
Area of negative loop of ei curve $= 0.08$ sq. in.

The above figures indicate that the area under the i^2R curve is equal to the difference of the areas of the positive and negative loops of the ei curve.

The curves of Fig. 93 indicate that, for the relative values of R and X assumed in constructing this figure, from time $= 0$ to time $= \frac{\pi}{2}$ both the resistance and the inductance are drawing energy from the source. From time $= \frac{\pi}{2}$ to time $= \pi$, the inductance gives up energy, all of which the resistance absorbs from time $= \frac{\pi}{2}$ to time $= \frac{3\pi}{4}$, as is evident from the fact that up to the time $\frac{3\pi}{4}$ the generator is still furnishing energy to the circuit (ei curve positive). From time $= \frac{3\pi}{4}$ to time $= 2\pi$, the inductance is giving up more energy than the resistance can dissipate, so that the surplus energy is given back to the source.

It is interesting to note that the area under one loop of the i^2X_L curve (0.36 sq. in.) if changed into the proper units, gives the maximum energy stored in the coil as $\frac{1}{2}Li^2$.

To determine by the calculus an expression for the average power when a voltage of the form $E_m \sin \omega t$ is impressed on a circuit containing resistance and inductance only, and a current of the form $I_m \sin (\omega t - \phi)$ results, the current lagging ϕ degrees behind the voltage, we need only evaluate the expression

$$\begin{aligned}
\text{Average power} &= \frac{1}{\pi}\int_0^\pi E_m \sin \omega t \times I_m \sin (\omega t - \phi) d(\omega t) \\
&= \frac{1}{\pi}\int_0^\pi E_m \sin \omega t \times I_m(\sin \omega t \cos \phi - \cos \omega t \sin \phi) d(\omega t) \\
&= \frac{1}{\pi}\int_0^\pi E_m \sin \omega t \times I_m(\sin \omega t \cos \phi) d(\omega t) \\
&\quad - \frac{1}{\pi}\int_0^\pi E_m \sin \omega t \times I_m(\cos \omega t \sin \phi) d(\omega t) \\
&= \frac{E_m I_m \cos \phi}{\pi}\int_0^\pi \sin^2 \omega t \, d(\omega t) \\
&\quad - \frac{E_m I_m \sin \phi}{\pi}\int_0^\pi \sin \omega t \cos \omega t \, d(\omega t).
\end{aligned}$$

Since $\int_0^\pi \sin \omega t \cos \omega t \, d(\omega t) = 0$ and since $\int_0^\pi \sin^2 \omega t \, d(\omega t) = \frac{\pi}{2}$ (page 16) we have that

$$P = \frac{E_m I_m \cos \phi}{2} = \frac{E_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos \phi = EI \cos \phi,$$

where E and I are the effective values of the voltage and current respectively, and $\cos \phi$ is the power factor.

51. Circuit Containing Several Resistances and Inductances in Series.—When current flows through a circuit containing several resistances and inductances in series, the IR drops will all be in phase with the current, and all the IX_L drops will be 90° ahead of the current. We may write then,

$$E = \sqrt{(IR' + IR'' + \ldots)^2 + (IX'_L + IX''_L + \ldots)^2}. \quad . \quad . \quad (59)$$

$$E = IZ = I\sqrt{(R' + R'' + \ldots)^2 + (X'_L + X''_L + \ldots)^2}. \quad . \quad . \quad (60)$$

$$Z = \sqrt{(R' + R'' + \ldots)^2 + (X'_L + X''_L + \ldots)^2}. \quad . \quad . \quad . \quad . \quad . \quad (61)$$

$$\cos \phi = \frac{R' + R'' + \ldots}{Z}, \quad \sin \phi = \frac{X'_L + X''_L + \ldots}{Z}. \quad . \quad . \quad . \quad . \quad . \quad (62)$$

52. Iron-core Inductance. Effective Resistance.—Whenever iron is used as a part of a magnetic circuit, and the flux is varied, hysteresis and eddy-current losses will occur in the iron. When an alternating current flows through an iron-core inductance, the flux will vary continuously with time, and a certain amount of power supplied to the circuit will be converted into heat.

The hysteresis loss for ordinary iron may be expressed, according to Steinmetz' empirical law, by

$$W_h = \eta f V B_m^{1.6}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (63)$$

where η = the hysteresis constant, or the loss per unit volume or weight, per cycle;
f = frequency in cycles per second;
V = volume or weight, depending on how η is expressed;
B_m = maximum flux density attained during the cycle.

The eddy-current loss for any material may be expressed by

$$W_e = \epsilon V t^2 f^2 B_m^2, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (64)$$

where ϵ = the eddy-current constant, or the loss per unit volume or weight, per cycle;
V = the volume or weight, depending on how ϵ is expressed;
t = thickness of laminations, in units corresponding to those of V;
f = frequency in cycles per second;
B_m = maximum flux density attained during the cycle.

The total watts dissipated by an iron-core inductance will then be

$$W = I^2 R_0 + W_h + W_e, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (65)$$

where R_0 is the *ohmic*, or *conductor*, resistance, or resistance as determined by continuous-current measurement. (The term *ohmic* resistance is not a good one, but is much used; it seems that *conductor* resistance is a better term.)

In case high frequency is used, even the conductor resistance itself is different from the value determined by continuous-current measurements. As the frequency increases, there is an increasing tendency for the current to use only the outer part of the conductor cross-section, resulting in a resistance increasing with frequency; this increase in resistance is said to be due to the *skin effect*.

It is evident, then, that the average rate at which heat energy is dissipated when an alternating current flows through an iron-core inductance, is not the same as the square of the effective value of the current multiplied by the resistance of the winding as determined by a continuous-current measurement, but is greater than this. In order to take account of hysteresis and eddy-current losses in our equations, we may consider that the iron-core inductance has an *effective*, *equivalent*, or *apparent* resistance, R, *such that this effective resistance multiplied by the square of the effective value of the current gives the true average rate at which heat energy is dissipated*, i.e.

$$\text{Watts} = I^2R,$$

or,

$$R \text{ (effective resistance)} = \frac{\text{watts}}{I^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

The effective resistance is only in very exceptional cases less than conductor resistance, or resistance by c.c. measurement. If there are no losses but that due to the resistance of the winding, the effective resistance is equal to the conductor resistance, but if other losses, such as iron loss (or others to be mentioned later) are present, the effective resistance may be much greater than this conductor resistance.

Inasmuch as hysteresis and eddy-current losses are dependent upon frequency, flux density, quality of iron, etc., it is obvious that the effective resistance of a circuit may vary as any or all of these quantities vary. The only way in which the effective resistance of a circuit, or part of a circuit, may be determined is by means of Eq. (66), the power dissipated in the circuit, or part thereof, being measured by a wattmeter, an instrument designed for this purpose.

Whenever we speak of the resistance of an alternating-current circuit we shall mean the effective resistance.

53. Example. — An air-core inductance coil ($L = 0.03$ henry, and resistance, $R_L = 2.0$ ohms), is placed in series with a non-inductive resistance ($R_R = 5.0$ ohms), across a 110-volt, 60-cycle source of power (Fig. 94). It is desired to know the current which will flow, the voltage across each part of the circuit, the power factor of the coil and of the entire circuit, and the power dissipated in the entire circuit and in each part of it.

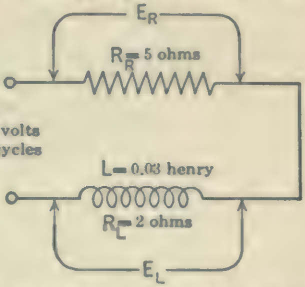


FIG. 94.—A resistance of 5 ohms in series with an inductance of 0.03 henry and 2 ohms resistance. Whereas this 2 ohms will be considered with the 5 ohms, it is to be remembered that actually this 2 ohms resistance cannot be dissociated from the coil.

The resistance, R, of the entire circuit is evidently the sum of the resistances of the two parts of the circuit, i.e.,

$$R = R_R + R_L = 2.0 + 5.0 = 7.0 \text{ ohms}.$$

The reactance, X_L, of the entire circuit, is that of the coil,

$$X_L = 2\pi fL = 2\pi \times 60 \times 0.03 = 377 \times 0.03 = 11.31 \text{ ohms}.$$

The impedance of the entire circuit is then (Fig. 95).

$$Z=\sqrt{(R_R+R_L)^2+(X_L)^2}=\sqrt{7^2+11.31^2}=\sqrt{49+127.9}=13.30 \text{ ohms.}$$

The impedance, Z_L, of the coil alone is

$$Z_L=\sqrt{R_L{}^2+X_L{}^2}=\sqrt{2^2+11.31^2}=\sqrt{4+127.9}=11.48 \text{ ohms.}$$

When the circuit is placed across a 110-volt, 60-cycle source of power, the current, I, will be

$$I=\frac{E}{Z}=\frac{110}{13.30}=8.27 \text{ amperes.}$$

The voltage across the resistance, $E_R=IR_R$, will be $8.27\times5=41.35$ volts, and that across the coil, $E_L=IZ_L$, will be $8.27\times11.48=94.94$ volts.

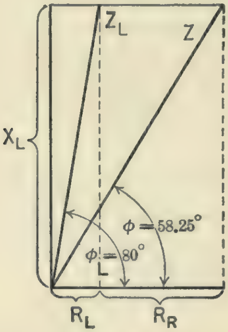


FIG. 95.

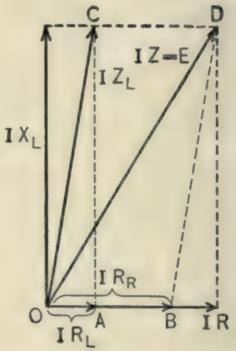


FIG. 96.

FIG. 95.—The impedance diagram of the circuit given in Fig. 94.
FIG. 96.—The voltage diagram for the circuit of Fig. 94.

These voltages will be measured by a voltmeter placed across the resistance and the inductance coil, respectively.

The total resistance drop of the circuit is $IR=IR_R+IR_L=8.27\times2+8.27\times5=16.54+41.35=57.89$ volts. The reactance drop of the circuit is $IX_L=8.27\times11.31=93.53$ volts. *These voltages cannot be obtained by any voltmeter measurements*, since, as noted before, the resistance and inductance of a coil cannot be dissociated one from the other. When combined at right angles, the total IR drop and the IX drop yield the impressed voltage (Fig. 96).

$$E=\sqrt{(IR)^2+(IX_L)^2}=\sqrt{57.89^2+93.53^2}$$
$$=\sqrt{3351+8748}=\sqrt{12099}=110 \text{ volts.}$$

The power factor of the entire circuit is, from the impedance diagram (Fig. 95),

$$\cos\phi=\frac{R_L+R_R}{Z}=\frac{7}{13.30}=0.5263,$$

and from the voltage diagram (Fig. 96),

$$\cos\phi=\frac{IR_R+IR_L}{E}=\frac{16.54+41.35}{110}=\frac{57.89}{110}=0.5263,$$

from which

$$\phi=58.25°.$$

The power factor of the coil alone is

$$\cos\phi_L=\frac{R_L}{Z_L}=\frac{2.0}{11.48}=0.1742,$$

or,

$$\cos\phi_L=\frac{IR_L}{E_L}=\frac{16.54}{94.94}=0.1742,$$

from which

$$\phi_L=80°.$$

The power dissipated by the entire circuit is then

$$P=EI\cos\phi=110\times8.27\times0.5263=478.8\text{ watts},$$

or,

$$P=I^2(R_R+R_L)=8.27^2\times7.0=478.8\text{ watts}.$$

The power dissipated in the inductance coil is

$$P_L=E_LI\cos\phi_L=94.94\times8.27\times0.1742=136.8\text{ watts},$$

or,

$$P_L=I^2R_L=8.27^2\times2.0=136.8\text{ watts}.$$

The power dissipated in the resistance, R_R, is

$$P_R=E_RI=41.35\times8.27=342.0\text{ watts},$$

or,

$$P_R=I^2R_R=8.27^2\times5.0=342.0\text{ watts}.$$

Evidently

$$P=P_R+P_L=342.0+136.8=478.8\text{ watts}.$$

54. Circuit Containing Capacitance and Resistance.—When a sine wave of voltage is impressed upon a series circuit containing resistance and capacitance, as in Fig. 97, a current will flow which will be of sine form; let us represent it by the equation, $i=I_m\sin 2\pi ft$. This current, flowing through the resistance, develops a resistance reaction

$$e_R=-iR=-I_mR\sin 2\pi ft,$$

which lies 180° behind the current. Obviously there will be a component

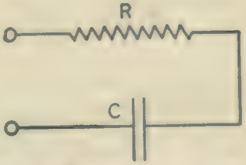


FIG. 97.—A supposedly perfect condenser in series with a pure resistance.

of the impressed voltage in phase with the current which overcomes this reaction; it is represented in Fig. 101 by the dotted curve.

We have seen (page 42) that if a sine wave of voltage, $e=E_m \sin 2\pi ft$, is impressed upon a capacitance, a cosine wave of current, $i=I_m \cos 2\pi ft$, leading the voltage wave by 90°, results. This current produces a capacitance reaction, the equation of which is

$$e_C=-\frac{I_m}{2\pi fC} \sin 2\pi ft=-I_m X_C \sin 2\pi ft.$$

If however, the current is assumed as being a sine wave, i.e., $i=I_m \sin 2\pi ft$, the wave of impressed voltage would have to be a cosine wave, which, in order to have the current lead by 90°, would have to be represented by the equation

$$e=-E_m \cos 2\pi ft,$$

and the capacitance reaction would have to be expressed by

$$e_C=\frac{I_m}{2\pi fC} \cos 2\pi ft=I_m X_C \cos 2\pi ft,$$

which leads the current by 90°.

The current which flows through the circuit of Fig. 97 is represented by the dotted curve in Fig. 101, the IR drop, or component of the impressed voltage which overcomes the resistance reaction, by the dashed curve, and the IX_C drop, or the component of the impressed voltage which overcomes the capacitance reaction, by the dash-dot curve. In constructing this figure, we have assumed that the values of frequency, capacitance, and resistance, are such as to make the resistance equal to the reactance, that is, equal to $\frac{1}{2\pi fC}$. The impressed voltage is then shown by the solid-line curve; it is found to lag 45° behind the current and may be represented by the equation

$$e=E_m \sin (2\pi ft-45°).$$

55. Voltage Diagram for a Circuit Containing Resistance and Capacitance.—The current being the same in all parts of the series circuit, it is again used as the reference vector. In Fig. 98 a vector I_m is drawn proportional to the maximum value of the current of Fig. 101. The vector representing the maximum value of the resistance reaction, $-I_mR$, is drawn 180° from the current, and the vector representing the maximum value of the capacitance reaction, $-I_mX_C$, is drawn 90° ahead of the current. The vector sum of $-I_mR$ and $-I_mX_C$ yields the impedance reaction, $-I_mZ$.

There being no other reactions present in the circuit, the impressed voltage is drawn equal and opposite to the impedance reaction.

In Fig. 99 the corresponding vector diagram in effective values is shown; the current being only a reference vector, its position only is indicated. The IR drop is then drawn in phase with the current, and the IX_C drop 90°

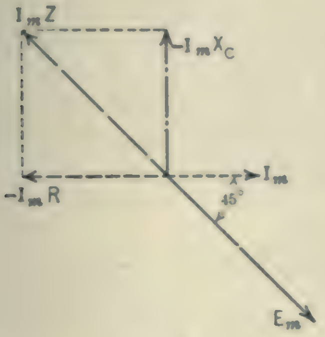


FIG. 98.

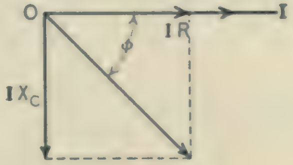


FIG. 99.

FIG. 98.—Vector diagram of reactions and impressed voltage for the circuit of Fig. 97.

FIG. 99.—Instead of using maximum voltages in constructing vector diagrams, it is customary to use effective values. This diagram gives the two components of the impressed force which is required to overcome the reactions of Fig. 98.

behind the current; the vector sum of IR and IX_C yields the impressed voltage. This relation may also be expressed by the equation

$$E = IZ = \sqrt{(IR)^2 + (IX_C)^2} = \sqrt{(IR)^2 + \left(\frac{I}{2\pi fC}\right)^2}, \quad . \quad . \quad . \quad (67)$$

which may also be written

$$E = IZ = I\sqrt{R^2 + X_C^2} = I\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

If the last equation is divided through by the current, I, we have

$$\frac{E}{I} = Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

or

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + X_C^2}} = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

56. Impedance Diagram for a Circuit Containing Resistance and Capacitance.—Equation (69) indicates that, to obtain the impedance, it is necessary to combine the resistance and reactance at right angles, the operation being indicated by the scalar diagram of Fig. 100. This figure may be said to have been obtained by dividing the IR, IX_C, and IZ drops of Fig. 99 by the current. It follows that, in the impedance diagram of Fig. 100, the capacitive reactance, X_C, should be drawn 90° behind the resistance, to make the diagram consistent with the others we have used.

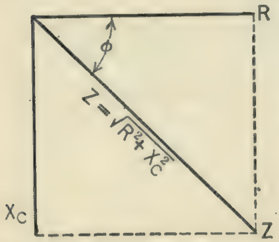


Fig. 100.—The impedance diagram for the circuit of Fig. 97.

If the angle between current and voltage is designated as ϕ, it will be seen that, from the voltage diagram (Fig. 99).

$$\left.\begin{aligned}\cos\phi&=\frac{IR}{IZ}=\frac{IR}{\sqrt{(IR)^2+(IX_C)^2}},\\ \sin\phi&=\frac{IX_C}{IZ}=\frac{IX_c}{\sqrt{(IR)^2+(IX_C)^2}},\\ \tan\phi&=\frac{IX_C}{IR},\end{aligned}\right\}, \qquad (71)$$

and from the impedance diagram (Fig. 100),

$$\left.\begin{aligned}\cos\phi&=\frac{R}{Z}=\frac{R}{\sqrt{R^2+X_C^2}},\\ \sin\phi&=\frac{X_C}{Z}=\frac{X_C}{\sqrt{R^2+X_C^2}},\\ \tan\phi&=\frac{X_C}{R}.\end{aligned}\right\} \qquad (72)$$

57. Losses in a Condenser.—When a condenser is used in an alternating-current circuit, there are various losses which occur. These losses may be due to actual leakage from one plate to the other through or around the dielectric, to dielectric hysteresis, or to I^2R losses in the conducting plates of the condenser. The dielectric hysteresis loss is due to the stressing of the dielectric by the voltage applied to it, alternately in one direction and in the other, and manifests itself by heating of the dielectric, just as magnetic hysteresis heats the iron.

In good condensers operated at ordinary commercial voltages and frequencies, the losses in the condenser are negligible; in our problems we shall so assume them unless otherwise stated. If it is desired to know the amount of the losses taking place in a condenser, they must be measured by a wattmeter or other equivalent means, and the effective series resistance of the condenser obtained by dividing the watts by the current squared, i.e.,

$$R = \frac{\text{watts}}{I^2}.$$

It is to be noted then, that if the losses of the condenser of Fig. 97 are negligible, so that the effective series resistance of the condenser is zero, the drop IX_C is the voltage which would be measured by a voltmeter when placed across the capacitance. In the circuit of Fig. 94 we found that if a voltmeter was placed across the inductance coil, it measured the IZ drop, or vector sum of the resistance and reactance drops of the coil; the resistance drop and reactance drop could not be separately measured, inasmuch as the resistance and reactance of the coil could not be dissociated one from the other.

58. Power in a Circuit Containing Resistance and Capacitance.—The product of the IX_C drop and the current, I, that is, $(IX_C)I$, in Fig. 99, cannot represent any real power because the angle between the voltage impressed on the condenser and the current which flows is 90°, thus making the power factor of the condenser equal to zero. High grade paraffine paper or oil condensers, such as are used in practice, have an angle of lead between 89° and 90°. The power factor is thus about 1% or even less. The product of the IR drop and the current, I, that is, $(IR)I$, or I^2R, must therefore indicate the total power actually used in the circuit of Fig. 97 since we have assumed that there was no power lost in the condenser.

From Fig. 99 we have, since $IR = E \cos \phi$,

$$\text{Power} = P = (IR)I = EI \cos \phi,$$

where $\cos \phi$, the power factor, is the cosine of the angle between the current and the voltage impressed in the circuit.

In Fig. 102 the volt-ampere curves corresponding to the curves of Fig. 101 are plotted, being obtained as in the case of the inductive circuit considered before, by plotting instantaneous products of current and IR drop, IX_C drop, and impressed voltage, respectively. It will be seen that the volt-ampere curve, representing the product of instantaneous values of current and the IX_C curve (dot-dash curve), has equal positive and negative loops, indicating that on the average the capacitance dissipates no power. The curve representing the product of instantaneous values of current and IR drop (full-line curve) is positive throughout, indicating that all of the power is dissipated. The volt-ampere curve, representing the product of instantaneous values of current and impressed voltage (broken-line curve), has both positive and negative loops, the former being larger than the negative loops. The difference in the areas of the positive

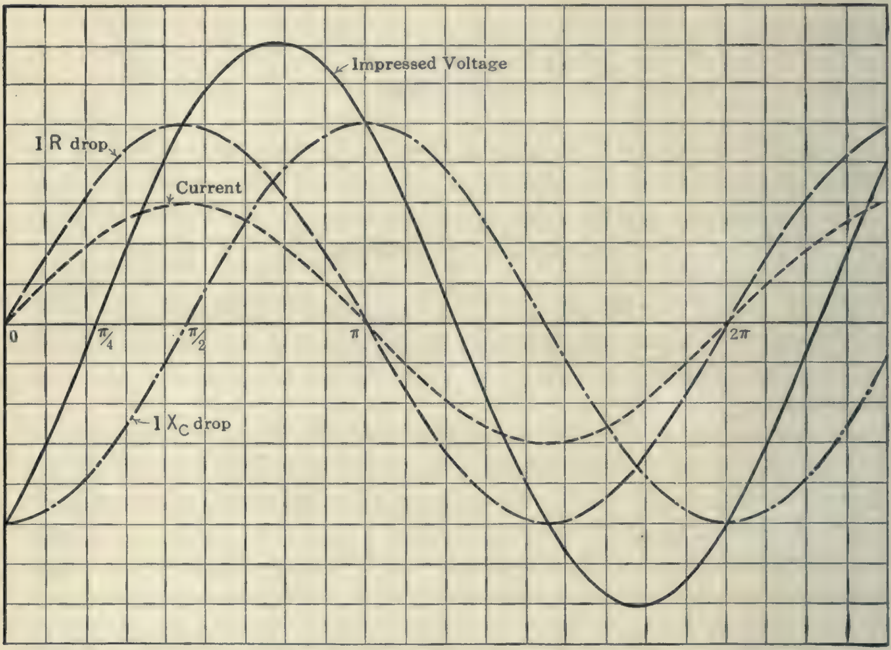


FIG. 101.—Curves of voltages for a condensive circuit having a resistance equal to its reactance.

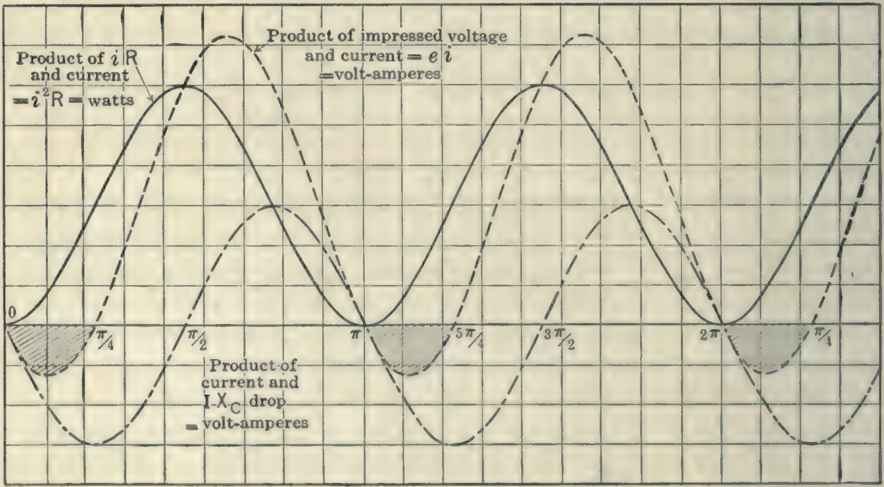


FIG. 102.—Volt-ampere curves for the condensive circuit used in Fig. 97; during part of the cycle energy is flowing back from the circuit into the machine.

and negative loops again represents the energy dissipated by the entire circuit, and is equal to the area under the full-line curve.

It will be seen that from time $=0$ to time $=\pi/4$, for the relative values of R and X_C assumed, the capacitance is giving up more energy than the resistance is dissipating, so that energy is flowing back to the source. From time $=\pi/4$ to time $=\pi/2$, the resistance is dissipating more energy than the capacitance is giving up, the surplus being furnished by the source. From time $=\pi/2$ to time $=\pi$, both the resistance and the capacitance are drawing energy from the line.

To determine by the calculus an expression for the average power when a voltage of the form $E_m \sin \omega t$ is impressed on a circuit containing resistance and capacitance only, and a current of the form $I_m \sin (\omega t+\phi)$ results, the current leading the voltage by ϕ degrees, we need to evaluate the expression

$$\text{Average power}=\frac{1}{\pi}\int_0^\pi E_m \sin \omega t \times I_m \sin (\omega t+\phi) d(\omega t).$$

This expression is the same as that integrated on page 93 with the exception that here we have $\sin (\omega t+\phi)$ instead of $\sin (\omega t-\phi)$; this difference in the algebraic sign of ϕ makes no difference in the result of the integration, so that we have

$$P=\frac{E_m I_m \cos \phi}{2}=\frac{E_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos \phi=EI \cos \phi.$$

59. Circuit Containing Several Resistances and Capacitances in Series.—From what was said regarding a circuit containing several inductances and resistances in series (page 93), it is evident that, for a circuit containing several resistances and capacitances in series,

$$E=\sqrt{(IR'+IR''+\ldots)^2+(IX'_C+IX''_C+\ldots)^2}. \quad . \quad . \quad (73)$$

$$E=IZ=I\sqrt{(R'+R''+\ldots)^2+(X'_C+X''_C+\ldots)^2}. \quad . \quad (74)$$

$$Z=\sqrt{(R'+R''+\ldots)^2+(X'_C+X''_C+\ldots)^2}. \quad . \quad . \quad . \quad (75)$$

$$\cos \phi=\frac{R'+R''+\ldots}{Z}, \quad \sin \phi=\frac{X'_C+X''_C+\ldots}{Z}. \quad . \quad . \quad . \quad (76)$$

60. Example.—As a laboratory test, a capacitance, $C=200$ microfarads $(=200\times 10^{-6}$ farads), is placed in series with a non-inductive resistance, $R=10$ ohms, across a 110-volt, 25-cycle source of power. It is desired to know the current which will flow, the voltage across each part of the circuit, the power factor of the circuit, and the power dissipated. We assume the condensers sufficiently free from dielectric hysteresis and leakage to permit us to neglect their losses.

The reactance, X_C, of the circuit is that of the capacitance,

$$X_C=\frac{1}{2\pi\times25\times200\times10^{-6}}=\frac{10^6}{157\times200}=31.9 \text{ ohms.}$$

The impedance, Z, of the circuit is (Fig. 103),

$$Z=\sqrt{10^2+31.9^2}=33.4 \text{ ohms.}$$

When the circuit is placed across a 110-volt, 25-cycle source of power, the current will be

$$I=\frac{E}{Z}=\frac{110}{33.4}=3.29 \text{ amperes.}$$

The voltage across the resistance, $E_R=IR$, will be $3.29\times10=32.9$ volts, and that across the capacitance, $E_C=IX_C$, will be $3.29\times31.9=104.9$ volts. The vector sum of these voltages is the impressed voltage (Fig. 104).

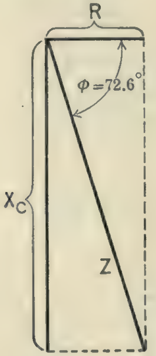


FIG. 103.

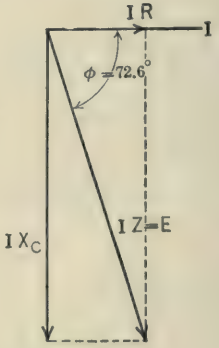


FIG. 104.

FIG. 103.—The impedance diagram for a capacitive circuit containing 10 ohms resistance in series with 31.9 ohms reactance.

FIG. 104.—The voltage diagram for the circuit used in Fig. 103.

$$E=\sqrt{32.9^2+104.9^2}=110 \text{ volts.}$$

The power factor of the circuit is, from the impedance diagram (Fig. 103),

$$\cos\phi=\frac{R}{Z}=\frac{10}{33.4}=0.299,$$

and from the voltage diagram (Fig. 104),

$$\cos\phi=\frac{IR}{IZ}=\frac{32.9}{110}=0.299,$$

from which $\phi=72.6°$.

The power dissipated is

$$P=EI\cos\phi=110\times3.29\times0.299=108.2 \text{ watts},$$

or,

$$P=I^2R=3.29^2\times10=108.2 \text{ watts}.$$

In case the losses in the condenser may not be neglected, it is to be noted that there will be an IR_C drop in the condenser. A voltmeter placed across the condenser would then read the impedance drop across the condenser, i.e., $IZ_C=\sqrt{(IR_C)^2+(IX_C)^2}$.

61. Circuit Containing Resistance, Inductance, and Capacitance in Series.—When a sine wave of voltage is impressed on a circuit containing a pure resistance, a pure inductance, and a pure capacitance in series, as in Fig. 105, a sine wave of current will flow, of the form, $i=I_m \sin 2\pi ft$, as indicated by the dotted curve of Fig. 109. This current when flowing through the resistance produces a resistance reaction, which will be balanced by the IR drop in phase with the current; this component of the impressed voltage is indicated by the dashed curve in Fig. 109. When flowing through the inductance, the current produces an inductance reaction, which will be balanced by the IX_L drop leading the current by 90°; this component of the impressed voltage is indicated by the dash-dot curve in Fig. 109. Similarly, when flowing through the capacitance, the current produces a capacitance reaction, which will be balanced by the IX_C drop, lagging 90° behind the current; this component of the impressed voltage is shown by the dash-double-dot curve of Fig. 109.

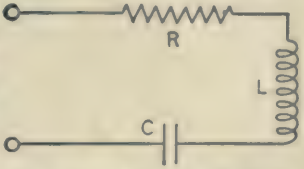


Fig. 105.—An imaginary circuit containing a pure resistance in series with a resistance-less inductor and a perfect condenser.

As the impressed voltage at any instant is the algebraic sum of all its components at that instant, its wave is readily constructed; it is indicated in Fig. 109 by the full-line curve. It will be seen that the IX_L and IX_C drops lie 180° apart, and that the resultant reactance drop of the circuit will always be the difference between them. The position of the impressed voltage with respect to the current depends then upon which of the reactance drops is the greater. If the IX_L drop is greater than the IX_C drop, the impressed voltage will be ahead of the current, but if the IX_C drop is greater than the IX_L drop, the impressed voltage will lie behind the current. For the relative magnitudes of reactance drops assumed in Fig. 109, the impressed voltage leads the current by the angle ϕ. If the current then is expressed by the equation, $i=I_m \sin \omega t$, the voltage must evidently be expressed by $e=E_m \sin(\omega t+\phi)$.

62. Voltage Diagram for a Circuit Containing Resistance, Inductance, and Capacitance in Series.—In both parts of Fig. 106, the current is used as the reference vector, the IR drop is drawn in phase with the current, the IX_L drop 90° ahead of the current, and the IX_C drop 90° behind the current. In Fig. 106a, the impressed voltage is found by finding first the resultant of IR and IX_L and then combining this resultant with IX_C. It is evident that, since IX_L and IX_C are 180° apart, we may combine their difference directly with the IR drop, as is done in Fig. 106b.

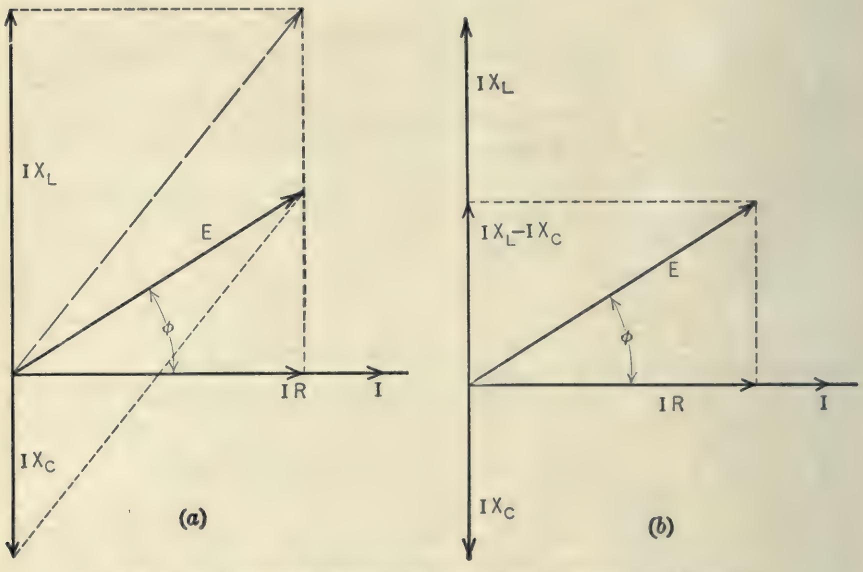


Fig. 106.—Showing the voltage diagram for the circuit of Fig. 105.

The operation indicated in Fig. 106 may be expressed by the equation

$$E=IZ=\sqrt{(IR)^2+(IX_L-IX_C)^2}=\sqrt{(IR)^2+\left(2\pi fLI-\frac{I}{2\pi fC}\right)^2}, \quad (77)$$

which may also be written

$$E=IZ=I\sqrt{R^2+(X_L-X_C)^2}=I\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}. \quad . \quad . \quad (78)$$

If the last equation is divided through by the current, I, we have that

$$\frac{E}{I}=Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}, \quad . \quad . \quad . \quad (79)$$

or that

$$I=\frac{E}{Z}=\frac{E}{\sqrt{R^2+(X_L-X_C)^2}}=\frac{E}{\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}}. \quad . \quad . \quad . \quad (80)$$

If the angle between the current and the voltage is again designated by ϕ, it will be seen that, from the voltage diagram (Fig. 106),

$$\left.\begin{aligned}\cos\phi&=\frac{IR}{IZ}=\frac{IR}{\sqrt{(IR)^2+(IX_L-IX_C)^2}},\\ \sin\phi&=\frac{IX_L-IX_C}{IZ}=\frac{IX_L-IX_C}{\sqrt{(IR)^2+(IX_L-IX_C)^2}}\\ \tan\phi&=\frac{IX_L-IX_C}{IR}.\end{aligned}\right\} \quad . \quad . \quad . \quad (81)$$

63. Significance of the Algebraic Sign of the Angle ϕ.—During the previous discussions we have at times used the expression $\sin(\omega t-\phi)$ and at others, $\sin(\omega t+\phi)$ in equations for the voltage or current, the angle ϕ being always the angle between the voltage and the current.

From the expression

$$\sin\phi=\frac{IX_L-IX_C}{\sqrt{(IR)^2+(IX_L-IX_C)^2}},$$

it is evident that ϕ may come out either positive or negative, according to which reactance drop is the greater.

In Fig. 106, for example, the current has been chosen as the reference vector and positive direction of rotation has been assumed as counter-clockwise. As the voltage vector, E, is distant, in a counter-clockwise direction, the angle ϕ from the reference vector, I, the angle ϕ must be positive; the expression for the voltage must then be $e=E_m\sin(\omega t+\phi)$, that for the current having been assumed as $i=I_m\sin\omega t$. This is indicated in Fig. 107*a*.

If the construction of Fig. 106 had given a voltage vector in the fourth quadrant, as a result of IX_C being greater than IX_L, then the voltage vector would be displaced from the current vector in a clockwise direction by the angle ϕ, and ϕ would be negative; the expression for the voltage would then be $e=E_m(\sin\omega t-\phi)$ as is indicated in Fig. 107*b*.

If the voltage vector had been used as the reference and expressed by $e=E_m\sin\omega t$, the proper expression for the current would be $i=I_m\sin(\omega t-\phi)$, as I is displaced in a clockwise direction from E. The vector diagram would then appear as in Fig. 107*c*. If IX_C were greater than IX_L the current would lead the voltage. Using the voltage vector as the reference and expressing it by $e=E_m\sin\omega t$, the expression for the current would be $i=I_m\sin(\omega t+\phi)$, as indicated in Fig. 107*d*.

64. Impedance Diagram for a Circuit Containing Resistance, Inductance, and Capacitance in Series.—Equation (79) indicates that to find the impedance of the series circuit containing resistance, inductance, and capacitance, it is necessary to combine, at right angles, the difference between the inductive and capacitive reactances, and the resistance, the operation being indicated by the scalar diagram of Fig. 108. As Fig. 108

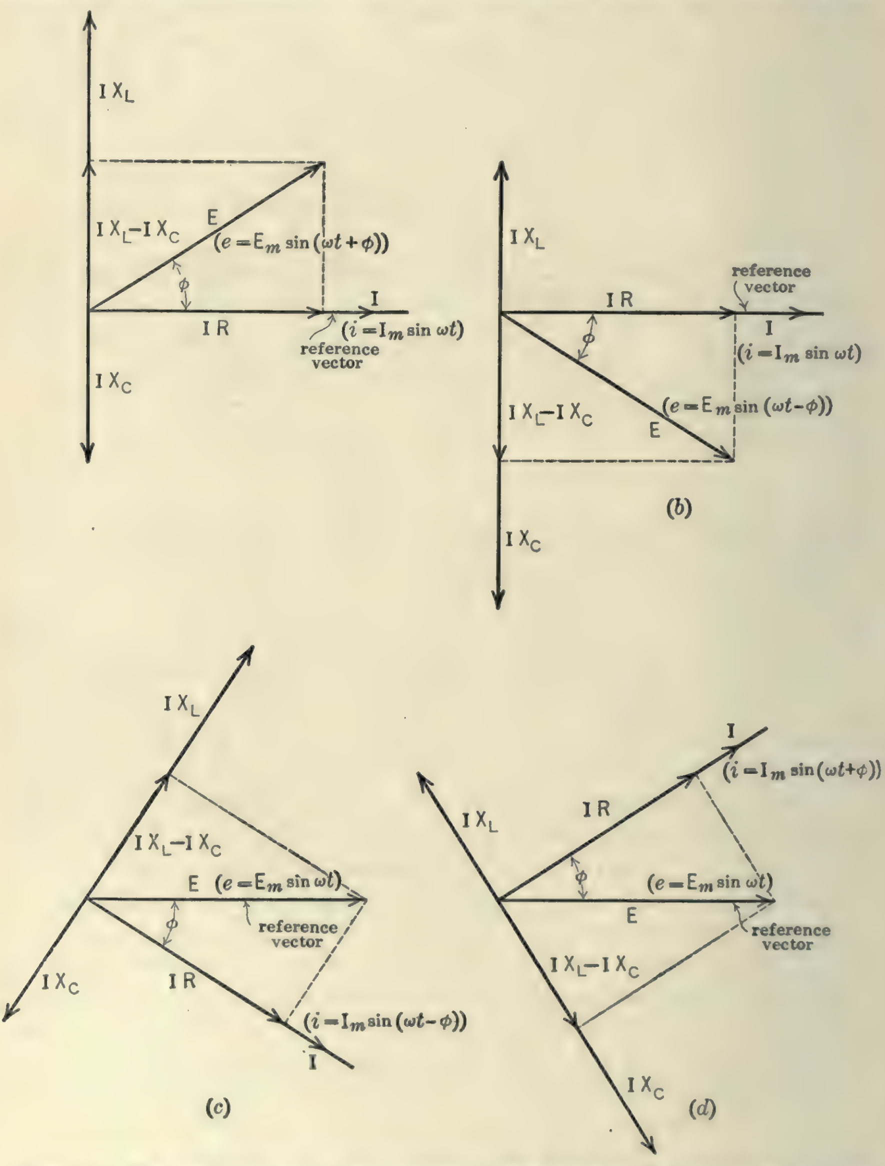


FIG. 107.—Vector diagram for the various voltages in a circuit consisting of inductance, capacitance, and resistance all in series.

is the result of dividing the IR, (IX_L-IX_C), and IZ drops of Fig. 106*b*, by the current, it follows that in the impedance diagram of Fig. 108, the inductive reactance, X_L, should be drawn 90° ahead of the resistance, and the capacitive reactance, X_C, 90° behind the resistance, to make the diagram consistent with the others we have used.

From the impedance diagram (Fig. 108) we have

$$\left.\begin{aligned}\cos\phi &= \frac{R}{Z} = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}},\\ \sin\phi &= \frac{X_L-X_C}{Z} = \frac{X_L-X_C}{\sqrt{R^2+(X_L-X_C)^2}},\\ \tan\phi &= \frac{X_L-X_C}{R}.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (82)$$

65. Power in a Circuit Containing Resistance, Inductance, and Capacitance in Series.—In Fig. 110 the volt-ampere curves corresponding to the curves of Fig. 109 are plotted, being obtained, as before, by plotting the instantaneous products of current and IR drop, IX_L drop, IX_C drop, and impressed voltage, respectively. As before, the curves representing the product of the current and the IX_L and IX_C drops are sine waves and have equal positive and negative loops, respectively. It will be seen, however, that these curves lie relatively 180° apart; when the inductance is absorbing energy, the capacitance is giving up energy, and *vice versa.* The net reactive volt-ampere curve for the entire circuit would be the difference between the two reactive volt-ampere curves shown. The energy dissipated by the entire circuit is indicated by the I^2R curve.

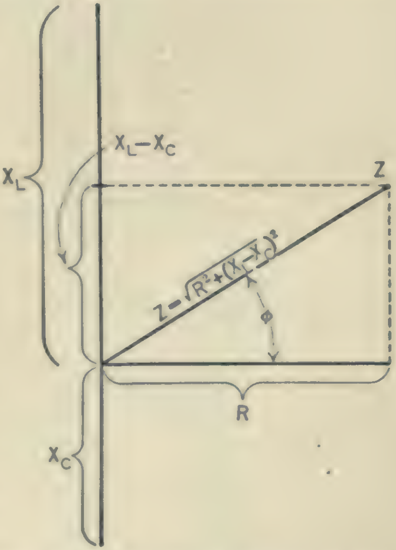


Fig. 108.—Impedance diagram of the circuit of Fig. 105.

Beginning with time $=0$, we see that both inductance and resistance are drawing energy from the source, while the capacitance is giving up energy. For the case considered, the inductance, beginning with time $=0$, absorbs more energy than the capacitance is giving up, so that the dashed curve (product of instantaneous values of current and impressed voltage) lies above the full-line, or I^2R curve. At time $=\pi/2$, neither inductance nor capacitance is drawing or giving up energy, so that the dashed and full-line curves intersect. After time $=\pi/2$, the inductance gives up its energy to both the capacitance and the resistance, but up to about time $= \frac{5\pi}{6}$ (for the case considered) the resistance and capacitance require more energy than the inductance can furnish, so that the circuit is still drawing energy from the source. Between time about $\frac{5\pi}{6}$ and time $= \pi$, the resistance and capacitance do not require all the energy being given up by the inductance, so that the surplus is given back to the source.

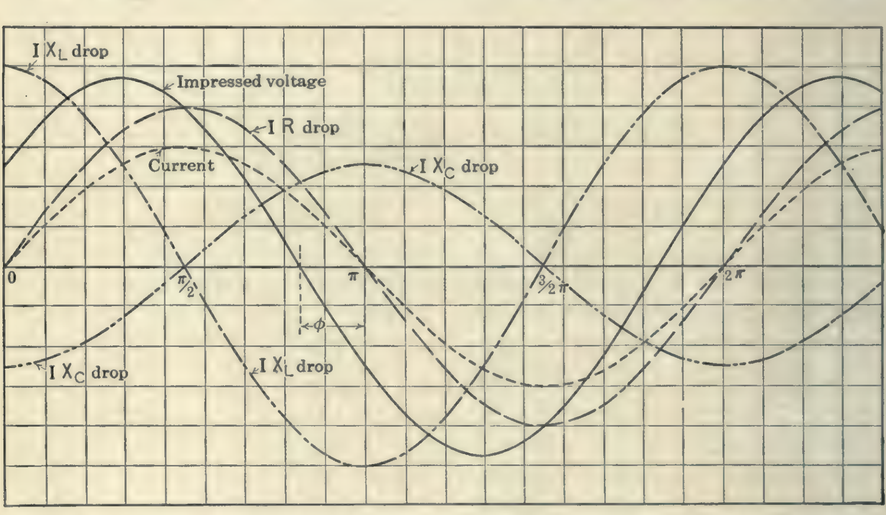


FIG. 109.—Curves of current and voltages in a circuit containing a resistance-less coil, a resistance, and a condenser in series.

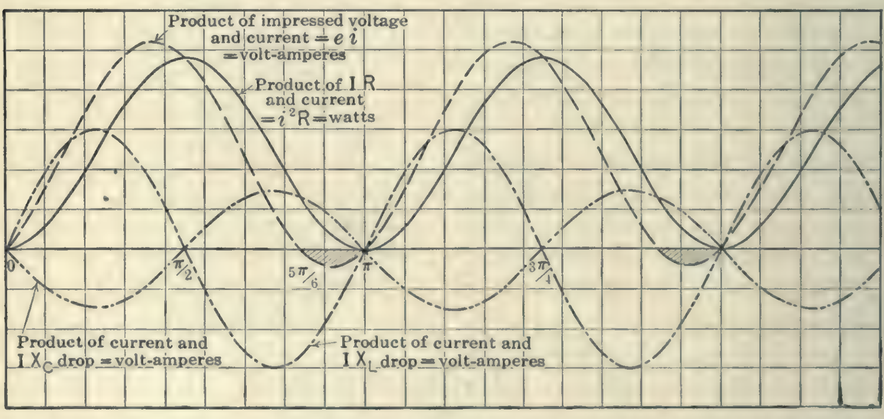


FIG. 110.—The volt-ampere and power curves for the circuit from which the curves of Fig. 109 were obtained.

To determine by the calculus an expression for the average power, when a voltage of the form $E_m \sin \omega t$ is impressed on a circuit containing resistance, inductance, and capacitance in series, we proceed as on pages 93 and 103. The resulting current will have the form $I_m \sin (\omega t-\phi)$ or $I_m \sin (\omega t+\phi)$ depending upon whether the current lags behind or leads the volt by the angle ϕ. When the inductive reactance is greater than the capacitive reactance, the current lags behind the voltage, and *vice versa*.

We need then to evaluate the expression

$$\text{Average power}=\frac{1}{\pi}\int_0^\pi E_m \sin \omega t \times I_m \sin (\omega t \pm \phi) d(\omega t).$$

This has been shown (pages 93 and 103) to reduce to

$$P=EI \cos \phi.$$

66. Circuit Containing Several Resistances, Inductances, and Capacitances in Series.—From what has been said before, it follows that

$$E=\sqrt{(IR'+IR''+\dots)^2+\{(IX'_L+IX''_L+\dots)-(IX'_C+IX''_C+\dots)\}^2}. \quad (83)$$

$$E=IZ=I\sqrt{(R'+R''+\dots)^2+\{(X'_L+X''_L+\dots)-(X'_C+X''_C+\dots)\}^2}. \quad (84)$$

$$Z=\sqrt{(R'+R''+\dots)^2+\{(X'_L+X''_L+\dots)-(X'_C+X''_C+\dots)\}^2}. \quad . \quad (85)$$

$$\cos \phi=\frac{R'+R''+\dots}{Z}, \quad \sin \phi=\frac{(X'_L+X''_L+\dots)-(X'_C+X''_C+\dots)}{Z}. \quad (86)$$

67. Example.—Two air-core inductance coils and a condenser are placed in series across a 110-volt, 60-cycle source of power, as in Fig. 111. The constants of coil No. 1 are $L_1=0.03$ henry, $R_1=3$ ohms; of coil No. 2, $L_2=0.01$ henry, $R_2=7$ ohms; of the condenser, $C=100$ microfarads, losses negligible. It is desired to know the current which will flow, the power factor of each part and of the entire circuit, the voltage across each part of the circuit, and the power dissipated in each part and in the entire circuit.

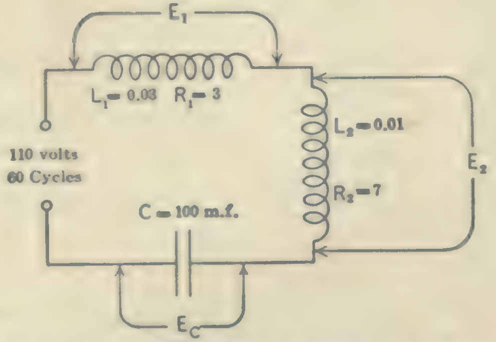


FIG. 111.—A possible actual circuit having coils in series with a condenser.

The reactances of the two coils are

$$X_1=2\pi\times60\times0.03=11.31 \text{ ohms},$$

$$X_2=2\pi\times60\times0.01= 3.77 \text{ ohms}.$$

The reactance (and impedance) of the capacitance is

$$X_C = \frac{10^6}{2\pi \times 60 \times 100} = 26.52 \text{ ohms.}$$

The impedances of the two coils are

$$Z_1 = \sqrt{3^2 + (2\pi \times 60 \times 0.03)^2} = \sqrt{3^2 + 11.31^2} = 11.70 \text{ ohms,}$$
$$Z_2 = \sqrt{7^2 + (2\pi \times 60 \times 0.01)^2} = \sqrt{7^2 + 3.77^2} = 7.95 \text{ ohms.}$$

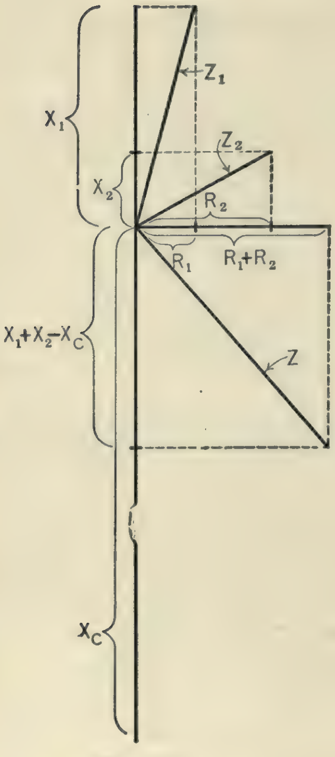


FIG. 112.

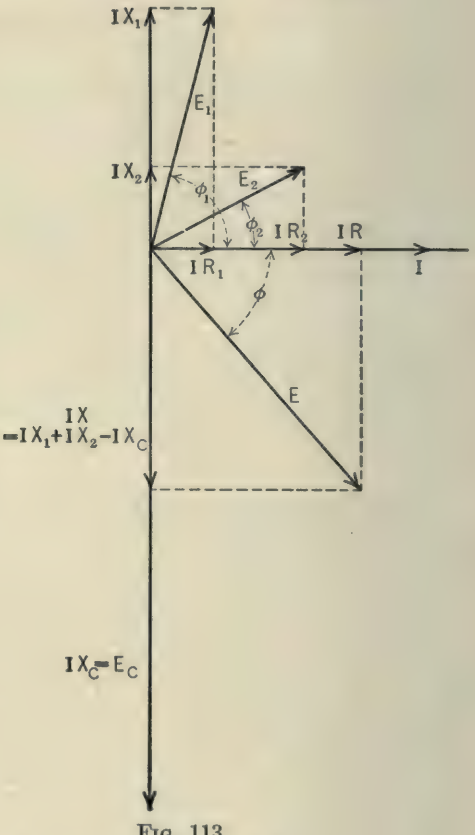


FIG. 113.

FIG. 112.—The impedance diagram of the circuit of Fig. 111.
FIG. 113.—The voltage diagram of the circuit of Fig. 111.

The impedance of the entire circuit is (Fig. 112),

$$\begin{aligned} Z &= \sqrt{(R_1 + R_2)^2 + (X_1 + X_2 - X_C)^2} \\ &= \sqrt{(3+7)^2 + (11.31 + 3.77 - 26.52)^2} \\ &= \sqrt{10^2 + (15.08 - 26.52)^2} = \sqrt{10^2 + (-11.42)^2} \\ &= \sqrt{100 + 130.8} = 15.2 \text{ ohms.} \end{aligned}$$

When the circuit is placed across a 110-volt, 60-cycle source of power, the current will be

$$I=\frac{E}{Z}=\frac{110}{15.2}=7.24 \text{ amperes.}$$

The power factor of coil No. 1 will be

$$\cos\phi_1=\frac{R_1}{Z_1}=\frac{3}{11.70}=0.2564, \qquad \text{so that} \quad \phi_1=75.15°.$$

The power factor of coil No. 2 will be

$$\cos\phi_2=\frac{R_2}{Z_2}=\frac{7}{7.95}=0.8805, \qquad \text{so that} \quad \phi_2=28.3°.$$

The power factor of the capacitance is zero and $\phi_C=90°$.

The power factor of the entire circuit is

$$\cos\phi=\frac{R_1+R_2}{Z}=\frac{10}{15.2}=0.6579, \qquad \text{so that} \quad \phi=48.9°.$$

The voltage across coil No. 1 will be

$$E_1=IZ_1=7.24\times 11.70=84.7 \text{ volts,}$$

which voltage leads the current by 75.15°. (Fig. 113.)

The voltage across coil No. 2 will be

$$E_2=IZ_2=7.24\times 7.95=57.6 \text{ volts,}$$

which leads the current by 28.3°.

The voltage across the capacitance will be

$$E_C=IZ_C=7.24\times 26.52=192.0 \text{ volts,}$$

which lags 90° behind the current.

The total resistance drop of the entire circuit is

$$E_R=IR=7.24\times 10=72.4 \text{ volts,}$$

and the total reactance drop is

$$\begin{aligned}E_X&=I(X_1+X_2-X_C)=7.24(11.31+3.77-26.52)\\&=7.24(-11.44)=82.8 \text{ volts,}\end{aligned}$$

which lags 90° behind the current.

As a check on our work, we may combine the voltages across the different parts of the circuit, as is indicated in Fig. 113; their vector sum is, of course, the impressed voltage. Thus

$$\begin{aligned}E&=\sqrt{(E_1\cos\phi_1+E_2\cos\phi_2)^2+(E_1\sin\phi_1+E_2\sin\phi_2-E_C\sin\phi_c)^2}\\&=\sqrt{(84.7\times 0.2564+57.6\times 0.8805)^2+(84.7\times 0.9666+57.6\times 0.4741-192.0)^2}\\&=\sqrt{(21.7+50.7)^2+(81.9+27.3-192.0)^2}\\&=\sqrt{72.4^2+(109.2-192.0)^2}=\sqrt{72.4^2+(-82.8)^2}\\&=\sqrt{5242+6856}=\sqrt{12098}=110.\end{aligned}$$

It will be noted that the component of the impressed voltage in phase with the current is the total resistance drop, and the component of the impressed voltage 90° out of phase with the current is the total reactance drop.

The power dissipated in coil No. 1 is

$$P_1 = E_1 I \cos \phi_1 = 84.7 \times 7.24 \times 0.2564 = 157.2 \text{ watts},$$

or

$$P_1 = I^2 R_1 = 7.24^2 \times 3 = 157.2 \text{ watts}.$$

The power dissipated in coil No. 2 is

$$P_2 = E_2 I \cos \phi_2 = 57.6 \times 7.24 \times 0.8805 = 366.6 \text{ watts},$$

or,

$$P_2 = I^2 R_2 = 7.24^2 \times 7 = 367.1 \text{ watts}.$$

The total power dissipated is

$$P = EI \cos \phi = 110 \times 7.24 \times 0.6579 = 524.0 \text{ watts},$$

or,

$$P = I^2 R = 7.24^2 \times 10 = 524.2 \text{ watts},$$

or,

$$P = P_1 + P_2 = 157.2 + 366.6 = 523.8 \text{ watts}.$$

The difference in the results for total power is due to lack of precision in the arithmetical work.

Example.—An iron-core inductance and a condenser with a high dielectric loss were placed in series across a 60-cycle source of power, and the circuit drew a current of 7.6 amperes, as indicated by an ammeter placed in the circuit. A voltmeter placed across the inductance coil read 81 volts, and when connected across the condenser read 100 volts. The power dissipated in both the iron-core inductance and the condenser was determined by a wattmeter; the inductance dissipated 577 watts and the condenser 52 watts. What voltage was impressed upon the circuit?

The effective resistance of the coil was

$$R_L = \frac{W_L}{I^2} = \frac{577}{7.6^2} = 10 \text{ ohms}.$$

The effective resistance of the condenser was

$$R_C = \frac{W_C}{I^2} = \frac{52}{7.6^2} = 0.9 \text{ ohms}.$$

The resistance drop of the coil was

$$IR_L = 7.6 \times 10 = 76 \text{ volts},$$

or,

$$IR_L = \frac{W_L}{I} = \frac{577}{7.6} = 76 \text{ volts}.$$

The resistance drop of the condenser was

$$IR_C = 7.6 \times 0.9 = 6.0 \text{ volts,}$$

or,

$$IR_C = \frac{W_C}{I} = \frac{52}{7.6} = 6.0 \text{ volts.}$$

With an impedance drop across the coil of 81 volts, the reactance drop of the coil was

$$IX_L = \sqrt{(IZ_L)^2 - (IR_L)^2} = \sqrt{81^2 - 76^2} = 28 \text{ volts.}$$

The reactance drop of the condenser was

$$IX_C = \sqrt{(IZ_C)^2 - (IR_C)^2} = \sqrt{100^2 - 6.9^2} = 99.8 \text{ volts.}$$

The impressed voltage was therefore

$$\begin{aligned} E &= \sqrt{(IR_L + IR_c)^2 + (IX_L - IX_c)^2} \\ &= \sqrt{(76 + 6.9)^2 + (28 - 99.8)^2} \\ &= \sqrt{82.9^2 + (-71.8)^2} = 110 \text{ volts.} \end{aligned}$$

Evidently the current was leading the voltage.

68. Resonance in a Series Circuit.—The general equation for current in a series circuit is

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

The impedance, which controls the magnitude and phase of the current for a given impressed voltage, is made up of three independent terms; of these the resistance is independent of the frequency, whereas the two components of the reactance terms depend directly on the frequency for their values. The manner in which these two reactance terms vary with frequency is indicated in Fig. 114; no matter what the values of L and C may be, these two reactance terms will vary in the manner shown, as the frequency is varied. It then appears that no matter what the actual values of L and C, there must be some frequency for which the two reactance terms are equal, so that the total reactance (the arithmetical difference of the two) will be zero.

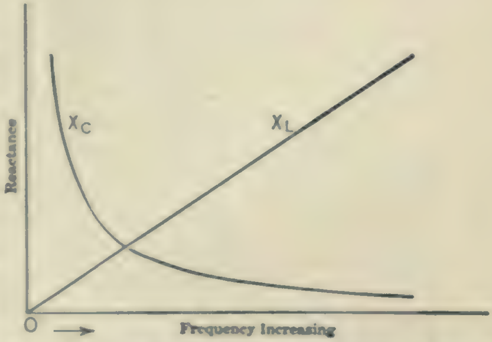


FIG. 114.—Showing how the reactance of a coil ($2\pi fL$) and that of a condenser $\left(\frac{1}{2\pi fC}\right)$ vary with frequency.

This frequency, called the *resonant frequency* of the circuit, is evidently given by the relation

$$2\pi fL=\frac{1}{2\pi fC}, \quad \dots\dots\dots \quad (88)$$

or,

$$f=\frac{1}{2\pi\sqrt{LC}}. \quad \dots\dots\dots \quad (89)$$

For this frequency the general equation for the current flow reduces to the simple form

$$I=\frac{E}{R}, \quad \dots\dots\dots \quad (90)$$

that is, the circuit, actually containing resistance, inductance, and capacitance in series, acts as though it had resistance only.

Dangerously high voltages may be set up in a series circuit at the resonant frequency. The drop across the coil, for example, is IX_L (on the assumption that the resistance of the coil is negligible in comparison to its reactance), and if we substitute the value of I from Eq. (90), this becomes

$$E_L=IX_L=\frac{E}{R}X_L=E\frac{2\pi fL}{R}. \quad \dots\dots \quad (91)$$

The reactance of a coil may be many times as great as the resistance of the series circuit of which it forms a part, so that the fraction $\frac{2\pi fL}{R}$ may have values as high as 10 or 20, at ordinary engineering frequencies. In radio circuits, because of the excessively high frequencies used, this ratio of reactance to resistance may easily be 100 or more. This means that *the drop across the coil may be as much as 100 times as great as the voltage impressed on the circuit.* Furthermore, as the reactance of the condenser is equal to that of the coil at the resonant frequency, it follows that the voltage across the condenser in a series circuit may be many times the impressed voltage of the circuit, and herein lies the danger of series resonance; the condensers are very likely to be broken down by excessive voltage, even though the impressed voltage is much less than the value for which they are guaranteed to be safe.

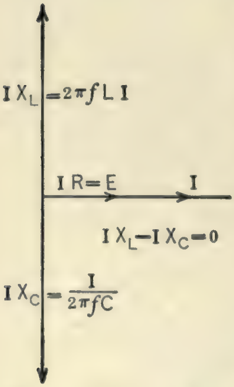


FIG. 115.—At some frequency the total reactance of a circuit having a coil in series with a condenser must be zero.

In Fig. 115 is shown the vector diagram of voltages in a series circuit in resonance; the two reactance drops are equal and opposite, making the total reactance drop zero, so that all of the impressed voltage may be used up in overcoming the *IR* reaction of the circuit. The current is thus in phase with the impressed voltage, and the power factor of the whole circuit is therefore unity. It is to be remembered, of course, that the power factor of the coil is not unity, neither is that of the condenser; in fact, the power factors of these two component parts of the circuit are generally very low. It is only when the whole circuit is considered, coil in series with the condenser, that the power factor is seen to be unity.

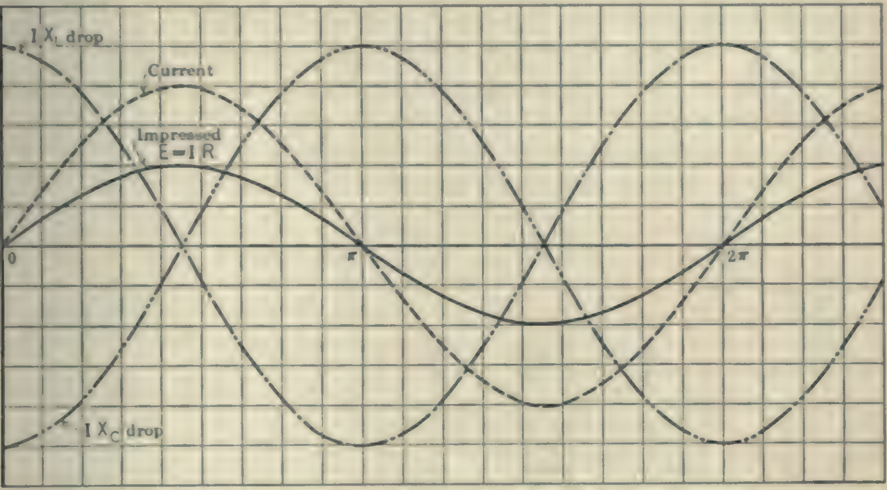


FIG. 116.—Voltage and current curves for a circuit in series resonance.

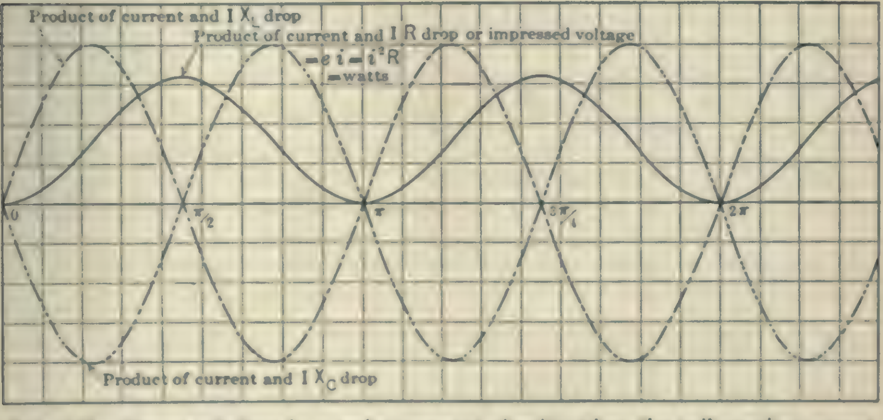


FIG. 117.—Power relations in a series resonant circuit; when the coil requires energy (its magnetic field being on the increase) the condenser is giving up energy at just the right rate to supply the coil, and vice versa.

In Fig. 116 the wave forms of the different voltages of a resonant circuit are plotted, and in Fig. 117, the corresponding volt-ampere curves are given. From the latter it will be seen that when the inductance *requires* energy, the capacitance is *giving up* energy, and, the circuit being in resonance, the condenser is giving up energy at exactly the same rate as the coil requires it; correspondingly, when the condenser requires energy the coil is giving it up. The stored energy in the circuit thus stays constant, sometimes being resident in the coil, sometimes in the condenser, and at other times being partly in each. The source of power supply is thus not called upon to furnish energy for either coil or condenser (after the circuit once reaches the steady state); all of the energy which flows from the source is used up in the resistances of the circuit and thus the power factor of the circuit is unity.

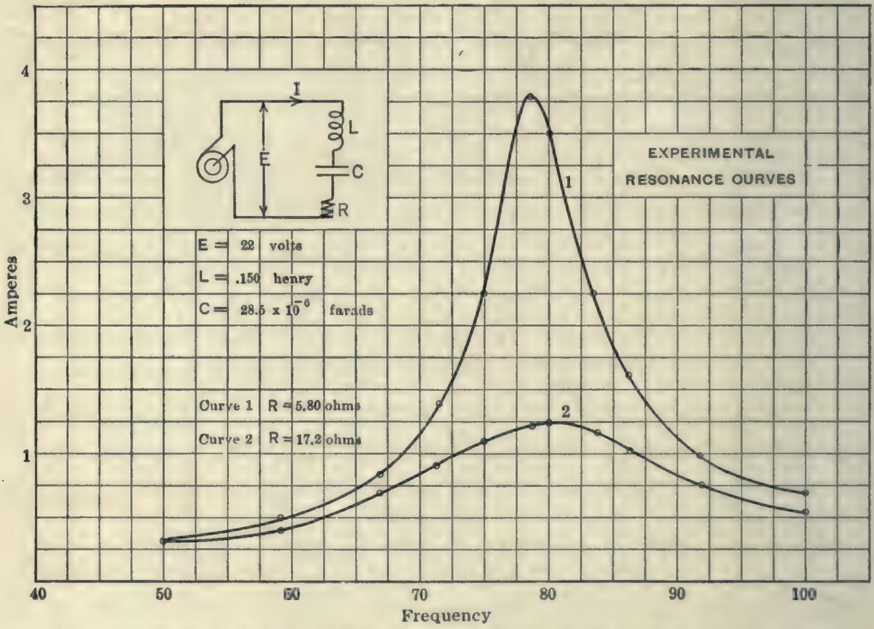


FIG. 118.—Showing how the current in a resonant circuit varies with frequency, the impressed voltage, inductance, and capacitance being constant; the resonance peak is more accentuated the lower the resistance.

69. Form of Resonance Curve.—If the voltage impressed on a series circuit (consisting of a coil in series with resistance and a condenser), is kept constant and either f, L, or C, is continuously varied, the current will increase to a maximum and then fall again, as is indicated in Fig. 118, showing the result of a test in which the frequency was varied; such a curve is called the resonance curve of the circuit. The sharpness of the curve depends upon the resistance in the circuit. Curve No. 1 shows the variation of current in a circuit containing 5.8 ohms; and curve No. 2 shows the variation when the resistance is trebled.

Resonant circuits are extensively used in radio receiving sets, their function being to receive certain frequency signals much more easily than other interfering signals. In such circuits the resonance may be very sharp, a change of frequency from the resonant value of only one per cent sufficing to cause the current to drop to a small fraction (perhaps only one-tenth), of its resonant value. The sharper the resonance curve, the easier it is for a radio set to " tune out " interfering signals.

70. Example of a Resonant Circuit.—Suppose we have a pure inductance of 0.1 henry, a capacitance of 70.4 microfarads and a resistance of 2 ohms connected in series to a 60-cycle, 110-volt line. How much current will flow and what will be the voltage across each part of the circuit?

$$I=\frac{110}{\sqrt{2^2+(37.7-37.7)^2}}=55 \text{ amperes.}$$

The drop across the inductance

$$=2\pi fLI=377\times 0.1\times 55=2073 \text{ volts.}$$

Across the condenser

$$\frac{55}{377\times 70.4\times 10^{-6}}=2073 \text{ volts.}$$

Across the resistance

$$55\times 2=110 \text{ volts.}$$

71. Solution of Parallel Circuits.—The parallel circuit is not in general quite as easy to solve as is the series circuit, this being due to the fact that the constants of the circuit are generally given in such units that the parallel circuit requires more arithmetical computation than does the series circuit. Consider a circuit made up of two inductive branches in parallel, as in Fig. 119. The same voltage, E, is impressed on both branches.

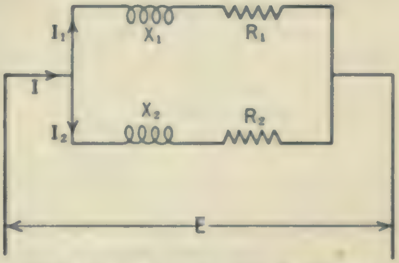


FIG. 119.—A simple case of parallel circuits.

The first point to notice is that each branch may be treated as a simple circuit, as though the other branch were not there. Thus, if a certain voltage E is impressed on branch X_1, R_1, a certain current will flow, its magnitude and phase being given by $I_1=\frac{E}{\sqrt{R_1^2+X_1^2}}$ and $\tan\phi_1=\frac{X_1}{R_1}$. Now *this current will be the same in magnitude and phase no matter how many paths are connected in parallel with it, provided the impressed voltage is not altered by connecting in the additional paths.*

Each of the currents, I_1 and I_2, having been calculated separately, the line current, I, is obtained by adding vectorially the two branch currents; this is indicated in Fig. 120. In case several paths, instead of only two, are connected in parallel, the procedure is just the same; each current is calculated from the impedance of the path in question, and then all branch currents are added vectorially to get the line current.

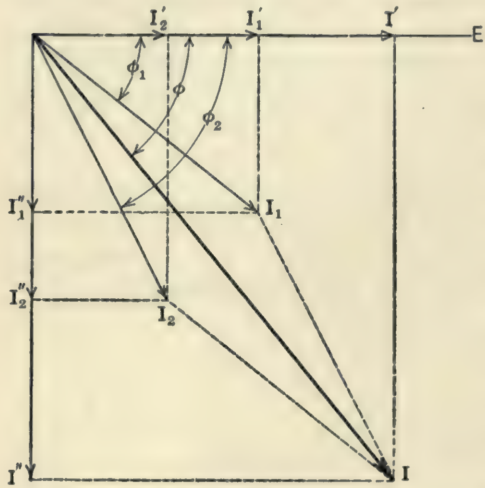


Fig. 120.—The current diagram for the circuit of Fig. 119; the line current is the vector sum of the branch currents.

The easiest way to solve the problem analytically is to resolve both I_1 and I_2 (Fig. 120) into their active and reactive components. The sum of the active components gives the active component of the line current, and the sum of the reactive components gives the reactive component of the line current. Referring to Fig. 120, we see that

$$\text{Active component of } I = I' = I_1 \cos \phi_1 + I_2 \cos \phi_2$$

$$= \frac{E}{Z_1} \times \frac{R_1}{Z_1} + \frac{E}{Z_2} \times \frac{R_2}{Z_2}$$

$$= \frac{ER_1}{Z_1^2} + \frac{ER_2}{Z_2^2}$$

$$= E\left(\frac{R_1}{Z_1^2} + \frac{R_2}{Z_2^2}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (92)$$

$$= E\left(\frac{R_1}{R_1^2 + X_1^2} + \frac{R_2}{R_2^2 + X_2^2}\right). \quad . \; . \; . \quad (93)$$

In the same way we find the reactive component of the line current.

$$\text{Reactive component of } I = I'' = I_1 \sin \phi_1 + I_2 \sin \phi_2$$

$$= \frac{E}{Z_1} \times \frac{X_1}{Z_1} + \frac{E}{Z_2} \times \frac{X_2}{Z_2}$$

$$= \frac{EX_1}{Z_1^2} + \frac{EX_2}{Z_2^2}$$

$$= E\left(\frac{X_1}{Z_1^2} + \frac{X_2}{Z_2^2}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (94)$$

$$= E\left(\frac{X_1}{R_1^2 + X_1^2} + \frac{X_2}{R_2^2 + X_2^2}\right) \quad . \; . \; . \; . \quad (95)$$

Evidently

$$I=\sqrt{(I')^2+(I'')^2}.$$

The power factor of the combined paths is given by the expression

$$\cos\phi=\frac{I'}{I},$$

and the impedance of the combined paths is given by

$$Z=\frac{E}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

The branched circuit may now be replaced by a single circuit; if the proper R and X are calculated for it, the single circuit will act electrically in exactly the same way as the actual branched circuit for which it has been substituted.

The impedance of the equivalent single circuit is given by the relation $Z=\frac{E}{I}$, and the two components of this equivalent impedance by

$$R=Z\cos\phi \quad \text{and} \quad X=Z\sin\phi.$$

A transformation of this kind is shown by Fig. 121. The branched circuit is solved as indicated above, to get the line current, and the phase of this current. The final answer will be the same no matter what voltage we assume is impressed on the branched circuit; the X and R of the single equivalent circuit, indicated in part b of Fig. 121, will prove to be the same for all voltages. To simplify the arithmetical work we assume 100 volts impressed on the branched circuit.

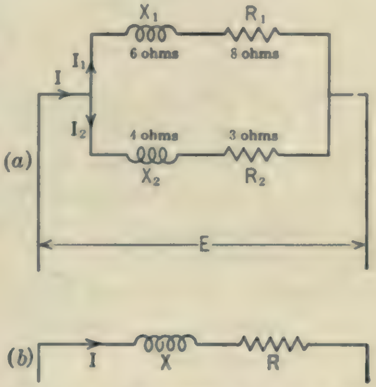


FIG. 121.—The general solution of circuits having branched sections requires that the parallel portions be replaced by equivalent series circuits; the circuit in (*a*) can be reduced to that in (*b*).

$$Z_1=\sqrt{8^2+6^2}=10 \text{ ohms.}$$

$$I_1=\frac{100}{10}=10 \text{ amperes.}$$

$$I'_1=I_1\cos\phi_1=10\times0.8=8 \text{ amperes.}$$

$$I''_1=I_1\sin\phi_1=10\times0.6=6 \text{ amperes.}$$

$$Z_2=\sqrt{3^2+4^2}=5 \text{ ohms.}$$

$$I_2=\frac{100}{5}=20 \text{ amperes.}$$

$$I'_2=I_2\cos\phi_2=20\times0.6=12 \text{ amperes.}$$

$$I''_2=I_2\sin\phi_2=20\times0.8=16 \text{ amperes.}$$

$$I' = I'_1 + I'_2 = 8 + 12 = 20 \text{ amperes.}$$

$$I'' = I''_1 + I''_2 = 6 + 16 = 22 \text{ amperes.}$$

$$I = \sqrt{(I')^2 + (I'')^2} = \sqrt{20^2 + 22^2} = 29.73 \text{ amperes.}$$

$$Z = \frac{E}{I} = \frac{100}{29.73} = 3.37 \text{ ohms.}$$

$$\cos \phi = \frac{I'}{I} = \frac{20}{29.73} = 0.6727.$$

$$\sin \phi = \frac{I''}{I} = \frac{22}{29.73} = 0.7400.$$

$$R = Z \cos \phi = 3.37 \times 0.6727 = 2.27 \text{ ohms.}$$

$$X = Z \sin \phi = 3.37 \times 0.7400 = 2.49 \text{ ohms.}$$

72. Admittance, Conductance, Susceptance.—So far, in dealing with alternating-current circuits, we have analyzed them from the idea of the impedance of the circuit, that is, the quantity by which the voltage must be *divided* to get the current. In dealing with parallel circuits, however, it is generally easier and shorter to deal with the quantity by which the voltage must be *multiplied* to get the current; this is evidently the reciprocal of the impedance. In paragraph 71 we used this method of solution.

We have

$$I = \frac{E}{Z} = E\frac{1}{Z} = EY, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (97)$$

where Y is called the *admittance* of the circuit, and is generally designated by the letter Y, as is done here. The expression for the value of Y will be given later.

We have

Active component of $I = I' = I \cos \phi$

$$= \frac{E}{Z} \times \frac{R}{Z} = E\frac{R}{Z^2} = EG. \quad . \; . \; . \; . \; . \; . \; . \quad (98)$$

The expression $\frac{R}{Z^2}$ is called the *conductance* of the circuit, and is generally designated by the letter G. It is the quantity by which the voltage must be multiplied to give the active component of current in the circuit.

In the same way

Reactive component of $I = I'' = I \sin \phi$

$$= \frac{E}{Z} \times \frac{X}{Z} = E\frac{X}{Z^2} = EB. \quad . \; . \; . \; . \; . \; . \; . \quad (99)$$

The expression $\frac{X}{Z^2}$ is called the *susceptance* of the circuit, and is generally designated by the letter B. It is the quantity by which the voltage must be multiplied to get the reactive component of current in the circuit.

Just as the reactance of a circuit has an algebraic sign, being positive or negative according as the circuit is inductive or condensive, so susceptance must be used with its proper sign.

In Fig. 122a, the current I'' leads the voltage E, the angle ϕ_1 being positive. We have

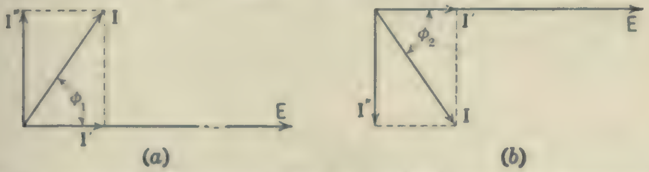
$$I''=I\sin\phi_1=E\frac{X}{Z^2}=E\times\text{susceptance}.$$

As $\sin\phi_1$ is positive, we must evidently consider the capacitive susceptance as positive.

In Fig. 122b, however, the conditions for an inductive circuit are shown, and we again have

$$I''=I\sin\phi_2=E\frac{X}{Z^2}=E\times\text{susceptance}.$$

As $\sin\phi_2$ is negative, ϕ_2 being in the fourth quadrant, we must consider inductive susceptance as negative. In many cases the sign of the susceptance is of no importance, as the expression used in solving the problem involves the susceptance only after it is squared. But it is very necessary to keep in mind the proper sign for the two kinds of susceptance when a problem involves both of them. Thus, if a condensive circuit is in parallel with an inductive circuit, the two reactive components of current actually tend to neutralize each other, a fact which is properly cared for in the solution of such problems if one susceptance is reckoned positive and the other negative.



(a) (b)

FIG. 122.—The current in a circuit may be resolved into two components, one in phase with the voltage and one at 90° to it.

As

$$I^2=(I')^2+(I'')^2,$$

it follows that

$$I^2=E^2\left(\frac{R}{Z^2}\right)^2+E^2\left(\frac{X}{Z^2}\right)^2,$$

or,

$$I=E\sqrt{\left(\frac{R}{Z^2}\right)^2+\left(\frac{X}{Z^2}\right)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

from which we see, by comparison with Eq. (97), that admittance of the circuit is given by

$$Y=\sqrt{\left(\frac{R}{Z^2}\right)^2+\left(\frac{X}{Z^2}\right)^2}. \qquad (101)$$

In solving problems by the help of the above expressions, it is to be noted that the impedance, Z, need never be calculated, as it is Z^2, not Z, which occurs in all the expressions.

In case a circuit consists of two or more branches in parallel, the conductance of the combined circuit is equal to the sum of the conductances of the several paths, and the combined susceptance is equal to the sum of the susceptances of the various paths. In the case of the conductances, the sum is always an arithmetical one, as all conductances (with rare exceptions) are positive; but in the case of susceptances, the individual susceptances may be either positive or negative according as the circuit is condensive or inductive as noted above. In this case the sum must therefore be an algebraic one, susceptances for condensive circuits being reckoned positive, and for inductive circuits being reckoned negative.

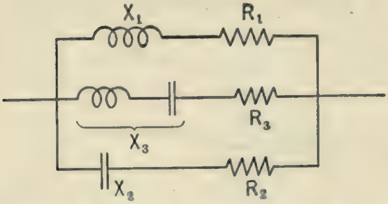


FIG. 123.—Such a circuit as this must be solved by the method of admittances, the conductances and susceptances for the various paths being respectively added.

We have then, for a circuit of three branches, as indicated in Fig. 123.

$$G_1=\frac{R_1}{R_1^2+X_1^2},\quad G_2=\frac{R_2}{R_2^2+X_2^2},\quad G_3=\frac{R_3}{R_3^2+X_3^2}. \qquad (102)$$

$$G=G_1+G_2+G_3. \qquad (103)$$

$$B_1=\frac{X_1}{R_1^2+X_1^2},\quad B_2=\frac{X_2}{R_2^2+X_2^2},\quad B_3=\frac{X_3}{R_3^2+X_3^2}. \qquad (104)$$

$$B=B_1+B_2+B_3. \qquad (105)$$

In this case (Fig. 123) B_1 is negative, B_2 is positive and B_3 may be either positive or negative according as the capacitive reactance of the branch is greater or less than the inductive reactance. In case the capacitive reactance is the greater, the algebraic sign of B_3 is positive, and *vice versa*.

The unit for either admittance, conductance, or susceptance is the *mho*, sometimes called the " reciprocal ohm."

The admittance of the combined paths is then given by

$$Y=\sqrt{G^2+B^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (106)$$

and the phase angle of the combined paths is given by one of the expressions

$$\tan \phi=\frac{B}{G}, \quad \cos \phi=\frac{G}{Y}, \quad \text{or} \quad \sin \phi=\frac{B}{Y}. \quad . \quad . \quad . \quad . \qquad (107)$$

73. Example.—As an example of this method of solving problems involving parallel circuits, we shall solve the two-path problem of Fig. 121, which has already been solved without actually calculating admittance and susceptance.

$$G_1=\frac{8}{8^2+6^2}=0.08 \text{ mho.}$$

$$G_2=\frac{3}{3^2+4^2}=0.12 \text{ mho.}$$

$$G=G_1+G_2=0.20 \text{ mho.}$$

$$B_1=\frac{6}{8^2+6^2}=0.06 \text{ mho.}$$

$$B_2=\frac{4}{3^2+4^2}=0.16 \text{ mho.}$$

$$B=B_1+B_2=0.22 \text{ mho.}$$

$$Y=\sqrt{0.20^2+0.22^2}=0.2973 \text{ mho.}$$

$$I=EY=100\times 0.2973=29.73 \text{ amperes.}$$

$$\tan \phi=\frac{0.22}{0.20}=1.1,$$

$$\sin \phi=\frac{0.22}{0.2973}=0.7400,$$

$$\cos \phi=\frac{0.20}{0.2973}=0.6727.$$

Having obtained the line current and power factor, the equivalent single circuit is calculated exactly as before.

74. Parallel Resonance.—It is to be noticed that in the case of parallel circuits having inductive reactance in some branches and condensive reactance in others, it is quite possible that the reactive current in the line may reduce to zero. This is indicated for a two-branched circuit in Fig.

124. The current in branch No. 1, I_1, leads the line voltage, having a reactive component I''_1. The current in branch No. 2, I_2, lags behind the line voltage and has a reactive current I''_2. As these two reactive currents are equal, and opposite in phase, the line current will have no reactive component, but will be exactly in phase with the line voltage, E. Such a circuit is said to be in resonance, and to distinguish it from the case of resonance in a circuit having coil and condenser in series, the circuit is said to be in parallel resonance.

If the resistance of each of the branch circuits is low compared with its reactance, the line current may be very much smaller than either of the branch currents. In the case of radio circuits, for example, the reactance of either path may be one hundred times as great as the resistance, so that the active current in the branch is only about 1 per cent of the total current. Such a case is suggested in Fig. 125. If the reactance of either path is one hundred times as great as its resistance, the line current for the resonant condition is only 2 per cent of the current in either branch.

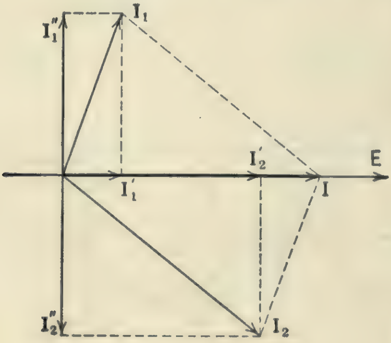


FIG. 124.

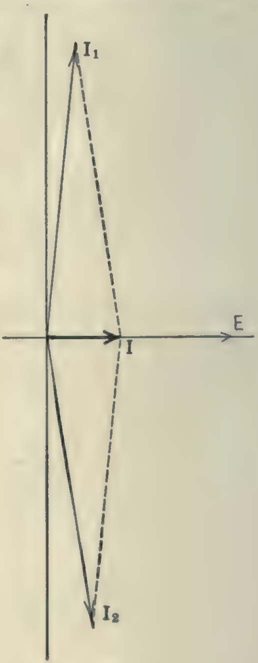


FIG. 125.

FIG. 124.—For a two-branched parallel circuit, it may happen that the reactive currents in the two branches just neutralize each other so that the line current is in phase with the voltage.

FIG. 125.—It may be that the line current, in the case of parallel circuits, is much less than the current in either branch; such a case is shown here.

From Eq. (95) it is evident that if there is to be no reactive current in the line of a two-branched circuit, we must have the condition that X_1 is minus and X_2 positive (or *vice versa*), and that

$$\frac{X_1}{R_1^2+X_1^2}=\frac{X_2}{R_2^2+X_2^2}. \quad \ldots\ldots \quad (108)$$

In case the resistances are small compared to the reactances, the solution for this condition of resonance gives essentially the same value for the resonant frequency as would be the case for series resonance, that is,

$$f = \frac{1}{2\pi\sqrt{LC}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

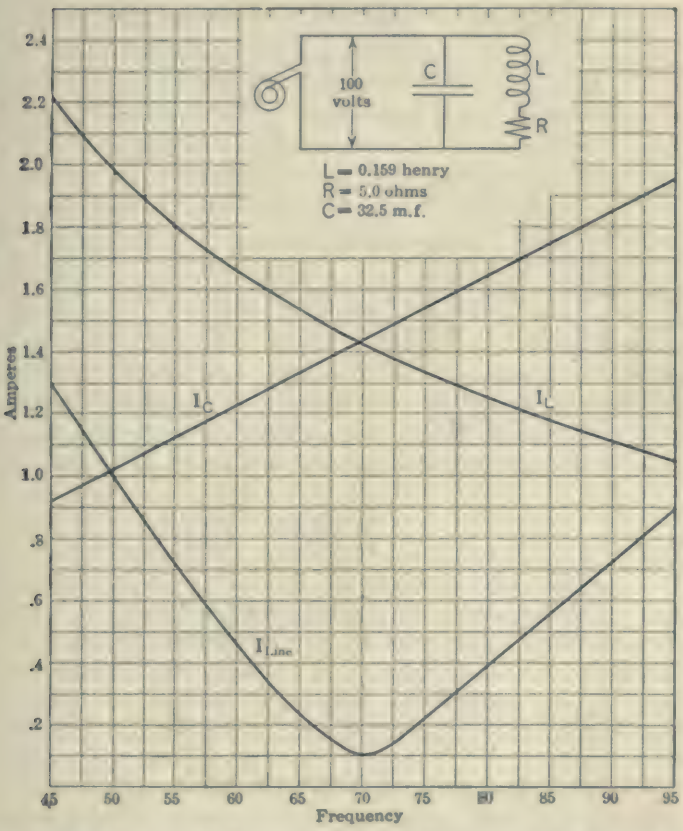


FIG. 126.—Showing how parallel resonance occurs; here are given the magnitudes of the line current and branch currents in the region of resonant frequency.

Hence, if a coil and condenser, when connected in series, are in resonance at a given frequency, they will also be in resonance for the same frequency (nearly) when connected in parallel.

For a pair of branched circuits, one having a coil and the other a condenser, if the resistance of both branches is small, the variation of current in either branch and in the line, as frequency is varied, is as shown in Fig. 126.

75. Solution of the General Circuit.—In the most general case, an alternating-current circuit may consist of portions with several branches in parallel, and these portions may be connected in series with other portions, consisting of single or branched circuits. A simple case of this kind is indicated in Fig. 127.

In such a case the portion between points *B* and *C* is first reduced to an equivalent single circuit by either of the two methods previously given. The circuit is then changed from the arrangement shown in Fig. 127*a*, to that shown in *b*. The values X_X and R_X are those of the series circuit which is electrically equivalent to the branched portion of Fig. 127*a*. The final simple circuit, shown in *c*, is obtained from *b* by putting $X = X_1 + X_X$ and $R = R_1 + R_X$. This same method of treatment holds good no matter how complicated the circuit may be. The parallel portions of the network must always be first reduced to a single circuit, and the resistances and reactances of the various portions of the complex single-branched circuit are then obtained by adding those of the separate parts.

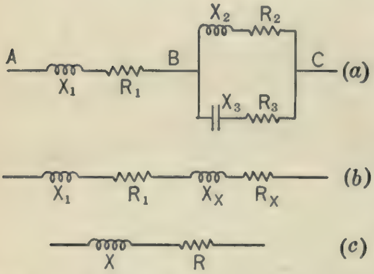


Fig. 127.—Showing how, by successive steps, a complicated circuit is reduced to a simple one.

PROBLEMS

1. An iron-core coil of 5000 turns has an average length of the magnetic circuit of 20 inches and an average cross-section of 100 square inches. What is its inductance in millihenries if the permeability of the iron is 500?

2. In a certain circuit the resistance is 10 ohms, the inductance is 0.026 henry, and the frequency is 25 cycles. What e.m.f. is necessary to cause a current of 5.4 amperes to flow? What current would flow if a continuous e.m.f. of this value were impressed?

3. A resistance of 1.2 ohms in series with a coil of 0.063 henry is connected to a 25-cycle source of 75 volts (effective). What is the current and angle of lag? Give the equation expressing the instantaneous value of the current.

4. If a current of 10 amperes flows through a circuit of 2 ohms resistance and 20 millihenries inductance when connected to a 110-volt a.c. line, what is the frequency? Reactance? Impedance?

5. The resistance of a circuit is 5 ohms, and with a 60-cycle, 185-volt e.m.f. impressed, a current of 20 amperes flows. What is the inductance, reactance, and power factor of the circuit?

6. An iron-core coil having 1000 feet of No. 10 copper wire in its winding, has a hysteresis loss of 26 watts, and an eddy current loss of 8 watts, when a current of 2.4 amperes is flowing. What is the effective resistance of the circuit?

7. In a certain iron-core coil having a conductor resistance of 120 ohms, a current of 0.245 ampere flows when connected to a 220-volt, 60-cycle line, the core loss being 28 watts. What is the value of L, and what is the angle of lag of the current?

8. An 8-ohm coil is connected to a 110-volt line in which $\omega = 377$, and the current lags 30° behind the voltage. Find the current, inductance, and reactance of the circuit.

9. Required the e.m.f. necessary to send 12.2 amperes through a circuit having 30 ohms resistance and an inductance drop of 21.6 volts.

10. In a circuit of 25 ohms resistance and 5 millihenries inductance, what is the largest value the frequency can have without reducing the current more than 2 per cent of the value it has in a continuous-current circuit?

11. A voltmeter has a resistance of 1250 ohms and an inductance of 23 millihenries. It is correct on a 60-cycle line. What is the maximum value the frequency can have without the error exceeding 1 per cent?

12. For an inductive circuit $R = 5$ ohms, $L = 0.03$ henry, $E = 150$ volts, $I = 12.3$ amperes. Frequency?

13. Two circuits having resistances of 3 and 20 ohms, and inductances of 0.12 and 0.036 henry, respectively, are in series. A current of 16.3 amperes is flowing at 60 cycles. What is the impressed voltage, and drop across each?

14. A 50-volt, 20-watt lamp is to be operated (at rated current and voltage) from a 110-volt, 25-cycle line by using a proper choke coil in series with it. If the coil has 15 ohms resistance, what must be its inductance? What will be the drop across the coil? What will be the power factor of the circuit and of the coil?

15. Suppose a resistance were used in Prob. 14, instead of the choke coil. What would be the increased cost per month if the lamp is operated 10 hours a day, power at $0.08 per kilowatt-hour?

16. Given two circuits having $R_1 = 5$ ohms, $R_2 = 20$ ohms, $L_1 = 0.01$ henry, $L_2 = 0.02$ henry. If an alternating current of 350 amperes with $\omega = 600$ be put through these two circuits in series, what e.m.f. must be used, and what will be the drop and angle of lag for each?

17. A 100-watt, 110-volt lamp is to be operated (at rated power and voltage) from a 220-volt, 60-cycle line by means of a suitable choke coil. The coil has a power factor of 0.12. What must be the inductance of the coil, and what is the power factor of the combination?

18. A circuit of $R = 8$, $X = 2$ is supplied over a transmission line of 0.6 ohm resistance and 0.7 reactance. If the generated e.m.f. is 1100 volts, what will be the e.m.f. at the load? What will be the regulation of the line? What will be the actual voltage drop in the line when supplying the load, and how much does the load voltage change if the load is open-circuited?

19. When connected to a 110-volt, 60-cycle line through a resistance of 2.5 ohms, a condenser draws 6.32 amperes. What is its capacity? Angle of lead of the current? Power used? Power factor?

20. How much resistance must be put in series with a 30-microfarad condenser connected to a 110-volt 500-cycle generator to limit the current to 3.2 amperes? What will be the drop across the condenser?

21. A 25-cycle, 100,000-volt, 160-mile transmission line (assumed single-phase for simplicity) has a capacity of 0.0123 microfarad per mile. What is the capacity reactance? What must be the rating (in kilovolt-amperes) of an alternator used to keep the line alive (no load on the line)?

22. Two circuits in parallel on a 2200-volt line have resistances of 5 and 12 ohms, and inductive reactances of 2 and 20 ohms, respectively. Required the line current and line power factor.

23. Fifteen 25-watt, 110-volt lamps are in parallel on a 110-volt 60-cycle line, to which is also connected a coil of 5 ohms resistance and 4 millihenries inductance. What is the line current, angle of lag, power factor, and total power?

24. A 5-ampere, 50-watt lamp is to be operated in a 10-ampere series lighting circuit by the use of a proper shunt choke coil. Assuming the resistance of the coil negligible, what must be the inductance of the coil?

25. An induction motor taking 8 amperes at a power factor of 78 per cent is operating on the same line with thirty 0.4-ampere incandescent lamps. What is the line current and power factor?

26. A coil of $R=15$ ohms and $L=0.02$ henry is connected in series with a 120-microfarad condenser of negligible resistance. The line voltage is 110 and the frequency is 80 cycles. What is the line current and power factor? What is the drop across the coil and what across the condenser?

27. A coil of 19 ohms resistance and 8 millihenries, in series with a condenser of 120 microfarads with 5 ohms equivalent series resistance is connected to a 60-cycle line. What impressed voltage is necessary to send 11 amperes through the circuit? What is the line power factor? What frequency would give a line power factor of unity? What current would the circuit draw at this frequency?

28. A coil of 25 ohms resistance passes a current of 2 amperes when connected to a 1000-volt, 60-cycle line. What capacity must be used in series with the coil to give a line power factor of unity? What would be the drop across the condenser in terms of the impressed voltage?

29. A condenser of 20 microfarads in series with a resistance of 3 ohms is connected in parallel with a coil of 40 millihenries and 6 ohms resistance. The line voltage is 110 and frequency is 80 cycles. What is the line current and line power factor?

30. The cost of a certain 20,000 kv.a. power plant complete is $125 per kv.a. If the current supplied by the power house (on the average) lags by 35°, what is the available output in kw.? What is the decrease in value of the plant from the value it would have if the load had a unity power factor?

31. A wattmeter indicates proportional to the cosine of the angle between the current in the current coil and that in the potential coil. Required the maximum inductance allowable in the potential circuit (which has a resistance of 1200 ohms) in order that the error with a 60-cycle current shall not exceed $\frac{1}{2}$ of 1 per cent, the maximum lag in the load current being measured, not exceeding 45°.

CHAPTER IV

THE POLYPHASE CIRCUIT

76. Superiority of Polyphase Power over Single-phase Power.—The polyphase system, whether two-, three-, or six-phase, is evidently more complicated than the single-phase system, yet practically all electrical power systems are three-phase.

It has been shown that in a single-phase circuit the power is pulsating. With a power factor equal to unity, the instantaneous power has a value of zero twice in each cycle; with a power factor less than unity, the instantaneous power is not only zero four times in each cycle, but is negative twice in each cycle. Thus the power received by a single-phase motor from its supply line is zero twice each cycle; as it is delivering power to its load at a practically uniform rate, it must have a certain amount of flywheel action. The motor evidently gives out its energy in mechanical form at an essentially constant rate, but obtains its electrical energy at a pulsating rate. During a part of each cycle, it must therefore store sufficient energy in its rotating parts, to be converted into mechanical form during that part of the cycle when no energy is being obtained from the source.

It will be shown that, in a balanced polyphase system, although the power of any one phase is zero, or even negative at certain instants, the total power of all the phases is constant.

There are three important reasons why polyphase power is superior to single-phase power: 1. Polyphase apparatus is smaller and weighs less than single-phase apparatus of the same capacity, and is in general less complicated; it is therefore cheaper to build than single-phase apparatus. 2. Polyphase machinery (induction motors, synchronous converters, etc.), has better operating characteristics than single-phase machinery. 3. Three-phase power-transmission lines of a given kw. capacity, voltage, and specified efficiency of transmission, require only three-quarters as much copper for the line as would be required by a single-phase line. These points will be discussed in detail in later sections of the text.

77. Two-phase, Four-wire System.—We have seen (page 64) that if the coils on the armature of an alternator are in two separate groups, so arranged as to give two equal voltages 90° apart, a two-phase alternator results. Thus, one phase consisting of one-half of the armature winding generates one of the two voltages, and the second phase, or other half of the winding, the second voltage. In practice the two phases of a two-phase alternator generally have no electrical connection within the machine, the four ends of the two phases being brought out separately through the frame, as in Fig. 59.

If the four leads coming out from the winding are continued to the load,

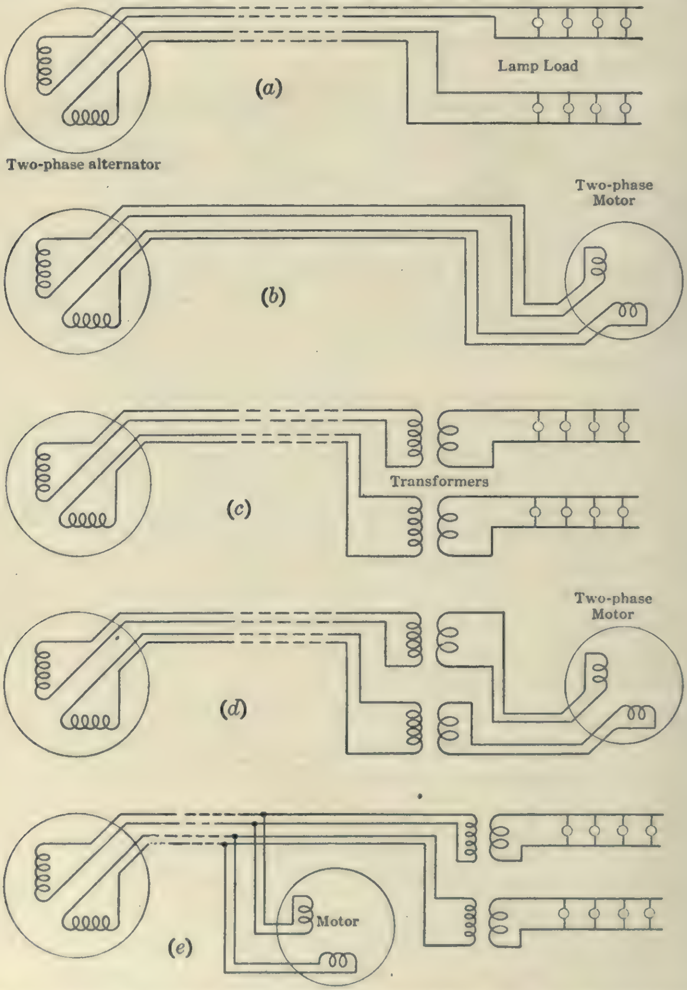


FIG. 128.—Various ways of supplying power from a two-phase four wire alternator.

the system is known as a two-phase, four-wire system, this being the general practice when a two-phase system of power distribution is used. The load on each phase may consist of a separate bank of lamps, as in Fig. 128*a*; one of the two phases of a two-phase motor, as in Fig. 128*b*; a transformer which in turn has a load connected to its secondary, as in Figs. 128*c* and *d*;

or any suitable combination, as in Fig. 128*e*. In Figs. 128*a* and *b* the voltage generated by the alternator is suitable for the lamps and motor respectively; in Figs. 128*c* and *d*, it is not suitable, so that the transformers are necessary. In Fig. 128*e* the voltage generated by the alternator is shown as suitable for the motor, but not for the lamps, hence these must be connected to the alternator through transformers.

78. Two-phase, Three-wire System.—If two of the terminals of a two-phase alternator, one from each phase, are connected together, and a similar connection is made at the load, two-phase power may be transmitted over three wires, as in Fig. 129. One wire is thus common to the two phases, and the system is known as a two-phase, three-wire system. Each phase of the load has impressed upon it the voltage generated by the respective phase of the alternator, so that the current taken by each phase of the load depends only upon its own resistance and reactance, and upon the value of the voltage impressed. The current taken by each phase of the load is thus independent of the other phase of the load.

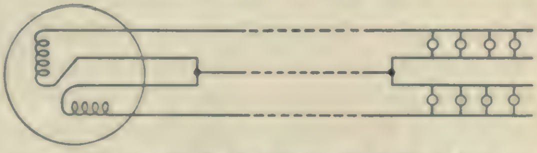


FIG. 129.—One method of connecting a two-phase lamp load to a two-phase alternator.

If both phases of the load draw current of the same numerical value, at the same power factor, the current drawn by one phase will lead that drawn by the other phase by 90° (Fig. 133). The current flowing in the common wire will then be the vector sum of two equal currents 90° apart; by reasoning similar to that on page 66, we have

$$I_c = I\sqrt{2} = 1.414I,$$

where I_c is the current in the common wire, and I is the current per phase, the same as flows in each outside wire.

When two-phase power is transmitted by three wires, the cross-section of the common wire must be 41.4 per cent greater than that of either of the two outside wires. The three-wire system is, however, more economical of copper than the four-wire system. If the cross-section required by the four-wire system is considered as 100 per cent, or that of each of the four wires as 25 per cent, the cross-section required by the three-wire system will be $(25\times2)+(25\times1.414)=50+35.3=85.3$ per cent.

In the three-wire system there is the disadvantage that the voltage between the two outside wires is equal to the voltage per phase multiplied by 1.414 (page 66). Better insulation is thus required for the three-wire system than is necessary in the four-wire system, and if grounds occur on the system, persons coming in contact with the system are liable to be subjected to higher voltage shocks than are possible in the four-wire system with the same voltage per phase.

79. Balanced Polyphase System.—A polyphase system is said to be *balanced* when the conditions in all phases are the same, that is, when the voltage impressed upon each phase, the current flowing in each phase, and the power factor of each phase are the same. When any one or more of these three quantities are not the same, the system is said to be *unbalanced.*

80. The Wattmeter.—From what has been said in the previous chapter, it is evident that the power used in an alternating-current circuit cannot be determined by employing a voltmeter and ammeter as would be done in a continuous-current circuit. If it were possible by some means to determine ϕ, the phase difference between voltage and current, then the power could be calculated from the known value of ϕ and the readings of these instruments, but to determine ϕ directly it is necessary to use either a laborious method of curve plotting or else special instruments, such as the oscillograph or power-factor indicator; these instruments are not generally available for this purpose.

An indicating instrument has been designed, however, which is constantly used on a.c. circuits, and will actually indicate the $EI \cos \phi$ value for any circuit; it is called the *wattmeter.* There are two coils in this instrument, one fixed and made of a few turns of comparatively large conductor, and the other movable and made of many turns of fine wire. This movable coil is mounted on a shaft, fitted with jeweled bearings, so that it may turn inside the stationary coil. It carries an indicating finger which moves over a suitably graduated scale.

It will be proved theoretically in a subsequent chapter and it may be proved experimentally in the laboratory, that such an instrument, when properly calibrated, actually reads the product of the current flowing through its current coil, the voltage impressed upon its potential coil, and the cosine of the angle between this current and voltage, the current and voltage being expressed in effective values. Fig. 180 shows a well-known type of portable wattmeter. The current coil terminates in the heavy binding-posts on the right of the instrument, and the potential coil is connected to the two smaller posts seen at the left side of the instrument.

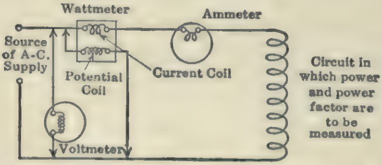


Fig. 130.—The normal connection of a wattmeter, ammeter, and voltmeter to measure power and power factor.

In Fig. 130 is shown the connection of an ammeter, voltmeter, and wattmeter to a circuit in which it is desired to know the power consumption and power factor. It will be seen that the current coil of the wattmeter is in series with the circuit and the potential coil connected across the line.

When line current flows through the current coil of a wattmeter there is set up a field which is proportional to the line current and in phase with it, and which alternates in direction with the frequency of the line current. The potential coil of the wattmeter generally has, in series with it, a high resistance which is wound non-inductively, so that when voltage is impressed on the potential circuit a current flows through the circuit which is proportional to the voltage and practically in phase with it. The field produced by the current flowing through the potential circuit alternates in direction with the frequency of the voltage, and since line current and voltage have the same frequency, the two fields of the wattmeter alternate in direction at the same frequency.

The actuating force of a wattmeter being derived from the interaction of the field of its current coil and the field of its potential coil, the instrument will have polarity. With its current and potential coils connected to a circuit as in Fig. 130, the needle may move up the scale, or indicate positively, but if such is the case, and its potential leads are connected to the line in reversed order, as in Fig. 131, the needle will move backwards, or indicate negatively.

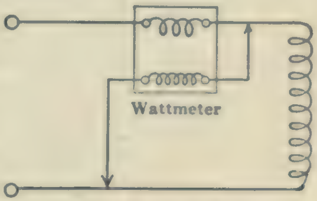


FIG. 131.—If the wattmeter of Fig. 130 deflects positively, it will deflect negatively (backward) if the potential leads are reversed, as shown here.

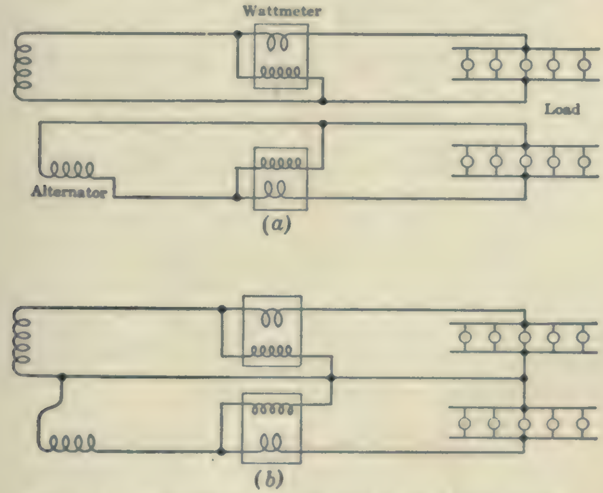


FIG. 132.—Wattmeter connections in two-phase circuits.

81. Measurement of Power in a Two-phase System.—It will be seen that a two-phase system may be considered as made up of two single-phase systems, the voltages of which are 90° apart. To measure the power in a two-phase, four-wire system, it is obviously necessary to use two wattmeters, one in each phase as in Fig. 132*a*; the sum of the two wattmeter readings will always give the total power delivered by the alternator, whether the system is balanced or not.

To correctly measure power in a two-phase, three-wire system under all conditions, two wattmeters are used as in Fig. 132*b*, the current coils of the wattmeters being placed one in each of the outer wires and their potential coils placed across, from the line in which the current coil is situated, to the common wire. The upper wattmeter carries the current of the upper phase, and has impressed upon its potential coil the voltage across the upper phase; the wattmeter therefore indicates only the power supplied by the upper phase. In the same way the lower wattmeter indicates only the power supplied by the lower phase.

It is possible to measure the power dissipated in a balanced two-phase, three-wire system by using only one wattmeter. As this method is useless when the system becomes unbalanced, it is never used commercially and is therefore not considered in this text.

82. Uniform Power in a Two-phase System.—We have seen that in a single-phase circuit, for the ordinary inductive load, the instantaneous power is given by the equation

$$\begin{aligned}p &= E_m \sin \omega t \times I_m \sin (\omega t - \phi),\\ &= E_m I_m \{\sin \omega t\ (\sin \omega t \cos \phi - \cos \omega t \sin \phi)\},\\ &= E_m I_m\ (\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi),\end{aligned}$$

This equation indicates that the instantaneous power varies with time. We have previously shown it to be pulsating at double the frequency of the current and voltage.

Consider a balanced two-phase system, the current in each phase lagging by the angle ϕ. If phase No. 2 leads phase No. 1 by 90°, and the voltage of phase No. 1 is expressed by the equation $e' = E'_m \sin \omega t$, the current in phase No. 1 will be indicated by the equation $i' = I'_m \sin (\omega t - \phi)$, and the instantaneous power by the equation

$$p' = E'_m \sin \omega t \times I'_m \sin (\omega t - \phi).$$

Since phase No. 2 leads phase No. 1 by 90° its voltage will be expressed by the equation $e'' = E''_m \sin (\omega t + 90°) = E''_m \cos \omega t$, its current by the equation $i'' = I''_m \sin (\omega t + 90° - \phi) = I''_m \cos (\omega t - \phi)$, and its power by

$$p'' = E''_m \cos \omega t \times I''_m \cos (\omega t - \phi).$$

Having assumed that the system was balanced, we have $E' = E''$, and that $I' = I''$.

The instantaneous power of the system will therefore be

$$\begin{aligned}
P=p'+p''&=E_m \sin \omega t\times I_m \sin (\omega t-\phi)+E_m \cos \omega t\times I_m \cos (\omega t-\phi)\\
&=E_mI_m\{\sin \omega t\ (\sin \omega t \cos \phi-\cos \omega t \sin \phi)\\
&\qquad+\cos \omega t\ (\cos \omega t \cos \phi+\sin \omega t \sin \phi)\}\\
&=E_mI_m\ (\sin^2 \omega t \cos \phi-\sin \omega t \cos \omega t \sin \phi\\
&\qquad+\cos^2 \omega t \cos \phi+\sin \omega t \cos \omega t \sin \phi)\\
&=E_mI_m \cos \phi\ (\sin^2 \omega t+\cos^2 \omega t)\\
&=E_mI_m \cos \phi,
\end{aligned}$$

which is constant, independent of time.

The last expression may be further reduced to

$$P=E_mI_m \cos \phi=2\frac{E_m}{\sqrt{2}}\,\frac{I_m}{\sqrt{2}} \cos \phi=2EI \cos \phi,$$

where E and I are the effective voltage and current per phase.

The power in a balanced two-phase system is therefore shown to be constant (as is the case for any balanced polyphase system) and equal to twice the power per phase, as we know it must be.

That the instantaneous power of a two-phase balanced system is constant may be seen from the power curves. In Fig. 133 are shown the voltage and current curves for a balanced system, each current lagging behind its respective voltage by 20° (i.e., $\phi=20°$); the corresponding curves of instantaneous power are shown in Fig. 134. It may be seen that the algebraic sum of the ordinates of the two power curves at any instant add up to a constant value, indicated in the figure as the "system power."

At time $=\pi/2$, when the power of phase No. 2 is zero (because the value of its voltage is zero), the system power is furnished by phase No. 1. If in Fig. 133 the maximum values of voltage and current are taken as 40 volts and 30 amperes respectively, the power of phase No. 1, at time $=\pi/2$, is

$$\begin{aligned}
p'&=40 \sin 90°\times 30 \sin (90°-20°)=40\times 30 \sin 70°\\
&=1200\times 0.9397=1127.6 \text{ watts.}
\end{aligned}$$

The power curves are seen to cross when the voltage of phase No. 1 is at its 55° value. The power indicated by phase No. 1 is then

$$\begin{aligned}
p'&=40 \sin 55°\times 30 \sin (55°-20°)\\
&=40 \sin 55°\times 30 \sin 35°=40\times 0.8192+30\times 0.5736\\
&=32.768\times 17.208=563.8 \text{ watts.}
\end{aligned}$$

The power indicated by phase No. 2 is then

$$p'' = 40 \sin (55° + 90°) \times 30 \sin (55° + 90° - 20°)$$
$$= 40 \sin 55° \times 30 \sin 35° = 563.8 \text{ watts.}$$

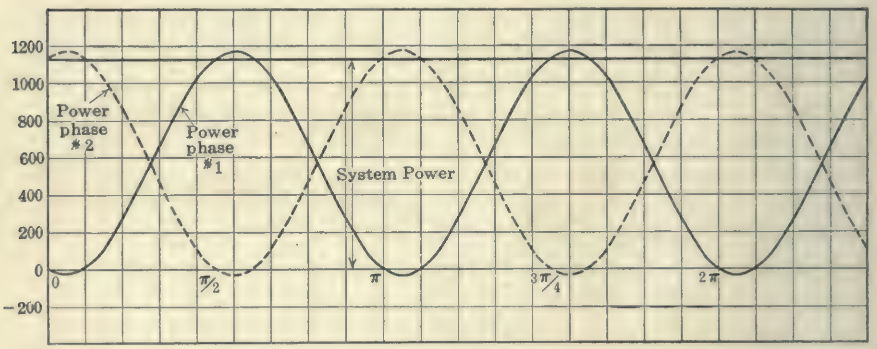


FIG. 133.—Voltage and current curves for a balanced two-phase load, the current in each phase lagging 20° behind its respective voltage.

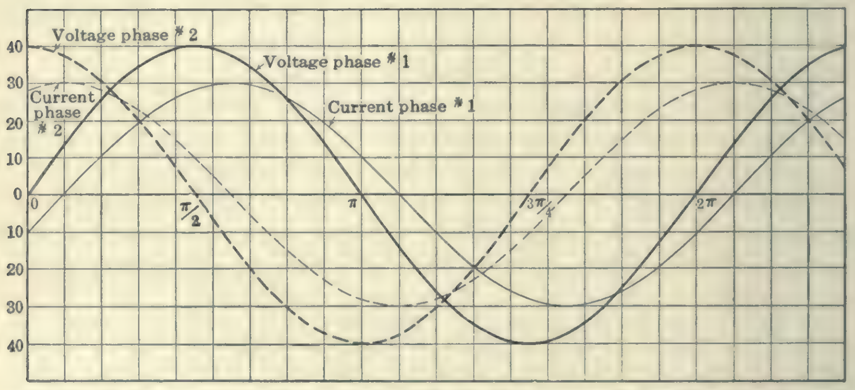


FIG. 134.—The power curves corresponding to the conditions of Fig. 133; although the power supply of each phase is pulsating the total power supply is uniform.

The effective current per phase is 30×0.707 = 21.21 amperes, and the effective voltage per phase is 40×0.707 = 28.28 volts. The average power per phase is $EI \cos \phi$, or

$$p = 21.21 \times 28.28 \cos 20° = 600 \times 0.9397 = 563.8 \text{ watts,}$$

or the total power is 1127.6 watts.

83. Three-phase, Y-connected System.—We saw that a simple three-phase alternator had three coils on its armature, and that by connecting together an end of each coil, the three ends selected being 120° apart, the machine became a Y-connected, three-phase alternator, delivering three equal voltages 120° apart at the open ends.

A Y-connected, three-phase load consists of three separate loads, with one end of each load connected to one end of each of the others. The three open ends are then connected to the three leads from an alternator. Such an arrangement, consisting of a Y-connected alternator and a Y-connected lamp load, is diagrammatically represented in Fig. 135, the lamp load being drawn similar to the letter Y, even though it would probably be arranged as in Fig. 136.

The relation between the voltage per phase of the alternator and the voltage between the lines was given in Eq. (48), (page 75), as

$$E=\sqrt{3}e \quad \text{or,} \quad e=\frac{E}{\sqrt{3}}. \qquad (110)$$

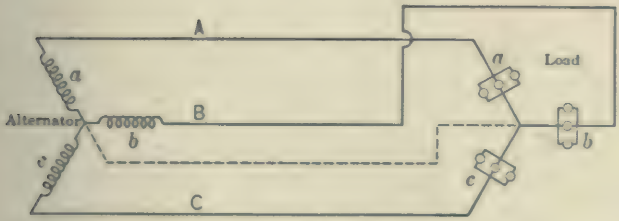


FIG. 135.

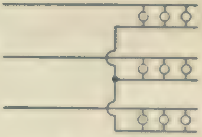


FIG. 136.

FIG. 135.—A Y-connected load being supplied with power from a Y-connected alternator. The central wire (neutral) shown in dashed line, is generally not used.

FIG. 136.—The way a Y-connected load of lamps actually appears.

If the Y-connected load is balanced, it is then symmetrical, and we should expect that the line voltages would resolve themselves into the phase voltages of the load in the same ratio that the phase voltages of the alternator combined to form the line voltages. That is, the same relation exists between the line voltages and the phase voltages of the load as exists between the line voltages and the phase voltages of the alternator. If the three line voltages supplying the load are equal to each other and are 120° apart, then the three phase voltages of a Y-connected load are also 120° apart and equal to each other, the relation between the numerical values of phase and line voltages being as in Eq. (110).

In Fig. 135 there is indicated a connection between the center of the Y of the alternator and the center of the Y of the load; this wire, sometimes used in practice but generally not, is called the *neutral wire*, as it joins the so-called " neutral " of the alternator to that of the load.

It is seen that this neutral wire may be considered as the common return wire for all three phases. The current which flows into it from load a is evidently in phase with the voltage of phase a, because we may consider phase a and its load a as independent of the other loads. The current which flows into the neutral wire from load b is likewise in phase with the voltage of alternator phase b, and there is similarly a current from load c in phase with the voltage of phase c. As the voltages of the different phases are equal and 120° apart, and the load resistances are all equal, the neutral wire must carry the vector sum of three equal currents 120° apart in phase. But the sum of three such currents is zero, and the neutral wire will therefore carry no current and is thus entirely unnecessary. If, however, the load is unbalanced, the equality of voltage on the three phases, that is $e=\frac{E}{\sqrt{3}}$, can be maintained only if such a neutral wire is used; for unbalanced loads the neutral wire does carry a current. The connection joining the neutral points of load and alternator should therefore be used when unbalanced lamp loads are likely to occur; such a wiring scheme is known as a *three-phase, four-wire system.*

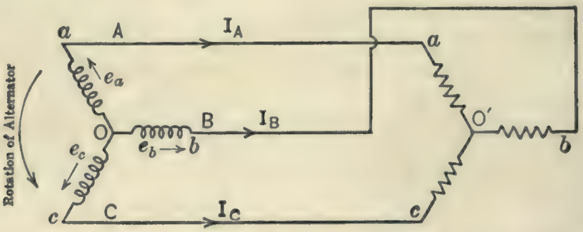


FIG. 137.—A three-phase circuit in which the voltage and current relations are to be analyzed.

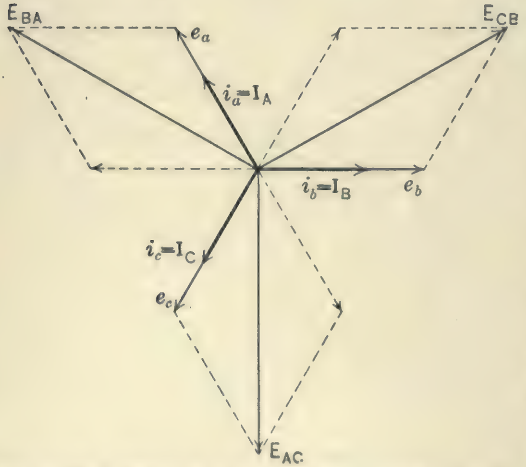


FIG. 138.—Voltage and current relations for the circuit of Fig. 137; line currents lag 30° behind their respective voltages.

84. Vector Diagram for a Non-inductive Three-phase, Y-connected System.—We assume in Fig. 137 that the alternator is revolving in such a direction that phase voltage $e_{ob}=e_b$ lags 120° behind $e_{oa}=e_a$, and that e_a lags 120° behind $e_{co}=e_c$, as is indicated in Fig. 138. As the load is resistive $I_A(=i_a)$ is in phase with e_a, $I_B(=i_b)$ is in phase with e_b, and $I_C(=i_c)$ is in phase with e_c.

If it is assumed that the positive direction of a current is that in which the current would be urged by a positive voltage, and the positive phase voltages are e_{oa}, e_{ob}, and e_{oc}, then the positive direction of the line currents is away from the alternator, as is indicated in Fig. 137. This is entirely consistent with what was said in Chapter II with respect to the voltages of an alternator; what we indicate by the arrows is the *positive directions of the currents whenever they may occur*, but not instantaneous values.

We might just as well assume currents *towards* the alternator as positive, provided *all of them are assumed in the same direction*; we could not, however, assume two currents *away from* the alternator and the other current *toward* the alternator, as these would not represent the three-phase currents of a three-phase balanced system.

The voltage between lines B and A, that is, the voltage E_{BA}, will then be the vector sum of the voltages e_{bo} and e_{oa}, or

$$E_{BA}=e_{bo}\oplus e_{oa}=\ominus e_b\oplus e_a,$$

similarly

$$E_{AC}=\ominus e_a\oplus e_c,$$

and

$$E_{CB}=\ominus e_c\oplus e_b.$$

Carrying out the vector construction indicated in Fig. 138, we find that voltage E_{BA} lags behind the voltage E_{AC} and this in turn lags behind E_{CB}. A non-inductive load having been assumed in Fig. 137, the current in phase a, $i_a(=I_A)$, is in phase with the phase voltage e_a, the current in phase b, $i_b(=I_B)$, is in phase with the phase voltage e_b, etc.

The vector diagram shows that the line currents, for the resistive load assumed, lag 30° behind their respective line voltages (when the phase voltages are taken in the order that e_b lags e_a lags e_c), that is, I_A lags 30° behind E_{BA}, I_B lags 30° behind E_{CB}, and I_C lags 30° behind E_{AC}. If instead of taking the three-line voltages as E_{BA}, E_{AC}, and E_{CB}, we had taken these voltages as E_{AB}, E_{BC}, and E_{CA}, we should have I_A leading the voltage E_{CA} by 30° and leading the voltage E_{AB} by 150°.

85. Suitable Notation for Three-phase Vectors.—It will be seen that the convention that we have used in constructing Fig. 138 is such that the subscripts of the voltages are taken in the sense that the first letter of the subscript is that of the voltage of the phase lagging 120° behind the voltage of the phase designated by the last letter of the subscript; thus the designation E_{BA} indicates that phase voltage e_b lags 120° behind e_a, E_{AC} indicates that e_a lags 120° behind e_c, etc.

Furthermore, the convention used is such as to associate the line current with the voltage which has the letter of that line as the last letter of its subscript; that is, current I_A lags 30° behind the line voltage E_{BA}, I_C lags 30° behind E_{AC}, and I_B lags 30° behind E_{CB}. Furthermore, currents away from the alternator are taken as positive.

This convention for designating the voltages makes voltage E_{BA} lag behind the voltage E_{AC}, and E_{AC} lag behind E_{CB}.

86. Vector Diagram for Reversed Phase Rotation.—If we now consider the phase rotation reversed from what it was in Figs. 137 and 138, as may be brought about by reversing the direction of rotation of the alternator, so that e_a lags behind e_b by 120°, and e_b lags behind ϵ_c, the conditions are as shown in Fig. 139.

Again I_A will lag behind the voltage with which it is to be associated, E_{CA}, by 30°, and the method of designating this voltage, E_{CA}, means that the voltage of phase c lags behind that of phase a by 120°. Similarly, the current I_B is associated with the voltage having B for its last subscript, namely E_{AB}, the subscripts of this voltage meaning that phase a lags behind phase b by 120°.

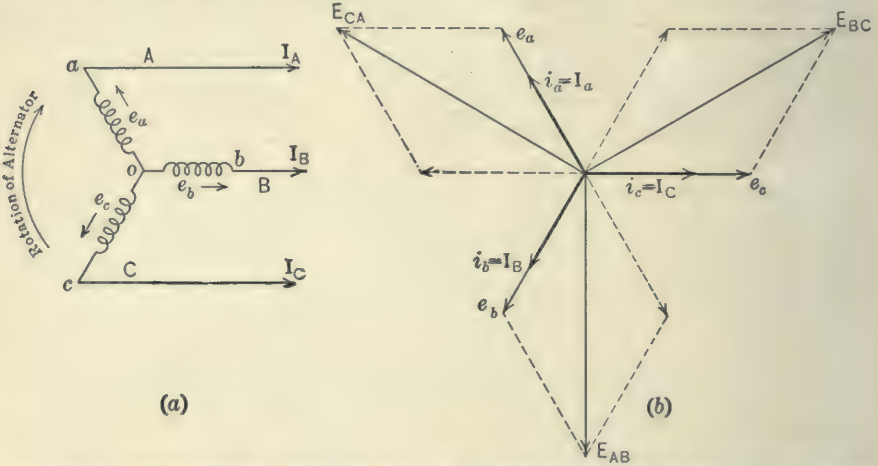


FIG. 139.—Even though the direction of rotation of the alternator is reversed the line currents still lag 30° behind their respective voltages, but the voltages are taken in a different sense than for Fig. 137.

Thus we have the rule that, for a resistive load, currents away from the power supply being considered positive, and voltages between lines being written in such a way that the sequence of the subscripts gives the phase rotation as shown above, the line currents lag behind their respective line voltages by 30°.

87. Power in a Three-phase, Y-connected System.—In a balanced three-phase system, the total power will evidently be three times the power per phase, i.e.,

$$P = 3ei \cos \phi.$$

where cos ϕ is the power factor of the load, that is, cosine of the angle *between the phase voltage and the phase current.*

Since $e=\frac{E}{\sqrt{3}}$ and $i=I$,

we have

$$P=3\times\frac{E}{\sqrt{3}}\times I\times\cos\phi=\sqrt{3}EI\cos\phi. \quad . \quad . \quad . \quad (111)$$

The last equation states, then, that the power in a balanced three-phase, Y-connected system is equal to the product of the line voltage, the line current, square root of three, and *the cosine of the angle between phase voltage and phase current.*

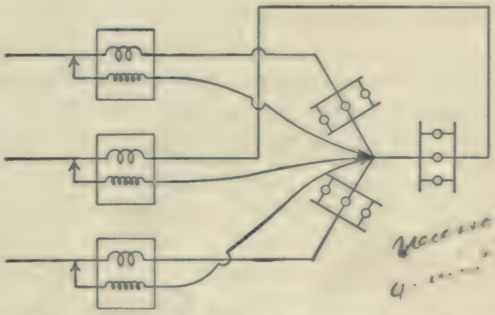


FIG. 140.—A possible scheme for measuring three-phase power when the neutral point of the load is available.

(for [illegible]

[illegible]

88. Measurement of Power in a Three-phase, Y-connected System. — *Three-wattmeter method.*—The power in a three-phase, Y-connected load, whether balanced or unbalanced, may evidently be obtained by the use of three wattmeters, connected as in Fig. 140. Here the current coil of a wattmeter is placed in series with each phase of the load, so that the current flowing through the current coil of the meter is that of the phase. The potential coils of the wattmeters have impressed upon them the phase voltage of their respective phases, so that each wattmeter indicates the power used in its respective phase. The sum of the three wattmeter

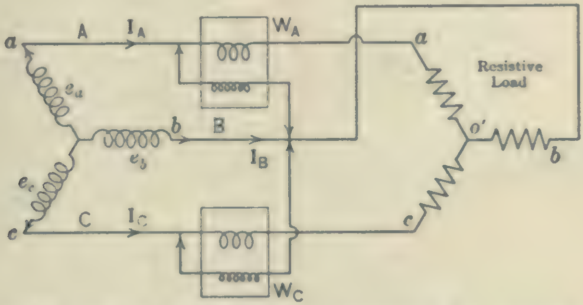


FIG. 141.—Standard method of wattmeter connection for measuring three-phase power, good for any power factor and condition of load.

readings gives the total power used; if the system is balanced the three readings are equal.

Two-wattmeter method.—The power in a three-phase system can be, and ordinarily is, measured by two wattmeters connected as is shown in Fig. 141. The current coils of the two meters are placed in series with two of the lines, and the potential coil of each instrument is connected from the line in which its current coil is placed, to the third line, or that line in which no wattmeter is placed. It is to be noted in Fig. 141, that the connections of the potential coils are symmetrical with respect to line *B*.

Since the vector sum of the three line currents is zero, we may consider any one line as the return path for the currents in the other two lines. Thus in Fig. 141 we may consider line *B* as the return path for the currents in line *A* and *C*. From this viewpoint all voltages must be considered toward line *B* or away from *B*. Thus if it is considered that there is impressed upon the potential circuit of the upper wattmeter, W_A, the voltage E_{BA}, that is, the voltage *from B to A*, then we must impress upon the potential coil of the lower wattmeter, W_C, the voltage E_{BC}, that is, the voltage *from B to C*.

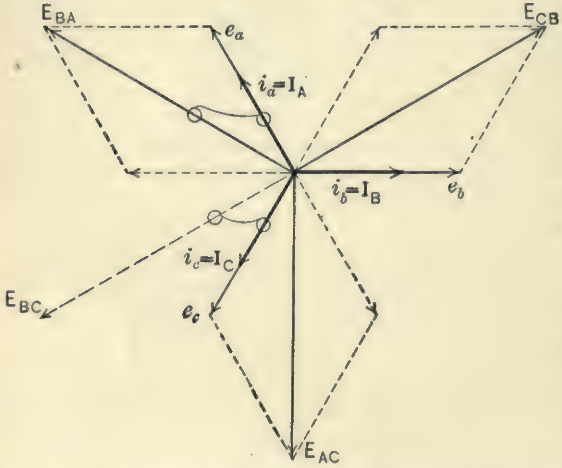


FIG. 142.—Showing the voltage and current relations for Fig. 141, from which the wattmeter readings can be calculated.

We know, however, that the reading of each wattmeter will be equal to the product of the current through its current coil, the voltage applied to its potential coil, and the cosine of the angle between this current and voltage. Thus the upper wattmeter, W_A, carries the line current I_A and has impressed upon its potential circuit the voltage E_{BA}. From the convention used in designating this current and voltage, or by reference to Fig. 142, which is the same vector diagram as that of Fig. 138, we see that I_A lags 30° behind E_{BA}. We see then that the power indicated by the upper wattmeter is

$$W_A = E_{BA} \times I_A \cos 30°.$$

The lower wattmeter, W_C, carries the line current I_C and has impressed upon its potential circuit the voltage E_{BC}. This voltage lies 180° from the voltage E_{CB} of the vector diagram, so that current I_C leads voltage E_{BC} by 30°. The power indicated by the lower wattmeter is then

$$W_C = E_{BC} \times I_C \cos 30°.$$

Since the system is balanced, line voltages are equal and line currents are equal, so that we may write

$$W_A + W_C = 2EI \cos 30° = 2EI\tfrac{1}{2}\sqrt{3} = \sqrt{3}EI.$$

Considering that the power factor of the circuit is unity, the last expression is identical with Eq. (111); it is thus evident that the power of a balanced, non-inductive, three-phase, Y-connected load can be measured by two wattmeters. For this balanced resistive load the two wattmeter readings are the same.

89. Two-wattmeter Method Analyzed from Instantaneous Values.—That the sum of the readings of the two wattmeters, placed in the lines as in Fig. 141, represents the true power, may be proved from instantaneous values. If i_a, i_b, i_c, and e_a, e_b, e_c represent the instantaneous currents and phase voltages, respectively, the instantaneous power will be equal to their products, regardless of power factor. That is, the total instantaneous power of the system is

$$p = e_a i_a + e_b i_b + e_c i_c.$$

But, in a Y-connected system without a neutral connection, the sum of the three currents is zero, or

$$i_a + i_b + i_c = 0,$$

and therefore

$$i_b = -i_a - i_c.$$

Substituting for i_b, we have

$$p = e_a i_a + e_b(-i_a - i_c) + e_c i_c$$

$$= i_a(e_a - e_b) + i_c(e_c - e_b)$$

We have already seen that $e_a - e_b$ is the instantaneous line voltage e_{BA}; similarly $e_c - e_b$ is the line voltage e_{BC}. The instantaneous force acting on the upper wattmeter is therefore proportional to

$$w_A = e_{BA} i_A,$$

and that of the lower wattmeter, to

$$w_C = e_{BC} i_C,$$

so that the sum of the forces acting on the two wattmeters at any instant is proportional to the instantaneous value of the three-phase power. As the wattmeter indicates proportional to the average force acting on it, and as this average force is shown to be such that, for both wattmeters, its average value is proportional to the average three-phase power, the sum of the two wattmeter readings must give the total three-phase power.

90. Wattmeter Reading Negative or Positive.—It may be seen from the vector diagram that if the voltages E_{AB} and E_{CB} had been impressed upon the potential circuits of the upper and lower wattmeters, respectively, instead of E_{BA} and E_{BC}, the indications of the meters would have been

$$W_A = E_{AB} I_A \cos 150° = E_{AB} I_A \ (-\cos 30°),$$

$$W_C = E_{CB} I_C \cos 150° = E_{CB} I_C \ (-\cos 30°).$$

Both readings would be negative, and the wattmeter needles would therefore move below zero. An observer would naturally reverse the potential connections of each meter, which would reverse the voltage applied to each by 180°, thus impressing the voltage E_{BA} upon the upper meter and the voltage E_{BC} upon the lower.

The question naturally arises, "Should these reversed readings be called negative?" The answer is obtained from a consideration of the power, for, as we have assumed a symmetrical connection, we must call both readings negative, or both positive and, unless some special condition arises, they are called positive. Later on we shall find that with the connection assumed, and for loads of low power factors, one wattmeter will indicate positive and the other negative; only then will one of them be called negative.

91. Problem.—Assume that each resistance in Fig. 137 is 5 ohms and that the voltage supplied by the alternator between lines is 110 volts. Calculate the power used in the circuit and the reading of each wattmeter.

The phase voltage is $\frac{110}{\sqrt{3}} = 63.5$ volts and the phase currents $\frac{63.5}{5} = 12.7$ amperes.

The total power is

$$P = 3ei = 3 \times 63.5 \times 12.7 = 2419.6 \text{ watts},$$

or,

$$P = \sqrt{3} EI = \sqrt{3} \times 110 \times 12.7 = 2419.6 \text{ watts}.$$

Each wattmeter will read

$$P' = EI \cos 30° = 110 \times 12.7 \times 0.866 = 1209.8 \text{ watts},$$

so that

$$P = 2 \times 1209.8 = 2419.6 \text{ watts}.$$

92. Balanced Inductive, Three-phase, Y-connected System.—Assume that the three resistances of Fig. 137 are replaced by three equal inductances, as in Fig. 143, such that the current in each phase lags behind the corresponding phase voltage by $\phi = 45°$. The phase rotation is assumed to be the same as in Fig. 137, that is, e_b lags 120° behind e_a and e_a lags 120° behind e_c, as is indicated in Fig. 144.

From the conventional notation used, we have

$$E_{BA} = \ominus e_b \oplus e_a,$$

$$E_{AC} = \ominus e_a \oplus e_c,$$

$$E_{CB} = \ominus e_c \oplus e_b.$$

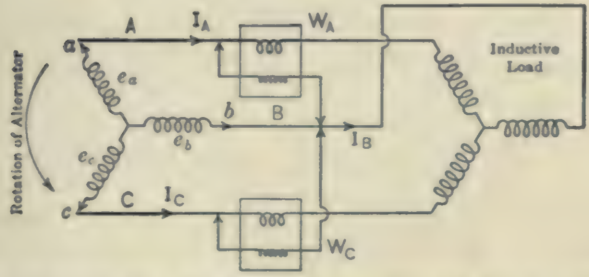


Fig. 143.—An inductive load, Y-connected, being supplied from a three-phase generator, with wattmeters properly connected for power measurement.

Carrying out the vector construction indicated, in Fig. 144, we check our convention by finding that the voltage E_{BA} lags behind E_{AC}, and that this in turn lags behind E_{CB}.

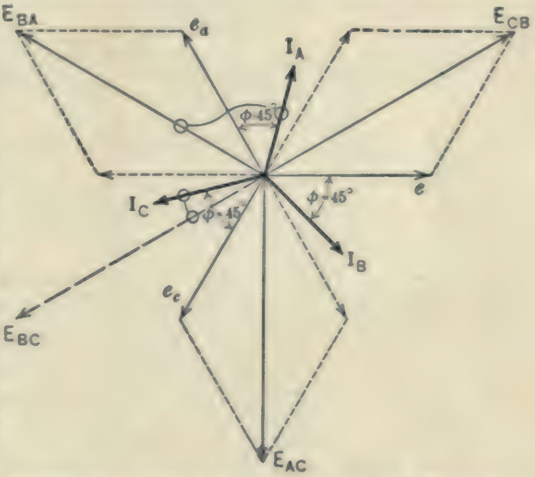


Fig. 144.—Voltage and current relations for the circuit of Fig. 143.

The currents are now drawn 45° behind their respective phase voltages, but according to the notation used we might have drawn them with reference to the line voltages with which they are associated. We found, for a balanced resistive load, that I_A would lag behind the voltage E_{BA} by 30°; hence for this inductive load it will lag behind the voltage E_{BA} by $30°+45°=75°$. Similarly, I_C will lag 75° behind the voltage E_{AC}, etc. In general, then, for a balanced circuit, with the notation of vectors adopted, the current I_A will lag behind the voltage E_{BA} by an angle $(30°+\phi)$, I_B will lag behind the voltage E_{CB} by the angle $(30°+\phi)$, and I_C will lag behind the voltage E_{AC} by the angle $(30°+\phi)$, where ϕ is the angle between the phase voltage and current; for inductive loads ϕ is to be considered positive and for condensive, or anti-inductive loads, it is to be considered negative.

The upper wattmeter, W_A, in Fig. 143, carries the line current I_A and has impressed upon its potential circuit the voltage E_{BA}; its indication will then be, from Fig. 144,

$$\begin{aligned} W_A &= E_{BA} \times I_A \times \cos(30°+\phi) \\ &= E_{BA} \times I_A \times \cos(30°+45°). \end{aligned}$$

The indication of the lower wattmeter, W_C, will be

$$\begin{aligned} W_C &= E_{BC} \times I_C \times \cos(30°-\phi) \\ &= E_{BC} \times I_C \times \cos(30°-45°). \end{aligned}$$

The sum of these two readings represents the true power, as may readily be proved. Since $E_{BA}=E_{BC}=E$, and $I_A=I_C=I$ we have

$$\begin{aligned} W_A+W_C &= EI\cos(30°+\phi)+EI\cos(30°-\phi) \\ &= EI\,(\cos 30° \times \cos\phi - \sin 30° \times \sin\phi \\ &\qquad + \cos 30° \times \cos\phi + \sin 30° \times \sin\phi) \\ &= 2EI\cos 30° \cos\phi = 2EI \times \tfrac{1}{2}\sqrt{3} \times \cos\phi \\ &= \sqrt{3}EI\cos\phi, \end{aligned}$$

which, as we know, is the power in a balanced, inductive, three-phase system.

It is to be carefully noted that the reading of either wattmeter *depends upon the cosine of the angle between the current through its current coil and the voltage impressed upon its potential coil.* This angle in a balanced system is either $(30°+\phi)$ or $(30°-\phi)$, where ϕ is the angle *between phase currents and voltages*; the cosine of ϕ is the power factor of the system.

93. Inequality of the Wattmeter Readings.—It will be seen that, whereas for a balanced, non-inductive loading the two wattmeter readings were the same, when the power factor is other than unity one wattmeter reading becomes greater than the other. Thus in the case just considered, the reading of W_A is smaller than that of W_C, since the angle $(30°+\phi)$ is larger than the angle $(30°-\phi)$, and therefore $\cos(30°+\phi)$ is smaller than $\cos(30°-\phi)$.

It is to be noted, however, that if the loading considered were condensive, so that the phase currents led the phase voltages, the reading of the upper wattmeter would be

$$W_A = E_{BA} \times I_A \cos(30°-\phi),$$

and that of the lower wattmeter

$$W_C = E_{BC} \times I_C \cos (30° + \phi),$$

as may be seen from Fig. 145, which is the vector diagram for a condensive load in which the phase currents lead their respective voltages by $\phi = 45°$. Evidently, under these conditions, the reading of W_A is greater than that of W_C.

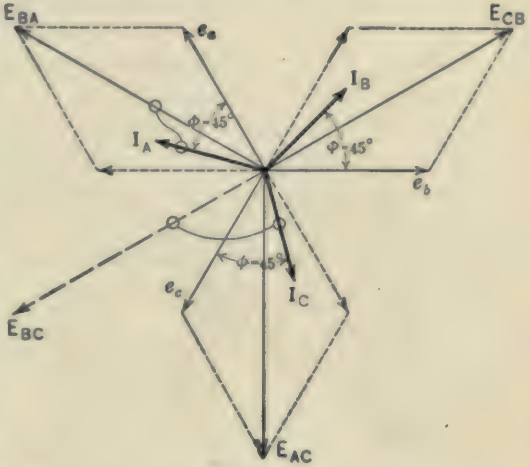


FIG. 145.—Voltage and current relations in a Y-connected capacitive load.

94. Problem.—Each load in Fig. 143 has 6 ohms resistance and 3 ohms inductive reactance. The alternator generates 110 volts per phase at 60 cycles. Determine the current in each line, the voltage between lines, the reading of each wattmeter, and the power factor of the system.

The current per phase and the line current $= \frac{e}{z} = \frac{110}{\sqrt{6^2+3^2}} = \frac{110}{6.708} = 16.4$ amperes.

The power factor per phase, and of the system, is

$$\cos \phi = \frac{6}{6.708} = 0.8944 \quad \text{and} \quad \phi = 26.6°.$$

Upon constructing a vector diagram similar to Fig. 144, we find the line voltages to be $E = 110 \times \sqrt{3} = 190.5$ volts.

The readings of the two wattmeters are

$$\begin{aligned} W_A &= 190.5 \times 16.4 \times \cos (30° + 26.6°) \\ &= 190.5 \times 16.4 \times 0.5505 \\ &= 1720 \text{ watts.} \\ W_C &= 190.5 \times 16.4 \times \cos (30° - 26.6°) \\ &= 190.5 \times 16.4 \times 0.9982 \\ &= 3119 \text{ watts.} \end{aligned}$$

The total power is

$$W = W_A + W_C = 1720 + 3119 = 4839 \text{ watts}$$
$$= \sqrt{3}EI \cos\phi = \sqrt{3} \times 190.5 \times 16.4 \times 0.8944 = 4839 \text{ watts}$$
$$= 3ei \cos\phi = 3 \times 110 \times 16.4 \times 0.8944 = 4840 \text{ watts}$$
$$= 3i^2r = 3 \times 16.4 \times 6 = 4841 \text{ watts.}$$

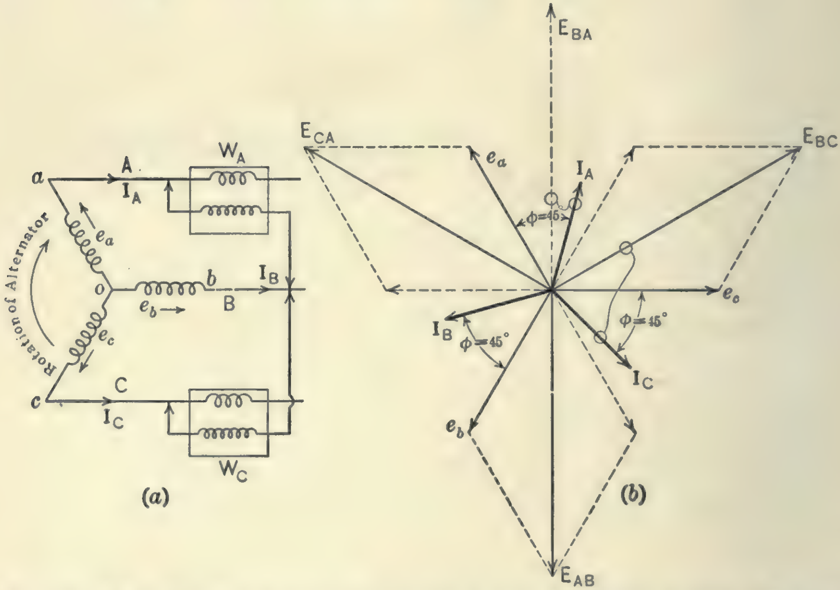


FIG. 146.—If the direction of alternator rotation is assumed opposite to that of Fig. 143, the voltage and current relations of Fig. 144 change to those given here.

95. Effect of Reversed Phase Rotation.—If now, with the same balanced loading, we consider the phase rotation reversed from what it was in Figs. 143 and 144, as it may be by reversing the direction of rotation of the alternator, so that e_a lags behind e_b by 120° and e_b lags behind e_c, the conditions will be as shown in Fig. 146. Even with reversed phase rotation, the positive direction of the phase voltages remains the same, i.e., from the neutral to the open ends of the phases. That this is correct may be seen from Fig. 147; a conductor starting in either direction from A will generate a voltage *out* from the paper. This direction is to be taken as positive, since a conductor is considered as generating positive voltage during the first half-cycle.

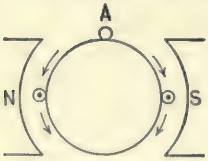


FIG. 147.—Reversing the direction of rotation does not reverse the positive direction of voltage through a coil; a conductor starting in either direction from A will generate voltage out of the paper.

It will be seen that the upper wattmeter, W_A, still has impressed upon its potential coil the voltage E_{BA}, while the voltage E_{BC} is still impressed upon the lower wattmeter, W_C. Reference to the vector diagram of Fig. 146 indicates that the effect of reversing the phase rotation has been to change the angle between the current I_A and the voltage E_{BA} from $(30°+\phi)$, (Fig. 144) to $(30°-\phi)$, (Fig. 146); similarly the angle between the current I_C and the voltage E_{BC} has been changed from $(30°-\phi)$, (Fig. 144) to $(30°+\phi)$ (Fig. 146).

Reversing the direction of phase rotation with a balanced load has the effect, therefore, of interchanging the readings of the two wattmeters, or of transferring the reading of one wattmeter to the other. Thus in the problem last presented, with reversed phase rotation wattmeter W_A would read 3119 watts, and wattmeter W_C would read 1720 watts. It is to be carefully noted, however, that although the readings are interchanged between the wattmeters, the readings are not reversed, i.e., it is not necessary to reverse the potential connections of the wattmeters in order to make them read on the scale.

96. Effect of Power Factors Equal to, and Less than, 0.5 upon the Wattmeter Readings.—With the reading of one of the two wattmeters, used to measure the power of a balanced load, equal to

$$W_1 = EI \cos (30°+\phi),$$

and that of the other equal to

$$W_2 = EI \cos (30°-\phi),$$

where ϕ is the angle between the phase voltages and currents, it is evident that for all values of ϕ between 0° and 60°, both wattmeters will have positive values, and that the total power is the sum of the two readings.

If the power factor is exactly equal to 0.5, lagging, the current lagging behind the phase voltage by 60°, the reading of the first wattmeter becomes

$$W_1 = EI \cos (30°+60°) = EI \cos 90° = 0.$$

The reading of the second wattmeter is equal to

$$W_2 = EI \cos (30°-60°) = EI \cos (-30°) = EI \cos 30°,$$

and this reading will represent the entire power of the system.

If, however, the power factor becomes less than 0.5, lagging, the currents lagging more than 60° behind the phase voltages, then the reading of the first wattmeter becomes negative, while that of the second still remains positive; the power in the circuit is then the *difference* between the two readings. To determine the reading of the wattmeter indicating negatively, the connections of its potential circuit are reversed to make its pointer read up the scale.

With a leading power factor of 0.5, it will be seen that the first wattmeter, W_1, will read $EI \cos 30°$, while the other wattmeter, W_2, will now read zero; with leading power factors less than 0.5, W_2 will read negatively.

97. Problem.—Consider that each load on Fig. 143 has 3 ohms resistance and 9 ohms reactance, the generated voltage per phase being 110 volts.

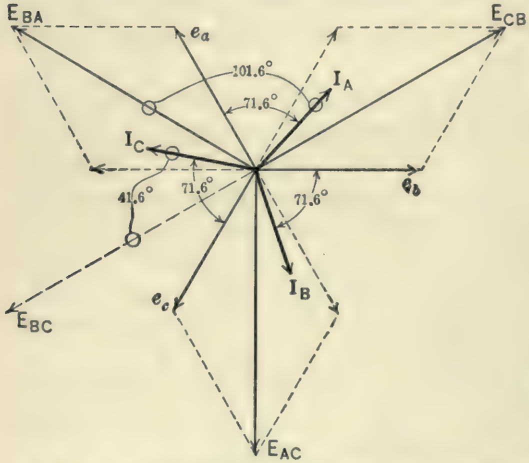


FIG. 148.—Proper phase relations for the circuit of Fig. 143, on the assumption that the reactance of each phase of the load is three times its resistance.

The current

$$= \frac{e}{z} = \frac{110}{\sqrt{3^2+9^2}} = \frac{110}{9.487}$$

$$= 11.6 \text{ amperes.}$$

The power factor is

$$\cos \phi = \frac{3}{9.487} = 0.316$$

and

$$\phi = 71.6°.$$

The line voltages $= 110 \times \sqrt{3} = 190.5$ volts.

The vector diagram for this circuit is given in Fig. 148; from it the readings of the two wattmeters are

$$\begin{aligned} W_A &= 190.5 \times 11.6 \times \cos (30° + 71.6°) \\ &= 190.5 \times 11.6 \times \cos 101.6° \\ &= 190.5 \times 11.6 \times (-\cos 78.4°) \\ &= 190.5 \times 11.6 \times (-0.2011) = -444 \text{ watts;} \\ W_C &= 190.5 \times 11.6 \times \cos (30° - 71.6°) \\ &= 190.5 \times 11.6 \times \cos (-41.6°) \\ &= 190.5 \times 11.6 \times 0.7478 = 1653 \text{ watts.} \end{aligned}$$

The total power is then, bearing in mind that W_A gives a negative reading,

$$W = W_C + W_A = 1653 - 444 = 1209 \text{ watts,}$$

but we know

$$W = 3EI \cos \phi = 3 \times 190.5 \times 11.6 \times 0.316 = 1209 \text{ watts.}$$

98. To Determine Whether a Wattmeter Reading is Positive or Negative.—In many three-phase measurements of power, we do not know at first whether the power factor is greater or less than 0.5, and so do not know whether the smaller of the two wattmeter readings should be called positive or negative. If it is possible to trace out the connections of the wattmeters used, and the wattmeters are identical in construction, it is generally possible to say whether a meter reading is positive or negative. In practice, however, the potential leads to a wattmeter are frequently of twisted cord, so that it is difficult to trace them.

If a known change can be made in the power factor, it is possible to tell from the changes in the wattmeter readings whether both of the wattmeter readings should have been called positive or not. Thus, if in the last problem considered, another Y-connected resistive load is connected in parallel with the first, the power factor of the total load would increase and the new system would require more power. If the reading of one of the wattmeters should originally have been called negative, when this resistive load is added, its reading will *decrease, or actually reverse.* Thus, if when resistive load is added, the smaller wattmeter reading decreases or reverses, it indicates that the reading of this meter should have been called negative in the original load. But if the smaller wattmeter reading increases when resistive load is added, it indicates that the reading of the meter should have been called positive in the original load.

There is another very simple test, which can usually be carried out, and which will determine at once whether a wattmeter reading is to be considered positive or negative. If the line in which the current coil of one of the wattmeters is placed, is opened, the system is reduced to a single-phase one, and the second wattmeter (if its original reading should have been called positive), with its potential connections left as before will indicate positively. Thus, if when this test is made the wattmeter reverses, its original reading should have been called negative; if it remains positive, it originally read positively.

If this test is applied to the circuit of the last problem, the readings of the wattmeters may easily be calculated. When line C is opened (Fig. 149a) the system is reduced to a single-phase one, loads a and b being in series across the voltage E_{BA}; naturally no current will flow through load c.

The current flowing through loads a' and b' over line A is then

$$I_A=\frac{190.5}{(3+3)^2+(9+9)^2}=\frac{190.5}{18.97}=10.04 \text{ amperes.}$$

The power factor is

$$\cos\phi=\frac{18}{18.97}=0.316 \quad \text{and} \quad \phi=71.6^\circ.$$

We see now that the current over line A is 71.6° behind *the voltage* E_{BA} (Fig. 150*a*), (not the voltage e_a, as was the case when line C was closed).

Wattmeter W_A then reads

$$W_A = E_{BA} \times I'_A \times \cos 71.6^\circ$$
$$= 190.5 \times 10.04 \times 0.316 = 605 \text{ watts},$$

which is the total power, since

$$W = (I'_A)^2 \times 2r = 10.04^2 \times 2 \times 3 = 604 \text{ watts}.$$

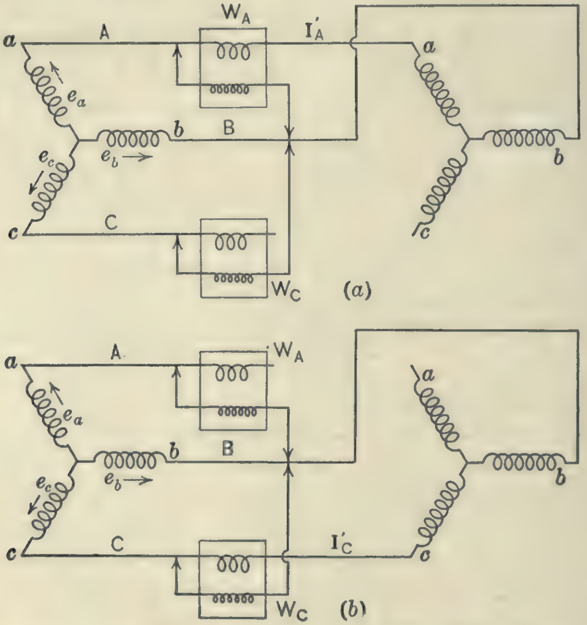


FIG. 149.—A scheme for testing for wattmeter polarity in a three-phase circuit. If, in (*a*), W_a still stays on the scale its reading in the three-phase connection should have been called positive, etc.

The reading of wattmeter W_A with line C open should evidently be positive on this single-phase load. Hence, as soon as line C is opened, the needle of W_A (which has been so connected as to read positively on the three-phase load) will swing below zero, indicating that the original reading was negative. This determination is sufficient, as we do not care about its numerical indication with line C open.

When line A is opened (Fig. 149*b*), loads b and c in series draw 10.04 amperes from the voltage E_{BC}, and the lower wattmeter, W_C, will indicate (Fig. 150*b*).

$$W_C = E_{BC} \times I'_C \times \cos 71.6^\circ = 605 \text{ watts},$$

the same as W_A when line C was opened. Hence, if this wattmeter was properly connected to the three-phase load for a positive reading (Fig. 149*a*), when line C is opened, its needle will still read on the scale, i.e., will not swing past zero.

One might ask why the voltage E_{BC}, and not the voltage E_{CB}, was

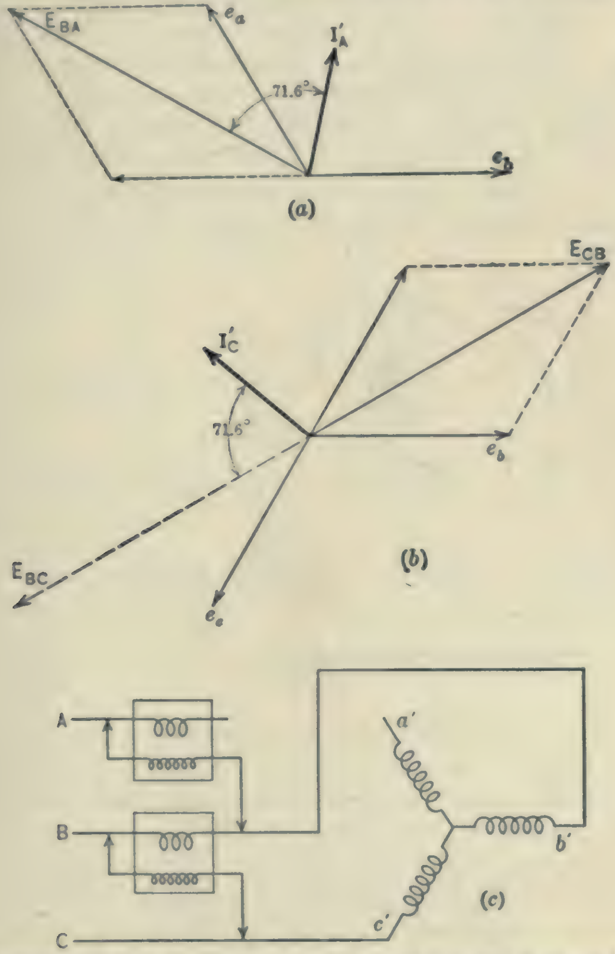


FIG. 150.—Vector diagram for explaining the results obtained when making the test indicated in Fig. 149.

considered as impressed upon loads b and c, when line A was opened. It must be realized that W_C does not possess the same symmetry with respect to its load as does W_A with respect to its load. If the voltage E_{CB} is to be considered, the wattmeter would have to be placed in line B, as is indicated in Fig. 150*c*. It must be noticed, however, that this connection of the two wattmeters would not be correct for reading the three-phase power.

99. Unbalanced Y-connected Load.—It very infrequently happens that a Y-connected load has the impedances of the various phases very different, either in magnitude, power factor, or both. It is evident that if the three-phase load is a piece of polyphase apparatus, such as a motor or similar machine, the phases are always balanced; when the load on a three-phase motor increases, the necessary increase in electrical power input is distributed equally between the different phases.

If a lighting load, such as the one illustrated in Fig. 151, should be connected in Y, it is evident that a badly unbalanced condition would be very likely to occur. The customers in street *A* might turn on lights and domestic apparatus to a much greater extent than those in streets *B* and *C*, with the result that the Y load would be unbalanced. In Fig. 151 the transformers are shown connected Y–Y, but it is to be noticed that the remarks about the effect of unbalanced load, this being Y-connected, hold good no matter how the supply transformers are connected. It is seen that were the Y-connection used for such a suburban load as that indicated in Fig. 151, street *A* could get no power at all if it so happened that no apparatus was being used in streets *B* or *C*; evidently such a condition is intolerable, and it is never used in practice.

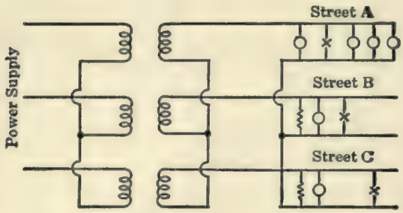


FIG. 151.

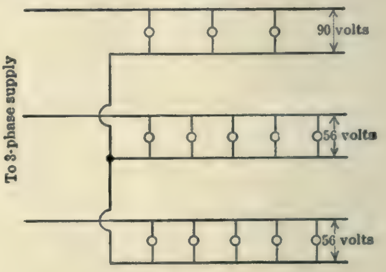


FIG. 152.

FIG. 151.—A possible condition if Y-connected networks were used in customers' circuits; the voltage supplied to one customer would depend entirely upon the load drawn by others.

FIG. 152.—Unbalanced Y-connected loads will give a non-uniform division of the line voltages.

In case an unbalanced load is connected in Y to a three-wire, three-phase power supply, the voltages on the different phases will not be the same; that phase having the highest impedance will have the highest voltage. A possible condition which might occur in the laboratory is indicated in Fig. 152; the upper lamp bank having only a few lamps connected in, does not have a voltage of $110/\sqrt{3}=63$ volts, as would be the case for the ordinary three-phase, Y-connected load, but might have 90 volts across it. In such a load the drop across each of the other lamp banks (each having the same number of lamps connected) would be 56 volts.

100. Graphical Solution for Unbalanced, Y-connected Resistive Load.—The analytical solution of unbalanced loads is generally much more tedious than a graphical or semi-graphical solution. For purely resistive loads a very simple construction enables the solution to be obtained for any

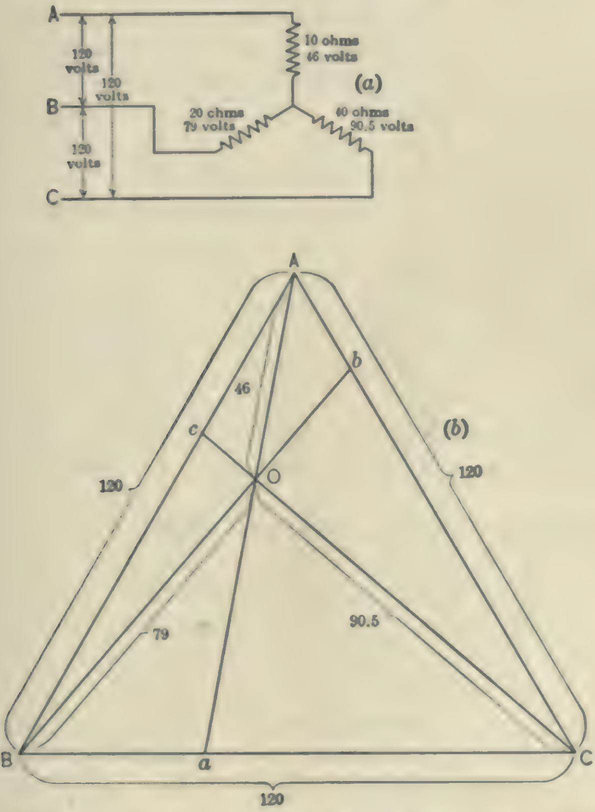


FIG. 153.—A simple graphical scheme for solving current and power relations in an unbalanced, Y-connected, resistive load.

condition of unbalance. This is indicated in Fig. 153, in which the upper diagram shows the unbalanced load, and the lower the graphical solution by means of which the voltage across each phase is obtained. The triangle *ABC* is the balanced three-phase voltage impressed on the load. The side *AB* is divided at point *c* in the proportion of 10 to 20, this being the ratio of the resistances connected between lines *A* and *B*. Side *BC* is divided at *a* into two parts in the ratio of 20 to 40, this being the ratio of the resistances of the loads connected between lines *B* and *C*. The two lines *C–c* and *A–a* intersect at *O*, and this point gives the voltage of the center point of the Y load, with respect to the three lines. As a check, the line *B–b* is constructed with the point *b* so located as to divide the side *CA* in the proportion of 40 to 10; the line *B–b* will intersect the other two at *O*, if the other two have been properly constructed.

Now, by measuring the distances from the three corners of the triangle to the point *O*, the voltage across each phase of the load is obtained to the same scale as that to which *AB* represents the line voltage. Knowing the voltage across each phase of the load, and the resistances of the phases, we at once obtain the current in each phase; and of course the phase current is also the line current.

In the above problem, the voltage across the 10-ohm phase is obtained from the voltage diagram as 46 volts; the current in line *A* is therefore $46 \div 10 = 4.6$ amperes and the power used in phase *A* is 212 watts. Across phase *B* the voltage is 79; the current in line *B* is therefore $70 \div 20 = 3.95$ amperes, and the power is 282 watts. Phase *C* has a voltage of 90.5 and hence a current of $90.5 \div 40 = 2.26$ amperes and power of 204 watts.

The phases of the three-phase voltages, and of the three line currents are properly given by the directions of the three lines in Fig. 153*b*, *O–A*, *O–B*, and *O–C*. It will be found that the three line currents found in the foregoing paragraph will, if laid off in these relative phases, add up vectorially to zero, as we know the currents of a three-wire system must always do.

101. Unbalanced Y-connected Reactive Loads.—The general case of the unbalanced Y-connected load does not lend itself to easy solution without the use of a complex arithmetical calculation, a scheme in which resistance is measured in real quantities and reactance in so-called imaginary quantities. For the solution of such problems, texts using complex arithmetical calculations should be consulted.

102. Three-phase, Δ-connected System.—We saw that a simple three-phase alternator having three coils on its armature separated by 120° (electrical), became Δ-connected if the beginning of one phase was joined to the end of the second phase, the beginning of the second phase was joined to the end of the third phase, and the beginning of the third phase was joined to the end of the first phase, that is, if the three coils were connected in series to form a closed circuit, the beginning of one phase being connected to the end of the preceding phase, etc. The machine then delivers three equal voltages, 120° apart, at the junction points of the phases, from which the leads of the machine are brought out.

A Δ-connected, three-phase load consists of three separate loads connected in the same order as were the coils of the simple alternator; the three junction points of the load are then connected to the leads from the alternator. Such an arrangement, consisting of a Δ-connected alternator and a Δ-connected lamp load, is diagrammatically represented in Fig. 154, the lamp load being drawn in the form of the letter Δ, even though it would probably be arranged as in Fig. 155.

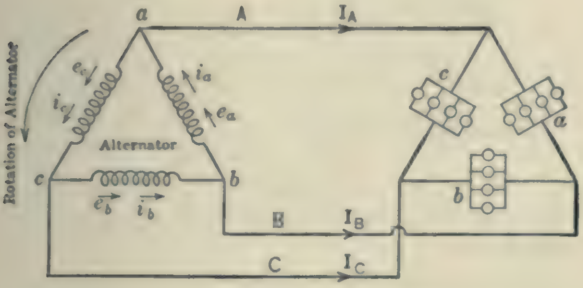


FIG. 154.

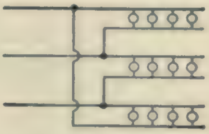


FIG. 155.

FIG. 154.—Schematic diagram of a three-phase lamp-bank load, delta-connected, being supplied from a delta-connected alternator.

FIG. 155.—Actual physical arrangement of a delta-connected lamp-bank load.

We assume in Fig. 154 that the alternator is revolving in such a direction that phase voltage e_b lags 120° behind e_a, and that e_a lags 120° behind e_c, as is indicated in Fig. 156. We know that the line voltage E_{BA} is the same as phase voltage e_a and that line voltages E_{CB} and E_{AC} are the same as phase voltages e_b and e_c, respectively. The load being symmetrical with respect to the alternator, the same relationship exists between the line voltages and the phase voltages of the load.

A resistive load having been assumed, the current flowing through load *a* will be in phase with the voltage impressed on that phase of the load; similarly, i_a, the current in phase *a* of the alternator, will be in phase with the phase voltage e_a, i_b will be in phase with e_b, and so on.

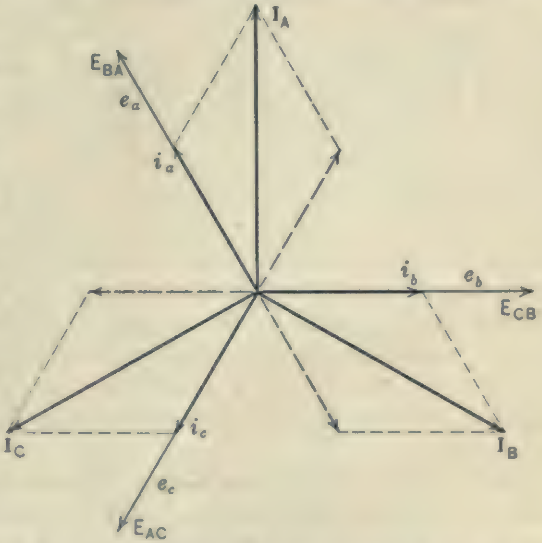


FIG. 156.—Voltage and current relations for the circuit given in Fig. 154.

It may be assumed that the positive direction of the line currents is either away from, or toward the alternator; but as before, they must all be assumed in the same direction. If the positive direction of the line currents is taken away from the alternator, as in Fig. 154, the line current I_A will be the vector difference of the phase currents i_a and i_c, or

$$I_A = i_a \ominus i_c = -i_c \oplus i_a,$$

and similarly

$$I_B = i_b \ominus i_a = -i_a \oplus i_b$$

$$I_C = i_c \ominus i_b = -i_b \oplus i_c.$$

The accuracy of the last equations may be tested by drawing the three-phase current waves and applying Kirchhoff's current law to instantaneous values at the junction points.

Carrying out the the vector construction indicated by the equations in the vector diagram, we find that in a balanced Δ-connected load the line and phase currents have the same numerical relation as exists between line and phase voltages in a Y-connected system, namely that

$$I = \sqrt{3}i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

where I is a line current and i is a phase current.

It is also to be seen that when the phase voltages are taken in such order that e_b lags e_a lags e_c: the line current I_A lags 30° behind the line voltage E_{BA}, I_B lags 30° behind E_{CB}, and I_C lags 30° behind E_{AC}.

103. Notation for Three-phase Vectors in a Δ-connected System.—The convention used for rotating vectors in a Δ-connected system should lead to the same conclusions as in a Y-connected system. In a Y-connected system, with the order of rotation of the phases as a, b, c, the designation E_{BA} indicates that phase voltage e_b lags 120° behind e_a; E_{AC} indicates that e_a lags 120° behind e_c, etc. In a Δ-connected system, with the same order of phase rotation, the designation E_{BA} should also indicate that phase voltage e_b lags 120° behind e_a, etc., and this will be so if by the voltage E_{BA} we mean that voltage which acts through the alternator from line B to line A, or from the end of phase b to the end of phase a, the end being that end of the phase winding toward which positive direction of voltage acts.

104. Vector Diagram for Reversed Phase Rotation.—If now we consider the phase rotation reversed from what it was in Figs. 154 and 156, as it would be if the direction of rotation of the alternator were changed, so that e_a lags behind e_b by 120°, and e_b lags behind e_c, the conditions are shown in Figs. 157 and 158. Reversing the direction of rotation of the alternator does not reverse the positive direction of the phase voltages, as was indicated in connection with a Y-connected alternator, on page 150.

The equations for the line currents are the same as before,

$$I_A = -i_c \oplus i_a$$

$$I_B = -i_a \oplus i_b$$

$$I_C = -i_b \oplus i_c$$

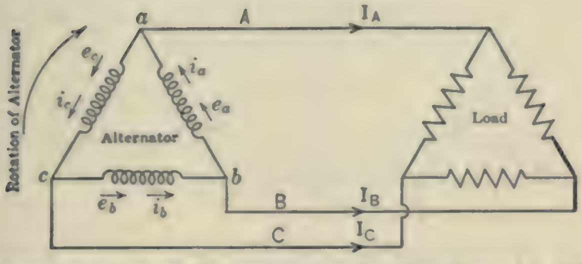


FIG. 157.—Current relations on the assumption that the alternator of Fig. 154 has its rotation reversed.

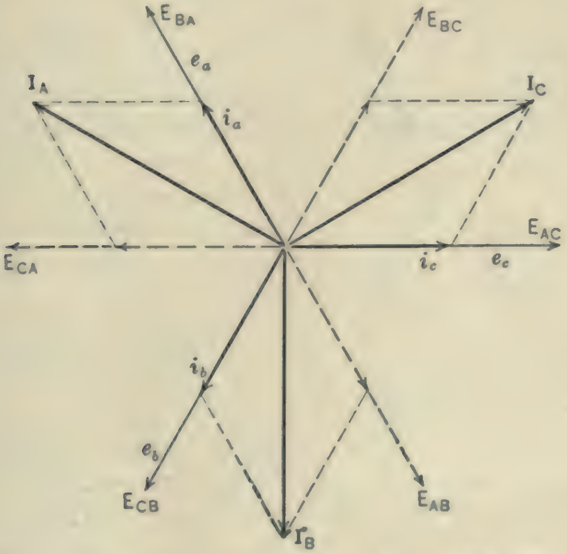


FIG. 158.—Vector relations for the circuit of Fig. 157.

The vector diagram for the circuit is shown in Fig. 158; the same conventional notation is used as before, that is the designation of a voltage by E_{CA} shows phase voltage e_c to be lagging 120° behind e_a, etc., and it appears from the vector diagram that for the resistive load, current I_A lags 30° behind the line voltage E_{CA}, etc., as was found for the Y-connected load.

Thus we again have the rule that for a Δ-connected resistive load, currents away from the power supply being considered positive, and line voltages being written in such a way that the subscripts give the phase rotation, the line currents lag behind their respective line voltages by 30°. A line is designated by the same subscript as is used for the phase whose positive voltage acts toward that line.

105. Power in a Three-phase, Δ-connected System.—In a balanced three-phase, Δ-connected system the total power will evidently be three times the power per phase, just as in a balanced Y-connected system, i.e.,

$$P=3ei\cos\phi,$$

where $\cos\phi$ is the power factor of the load, i.e., cosine of the angle between the phase voltage and the phase current.

Since in a Δ-connected system

$$e=E \quad \text{and} \quad i=\frac{I}{\sqrt{3}},$$

we have

$$P=3\times\frac{I}{\sqrt{3}}\times E\times\cos\phi=\sqrt{3}EI\cos\phi.$$

This equation is the same as Eq. (111); it states that in any balanced three-phase system, whether Y- or Δ-connected, the total power is equal to the product of the line voltage, the line current, the square root of three, and the cosine of the angle between the phase voltage and the phase current.

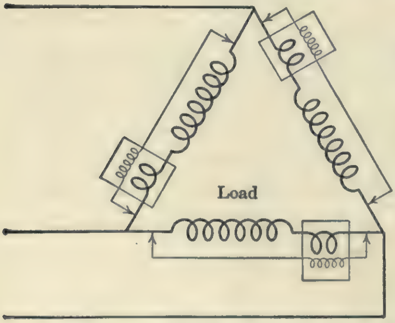


FIG. 159.—The most obvious method of measuring power in a Δ-connected load.

106. Measurement of Power in a Three-phase, Δ-connected System.—*Three-wattmeter Method.* The power in a three-phase, Δ-connected system, whether balanced or unbalanced, may evidently be obtained by the use of three wattmeters, connected as in Fig. 159. Here the current coil of a wattmeter is placed in series with each phase of the load and the potential coils have impressed upon them the phase voltage of their respective phases; each wattmeter gives the power used in its respective phase.

Two-wattmeter Method.—The power of any three-phase system, whether Y- or Δ-connected, can be measured by two wattmeters. We have already shown this to be true for a Y-connected load, the wattmeters being connected as in Fig. 141. It should be evident that the same connection of the wattmeters will measure the total power in a balanced Δ-connected resistive load, inasmuch as the wattmeters deal only with line values, and we have shown that the power in a balanced three-phase system, whether Y- or Δ-connected, is $P = \sqrt{3}EI \cos \phi$, and

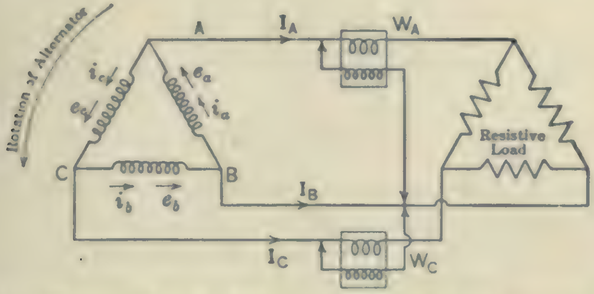


FIG. 160.—The actual method of measuring power in a Δ-connected load.

also that the same phase relation exists between line currents and line voltages in both systems.

In Fig. 160 is shown a Δ-connected resistive load, the power taken by which is to be measured by two wattmeters, placed in the lines in the same way as was done to measure the power in a Y-connected system.

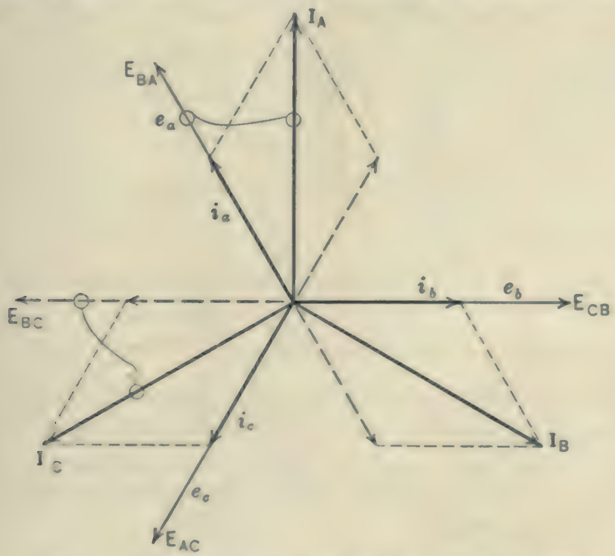


FIG. 161.—Vector relations for the circuit of Fig. 160.

The vector diagram for the circuit is shown in Fig. 161, it being the same as that of Fig. 156. It will be seen that W_A carries the current I_A and has impressed upon its potential coil the voltage E_{BA}; similarly,

W_C carries current I_C, with the voltage E_{BC} impressed upon its potential coil. From Fig. 161, the readings of the wattmeters are

$$W_A = E_{BA} I_A \cos 30°$$

$$W_C = E_{BC} I_C \cos 30°.$$

These readings are the same as in a Y-connected resistive loading. We have, then,

$$W_A + W_C = \sqrt{3} EI,$$

which we know to be the power in a three-phase resistive load.

That the readings of two wattmeters placed as in Fig. 160 represent the true power in a Δ-connected system, may also be proved from instantaneous values. The procedure is similar to that used for a Y-connected system, except that we use the relation that the sum of the instantaneous values of the three phase voltages is zero, i.e.,

$$e_a + e_b + e_c = 0.$$

107. Problem.—Assume that each resistance in Fig. 160 is 4 ohms, and that the voltage supplied by the alternator between lines is 120 volts. Calculate the power used in the circuit, and the reading of each wattmeter.

The system being balanced, each phase current is, $i = \frac{120}{4} = 30$ amperes, and each line current is, $I = \sqrt{3}i = \sqrt{3} \times 30 = 51.96$ amperes.

The total power is

$$P = 3ei = 3 \times 120 \times 30 = 10{,}800 \text{ watts},$$

$$P = \sqrt{3}EI = \sqrt{3} \times 120 \times 51.96 = 10{,}800 \text{ watts}.$$

Each wattmeter will read

$$P' = EI \cos 30° = 120 \times 51.96 \times 0.866 = 5400 \text{ watts},$$

so that

$$P = 2P' = 2 \times 5400 = 10{,}800 \text{ watts}.$$

108. Balanced Inductive, Three-phase, Δ-connected System.—Assume that the three resistances of Fig. 160 are replaced by three equal inductances, as in Fig. 162, such that the current in each phase lags behind the corresponding phase voltage by $\phi = 45°$. The phase rotation is assumed as *a, b, c,* as in Fig. 160.

From the conventional notation we have

$$I_A = -i_c \oplus i_a$$

$$I_B = -i_a \oplus i_b$$

$$I_C = -i_b \oplus i_c.$$

Carrying out the vector construction indicated, in Fig. 163, we find that for the inductive load assumed, the line current I_A lags behind the voltage E_{BA} by $30° + 45° = 75°$, that I_B lags 75° behind E_{CB}, and that I_C lags 75° behind E_{AC}.

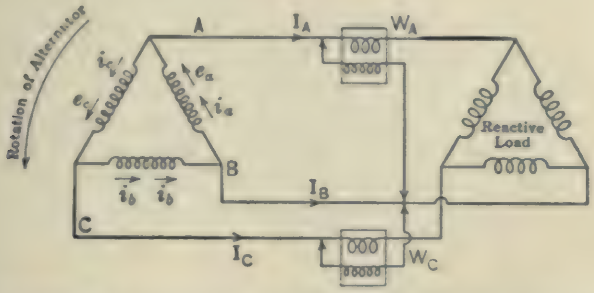


Fig. 162.—A reactive, Δ-connected load, with wattmeters for power measurement.

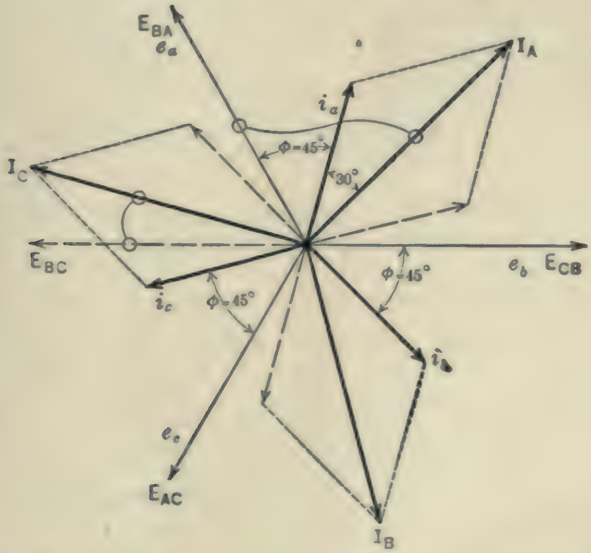


Fig. 163.—Vector diagram for voltages and currents of Fig. 162.

The readings of the wattmeters (Fig. 162) are then

$$W_A = E_{BA} I_A \cos(30° + \phi) = E_{BA} I_A \cos(30° + 45°)$$

and

$$W_C = E_{BC} I_C \cos(30° - \phi) = E_{BC} I_C \cos(30° - 45°).$$

These readings are identical with those for the Y-connected system considered on page 146, in which the power factor and the order of phase rotation were the same. We have then, from the proof there given,

$$W_A+W_C=\sqrt{3}EI\cos\phi,$$

which, as we know, is the power in a balanced three-phase system.

It will be realized that all that was said as to the inequality of the wattmeter readings varying with power factor in a Y-connected system, applies equally well to a Δ-connected system. Whenever the power factor of a three-phase balanced load is less than unity, and the power is measured by two wattmeters, one of them will read less than the other; with power factors greater than 0.5, both wattmeters read positively (or negatively); but with power factors less than 0.5, the smaller wattmeter reading must be considered negative if the larger is positive (and *vice versa*).

109. Problem.—Each load in Fig. 162 has 6 ohms resistance and 3 ohms inductive reactance. The alternator generates 110 volts per phase. Determine the line currents, the reading of each wattmeter, and the power factor of the system.

The current per phase is

$$i=\frac{e}{z}=\frac{110}{\sqrt{6^2+3^2}}=\frac{110}{6.708}=16.4\text{ amperes.}$$

The current per line is

$$I=\sqrt{3}i=\sqrt{3}\times 16.4=28.4\text{ amperes.}$$

The power factor is

$$\cos\phi=\frac{6}{6.708}=0.8944\quad\text{and}\quad\phi=26.6°.$$

The readings of the two wattmeters are

$$\begin{aligned}W_A&=110\times 28.4\times\cos(30°+26.6°)\\&=110\times 28.4\times 0.5505=1720\text{ watts;}\\W_C&=110\times 28.4\times\cos(30°-26.6°)\\&=110\times 28.4\times 0.9982=3119\text{ watts.}\end{aligned}$$

The total power is

$$\begin{aligned}W&=W_A+W_C=1720+3119=4839\text{ watts}\\&=\sqrt{3}EI\cos\phi=\sqrt{3}\times 110\times 28.4\times 0.8944=4839\text{ watts.}\end{aligned}$$

The loads used Δ-connected in this problem are the same as those used Y-connected in the problem on page 149, with the same voltage impressed per phase. It will be seen that the wattmeter readings are identical.

110. Problem.—Consider that the same loads ($r=3$ ohms, $x=9$ ohms) used in the problem on page 152, are connected in delta, the impressed voltage per phase being again 110 volts.

The current per phase is

$$i=\frac{110}{\sqrt{3^2+9^2}}=\frac{110}{9.487}=11.6 \text{ amperes.}$$

The power factor is again

$$\cos\phi=0.316, \quad \text{and} \quad \phi=71.6°.$$

The line currents are

$$I=\sqrt{3}i=\sqrt{3}\times 11.6=20.09 \text{ amperes.}$$

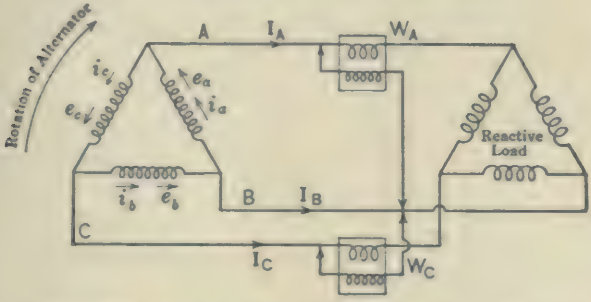


FIG. 164.—To consider the effect on the wattmeter readings of reversing the rotation of the alternator of Fig. 162.

A vector diagram constructed for this circuit will show that the readings of the wattmeters are

$$\begin{aligned}W_A&=110\times 20.09\times\cos(30°+71.6°)\\&=110\times 20.09\times(-\cos 78.4°)\\&=110\times 20.09\times(-0.2011)=-444 \text{ watts;}\\W_C&=110\times 20.09\times\cos(30°-71.6°)\\&=110\times 20.09\times\cos(-41.6°)\\&=110\times 20.09\times 0.7478=1653 \text{ watts.}\end{aligned}$$

The total power is then

$$W=W_A+W_C=1653-444=1209 \text{ watts,}$$

or,

$$W=\sqrt{3}EI\cos\phi=\sqrt{3}\times 110\times 20.09\times 0.316=1209 \text{ watts.}$$

111. Effect of Reversed Phase Rotation.—If now, with the same balanced inductive loading, we consider the phase rotation reversed from what it was in Figs. 162 and 163, as it may be by reversing the direction of rotation of the alternator, so that e_a lags behind e_b by 120° and e_b lags behind e_c, the conditions will be shown in Figs. 164 and 165.

It will be seen that the voltages E_{BA} and E_{BC} are still impressed upon the potential coils of the upper wattmeter, W_A, and the lower wattmeter, W_C, respectively. From the vector diagram of Fig. 165, we find that the effect of reversing the phase rotation has been to change the angle between the current I_A and the voltage E_{BA} from $(30° + \phi)$, (Fig. 163), to $(30° - \phi)$, (Fig. 165); similarly the angle between the current I_C and the voltage E_{BC} has been changed from $(30° - \phi)$, (Fig. 163), to $(30° + \phi)$, (Fig. 165). Exactly the same result was obtained when the phase rotation was reversed with a Y-connected system.

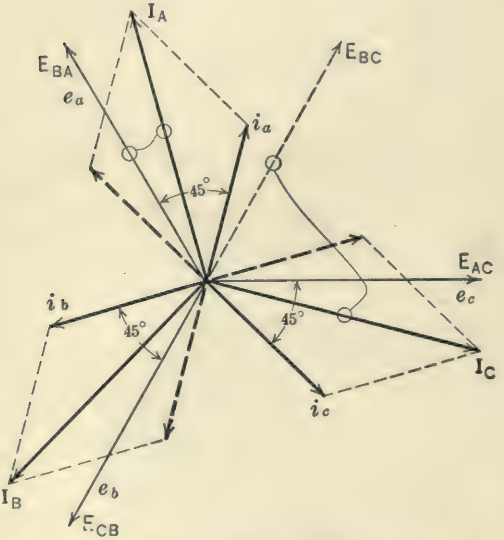


Fig. 165.—Vector diagram for conditions given in Fig. 164.

Reversing the direction of phase rotation with a balanced load, whether Y- or Δ-connected, has the effect, therefore, of interchanging the readings of the two wattmeters.

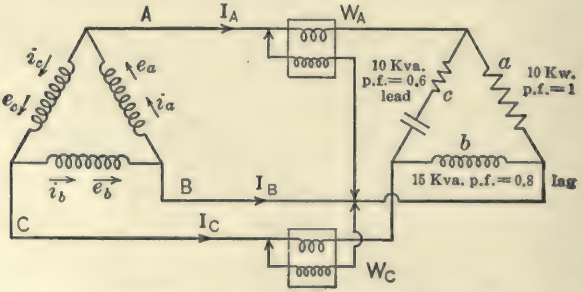


Fig. 166.—A badly unbalanced three-phase load, such as might be set up in the laboratory but which would never be encountered in practice.

112. Unbalanced Δ-connected Load.—Unbalanced Δ-connected loads are infrequent; when they occur unequal line currents result. As the solution of such circuits is not difficult, the following problem is worked out.

Three loads are connected in delta, as in Fig. 166; load *a* takes 10 kw. at unity power factor, load *b* takes 15 kv.a. at a lagging power factor of 0.8, and load *c* requires 10 kv.a. at a leading power factor of 0.6. The line voltages are each 100 volts and are 120° apart, the phase rotation being *a*, *b*, *c*. It is required to calculate the three line currents and the readings of the two wattmeters.

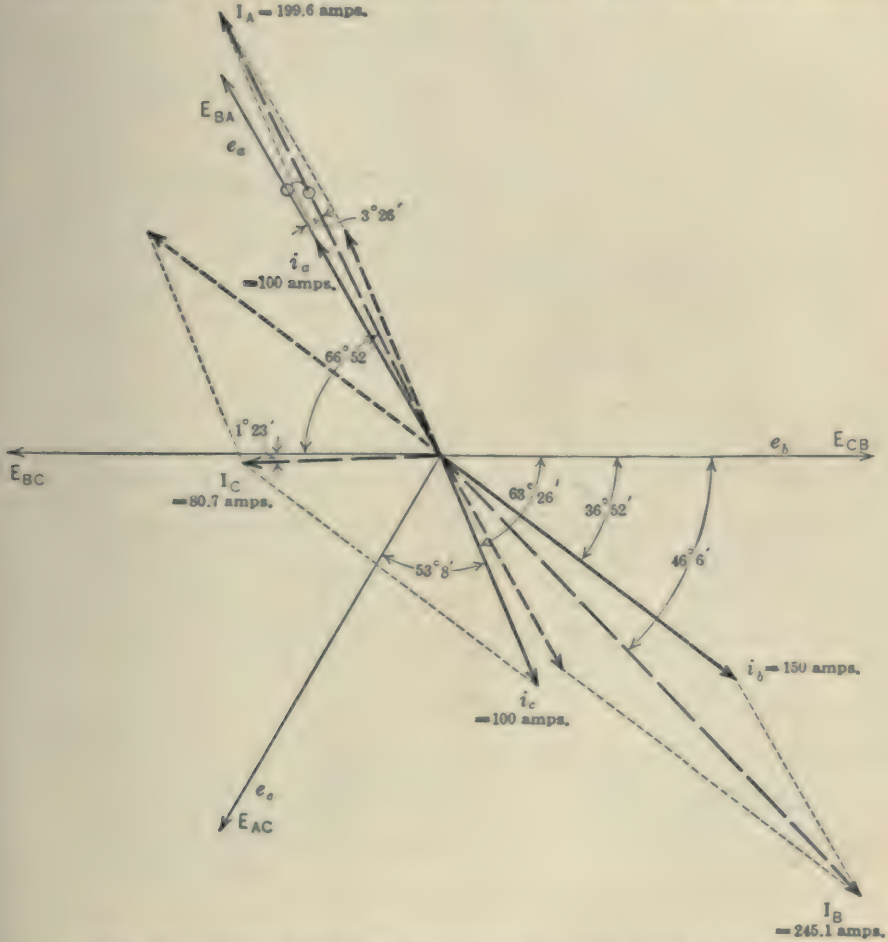


FIG. 167.—Current and voltage diagram for the load shown in Fig. 166.

The power in each phase is

$$p_a = 10 \text{ kw.}, \qquad p_b = 15 \times 0.8 = 12 \text{ kw.}, \qquad p_c = 10 \times 0.6 = 6 \text{ kw.},$$

and the total power is

$$p_a + p_b + p_c = 10 + 12 + 6 = 28 \text{ kw.}$$

The phase currents are

$$i_a=\frac{10{,}000}{100}=100 \text{ amperes,}$$
$$i_b=\frac{15{,}000}{100}=150 \text{ amperes,}$$
$$i_c=\frac{10{,}000}{100}=100 \text{ amperes.}$$

The angles between the phase currents and their respective phase voltages are

$$\phi_a=\cos^{-1} 1.0=0^\circ.$$
$$\phi_b=\cos^{-1} 0.8 \text{ lag } =36^\circ\ 52' \text{ lagging,}$$
$$\phi_c=\cos^{-1} 0.6 \text{ lead}=53^\circ\ \ 8' \text{ leading.}$$

A vector diagram may now be constructed and the phase currents located, as in Fig. 167. The equations for the line currents being

$$I_A=-i_c \oplus i_a,$$
$$I_B=-i_a \oplus i_b,$$
$$I_C=-i_b \oplus i_c,$$

their values may be calculated as follows.

$$I_A=\sqrt{\{(-i_a\times\cos 60^\circ)+(-i_c\times\cos 66^\circ 52')\}^2+\{i_a\times\sin 60^\circ+i_c\times\sin 66^\circ 52'\}^2}$$
$$=\sqrt{(-100\times0.5000-100\times0.3929)^2+(100\times0.8660+100\times0.9196)^2}$$
$$=\sqrt{(-50-39.29)^2+(86.60+91.96)^2}=\sqrt{(-89.29)^2+178.56^2}$$
$$=199.64 \text{ amperes.}$$

$$\cos\alpha=\frac{89.29}{199.64}=0.4472, \quad \alpha=63^\circ\ 26',$$

where α is the angle between I_A and the horizontal axis.

$$I_B=\sqrt{\{i_a\times\cos 60^\circ+i_b\times\cos 36^\circ\ 52'\}^2+\{(-i_a \sin 60^\circ)+(-i_b \sin 36^\circ\ 52')\}^2}$$
$$=\sqrt{(100\times0.500+150\times0.800)^2+(-100\times0.866-150\times0.600)^2}$$
$$=\sqrt{(50+120)^2+(-86.6-90)^2}=\sqrt{170^2+(-176.60)^2}=245.12 \text{ amperes,}$$

$$\cos\beta=\frac{170}{245.12}=0.6935, \quad \beta=46^\circ\ 6',$$

where β is the angle between I_B and the horizontal axis.

$$I_C=\sqrt{\{(-i_b\times\cos 36^\circ\ 52')+i_c\times\cos 66^\circ\ 52'\}+\{i_b \sin 36^\circ\ 52'-i_c \sin 66^\circ\ 52'\}^2}$$
$$=\sqrt{(-150\times0.800+100\times0.3929)^2+(150\times0.600-100\times0.9196)^2}$$
$$=\sqrt{(-120+39.29)^2+(90-91.96)^2}=\sqrt{(-80.71)^2+(-1.96)^2}$$
$$=80.73 \text{ amperes,}$$

$$\tan\gamma=\frac{1.96}{80.71}=0.242, \quad \gamma=1^\circ\ 23', \quad \cos\gamma=0.9997,$$

where γ is the angle between I_C and the horizontal axis.

As a check on the accuracy of our work, we should find that the sum of the horizontal and vertical components of the three line currents add up to zero respectively.

	Horizontal Component	Vertical Component
I_A	-89.29	$+178.56$
I_B	$+170.00$	-176.60
I_C	-80.71	$-\ \ 1.96$
	00.00	00.00

The wattmeter readings are

$$W_A = E_{BA} I_A \cos 3^\circ\, 26',$$
$$W_C = E_{BC} I_C \cos 1^\circ\, 23',$$
$$W_A = 100 \times 199.64 \times 0.9982 = 19930 \text{ watts}$$
$$W_C = 100 \times \ 80.73 \times 0.9997 = \ 8070 \text{ watts}$$
$$\text{Total power} \qquad = 28000 \text{ watts}$$

It is to be noted that in the above solution the calculations have been carried out with far greater accuracy than is necessary. In most engineering problems the accuracy obtained with the ordinary slide rule is sufficient.

If the direction of rotation of the alternator were reversed, so that the phase rotation became *a*, *c*, *b*, the line currents and the wattmeter readings would be changed, as may be seen from the following solution.

The phase currents and phase power factors remaining the same, and the equations for the line currents being identical with those used before, we may proceed to construct the vector diagram for the circuit, as in Fig. 168.

$$I_A = \sqrt{(-i_a \times \cos 60^\circ - i_c \times \cos 53^\circ\, 8')^2 + (i_a \times \sin 60^\circ - i_c \times \sin 53^\circ\, 8')^2}$$
$$= \sqrt{(-100 \times 0.500 - 100 \times 0.600)^2 + (100 \times 0.8660 - 100 \times 0.8000)^2}$$
$$= \sqrt{(-50-60)^2 + (86.60 - 80.00)^2} = \sqrt{(-110)^2 + 6.6^2} = 110.20 \text{ amperes},$$
$$\tan \alpha = \frac{6.6}{110} = 0.0600, \quad \alpha = 3^\circ\, 26',$$

where α is the angle between I_A and the horizontal axis.

$$I_B = \sqrt{(i_a \times \cos 60^\circ - i_b \times \cos 23^\circ\, 8')^2 + (-i_a \times \sin 60^\circ - i_b \times \sin 23^\circ\, 8')^2}$$
$$= \sqrt{(100 \times 0.500 - 150 \times 0.9196)^2 + (-100 \times 0.8660 - 150 \times 0.3929)^2}$$
$$= \sqrt{(50 - 137.94)^2 + (-86.60 - 58.94)^2} = \sqrt{(-87.94)^2 + (-145.54)^2}$$
$$= 170.00 \text{ amperes},$$
$$\cos \beta = \frac{87.94}{170.00} = 0.5173, \quad \beta = 58^\circ\, 51',$$

where β is the angle between I_B and the horizontal axis.

$$
\begin{aligned}
I_C &= \sqrt{(i_b \times \cos 23^\circ\, 8' + i_c \times \cos 53^\circ\, 8')^2 + (i_b \times \sin 23^\circ\, 8' + i_c \times \sin 53^\circ\, 8')^2} \\
&= \sqrt{(150 \times 0.9196 + 100 \times 0.6000)^2 + (150 \times 0.3929 + 100 \times 0.8000)^2} \\
&= \sqrt{(137.94 + 60)^2 + (58.94 + 80)^2} = \sqrt{197.94^2 + 138.94^2} = 241.83 \text{ amperes}
\end{aligned}
$$

$$
\cos \gamma = \frac{197.94}{241.83} = 0.8185, \quad \gamma = 35^\circ\, 4'.
$$

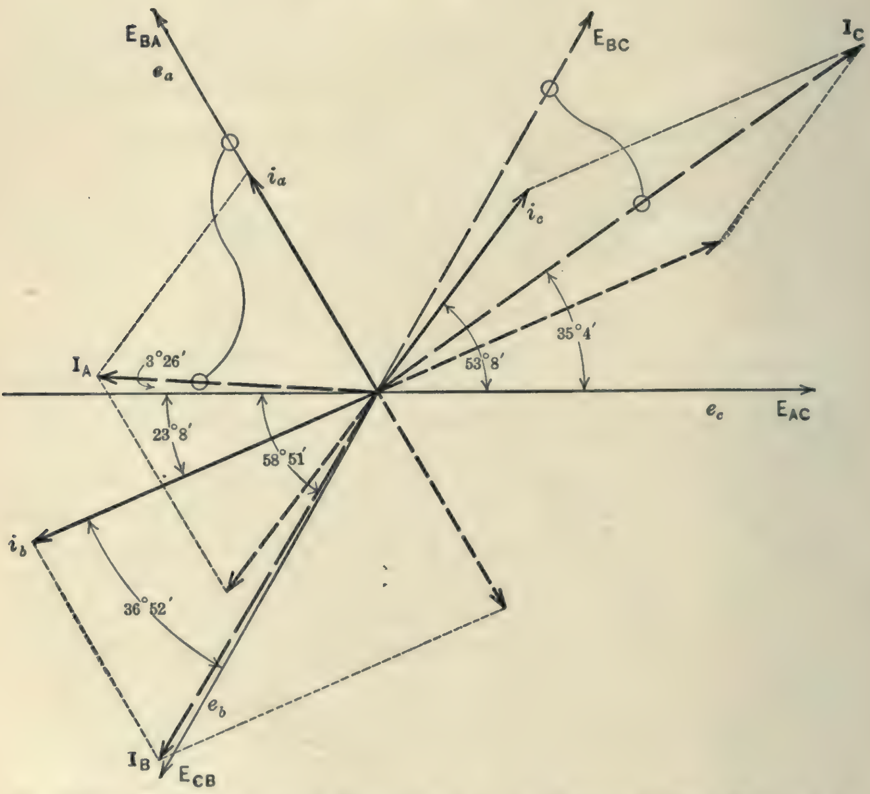


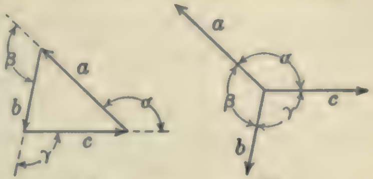
FIG. 168.—Current and voltage diagram for the load shown in Fig. 166, when the alternator has its rotation reversed from that assumed in Fig. 167.

The wattmeter readings become

$$
\begin{aligned}
W_A &= E_{BA} I_A \cos 56^\circ\, 34' = 100 \times 110.20 \times 0.5510 = 6070 \text{ watts} \\
W_C &= E_{BC} I_C \cos 24^\circ\, 56' = 100 \times 241.83 \times 0.9068 = 21930 \text{ watts} \\
&\text{Total power} \qquad\qquad\qquad\qquad\qquad\quad 28000 \text{ watts}
\end{aligned}
$$

The problem indicates that when the phase rotation is reversed with an unbalanced Δ-connected load, both the line currents and the wattmeter readings are completely changed.

113. Unbalanced Δ-connected Load with Unbalanced Line Voltages.—It may happen that the three line voltages of a three-phase alternator, when supplying a badly unbalanced load, are neither equal nor 120° apart. If the load is Δ-connected, the phase relationship of the three line (or phase) voltages may readily be determined, inasmuch as the three line voltages must, when drawn as a topographic diagram, form a closed triangle, as in Fig. 169*a*. Knowing the sides of the triangle, the angles, α, β, and γ may be determined, and the voltages replotted as a polar diagram, as in Fig. 169*b*.



(*a*) (*b*)

FIG. 169.—In a badly unbalanced three-phase system, the line voltages may be neither equal nor 120° apart. Their magnitudes being known, however, their relative phases are determined as shown here.

PROBLEMS

1. Three single-phase circuits of $R=5$, $X=10$, are connected in delta to a 110-volt, three-phase alternator. What are the line currents? What is the power factor of the load? What is the total power taken by the load?

2. Answer the same questions as above, if the three loads are Y-connected to the same alternator.

3. A two-phase, 110-volt, three-wire system is supplying 8.5 kw. on one phase and 3.7 kw. on the other, both loads being of unity power factor. What is the current in the common wire?

4. Phase No. 1 of Prob. 3 leads phase No. 2. A load of 6.05 kw. at lagging power factor of 0.85 is supplied by phase No. 1 and 15.6 kw. at lagging power factor of 0.50 by phase No. 2. What is the current in the common wire? A wattmeter with its current coil in the common wire and potential coil connected across the outside wires would read how much? How much if its potential leads are connected across phase No. 1, and how much if connected across phase No. 2?

5. The wattmeter of a 110-volt, two-phase, three-wire system has its current coil in the common wire. Phase No. 1 leads phase No. 2. A load of 3.2 kw. and $\cos\phi=0.65$ lagging, is being supplied by phase No. 1? How much load of $\cos\phi=0.3$ must be put on phase No. 2 to make the wattmeter read zero, its potential leads being across phase No. 1. What will the wattmeter read now, if its potential leads are connected across phase No. 2?

6. The current in an unbalanced delta-connected resistive load are, 80 amperes, phase No. 1, 90 amperes, phase No. 2 and 100 amperes, phase No. 3. Phase No. 1 leads No. 2, and phase No. 2 leads No. 3. What are the line currents and by what angle are they separated?

7. A 110-volt, delta-connected, three-phase load has resistances and inductive reactances as follows: Phase No. 1, $R=3$, $X=3$, phase No. 2, $R=6$, $X=10$, phase No. 3, $R=1$, $X=5$. Phase No. 1 leads No. 2, and phase No. 2 leads phase No. 3.

Determine the currents in the load circuits and in the three lines. What is the total power used?

8. In the above problem what will two properly connected wattmeters read, the line joining phases Nos. 1 and 2 serving as the common potential point for the two wattmeters?

9. A three-phase induction motor operating on a 440-volt line has a power factor (lagging) of 0.85 and efficiency of 0.80. It is delivering 10 h.p. What is the line current and the current in the motor conductors, this being delta-connected?

10. Two wattmeters are properly connected in a balanced three-phase line to measure total power. One gives a reading only 25 per cent that of the other. What is the power factor of the load?

11. The three phases of a Y-connected, resistive load have resistances of 5, 8, and 10 ohms. The load is connected to a 110-volt three-phase, Y-connected generator. What is the current in each wire, power used, and voltage to neutral from each wire?

12. If the neutral point of the generator and load of Prob. 11 are connected together, what will be the current in each of the four wires?

13. A three-phase, delta-connected load of 5 ohms resistance in each phase is supplied with power from a 110-volt, three-phase generator. The lines connecting load and generator have each 2 ohms resistance. What is the line current and current in each? (Note—Change the delta load to an equivalent Y-connected load to solve for load voltage and then restore the delta load for calculating currents.)

14. Solve Prob. 13 if in addition to the 5 ohms resistance, each load circuit has 5 ohms inductive reactance also.

15. Solve Prob. 14 if each of the supply lines has a reactance of 1 ohm inserted into it.

16. On the basis of equal I^2R in the armature, how much current could a 100 kv.a., 110-volt, three-phase alternator deliver if loaded single-phase only?

CHAPTER V

ALTERNATING-CURRENT METERS

114. Classes of Meters.—The quantities generally measured by alternating-current meters are current, voltage, power and energy. The instruments used for these purposes are the ammeter, voltmeter, wattmeter, and watt-hour meter, respectively. The first three instruments are generally of the indicating type, the operator actually reading the position of the pointer when the magnitude of the measured quantity is to be obtained. The watt-hour meter is an energy meter; it integrates the total energy fed through it by means of a motor operating a train of gears, thus making a permanent record of the number of revolutions of the moving element.

It is sometimes necessary to keep a continuous record of some quantity; for example, in conducting a life test on incandescent lamps, an accurate record of the line voltage must be kept if the test is to be of any value. In such a case, a recording instrument is used; its moving coil (generally much more massive than that of an indicating instrument) carries, instead of a pointer, a pen which writes on a moving chart. Either ammeter, voltmeter, or wattmeter may be built in the recording form.

Besides the four types of meters mentioned above, there are several others which find more or less continual application in alternating-current testing and machine operation. Among these are the power-factor meter, reactive-factor meter, reactive-power meter, frequency meter, instrument transformers, synchroscope, oscillograph, maximum-demand meter, and many others.

115. Requirements for Indicating Meters.—To make a meter indicate, there must evidently be an actuating force varying in some way with the magnitude of the quantity to be measured. In order that the needle, or pointer, may come to rest at some definite point on the scale, some counter force, or restoring force, must be provided, and the intensity of this restoring force must increase as the pointer moves over the scale. The moving coil must be carried in bearings of some kind or other, and these must offer as little friction as possible. In order to prevent the pointer from swinging too far as it moves back and forth over the scale, some kind of damping force (other than friction) must be brought into play which will make the pointer quickly settle down to its proper position, so that readings may be obtained. A uniform scale would generally be desirable, but in alternating-current instruments it is practically never possible; all voltmeters and ammeters for use on alternating-current circuits have scales much condensed for the lower regions, and very open at the upper part of the scale. A typical ammeter scale is shown in Fig. 170; it is seen that the deflection produced by one ampere as the current increases from 4 to 5 amperes is much greater than that produced when the current increases from zero to 1 ampere. This defect in these meters is due to the nature of the actuating force, which makes the meter deflect, as will be pointed out later.

116. An Essential Difference between A.C. and C.C. Voltmeters and Ammeters.—As was noted in Chapter VI, Volume I of this text, practically all c.c. voltmeters and ammeters are of the D'Arsonval type; the essential part of these meters consists of a small coil in the field of a permanent magnet. Such a meter will reverse its deflection if the current through it is reversed; as an alternating current is continually reversing its direction of flow, a meter of the D'Arsonval type could not deflect on an alternating-current circuit. The pointer would remain at zero in a more or less vibratory condition, no matter how much current was passed through it.

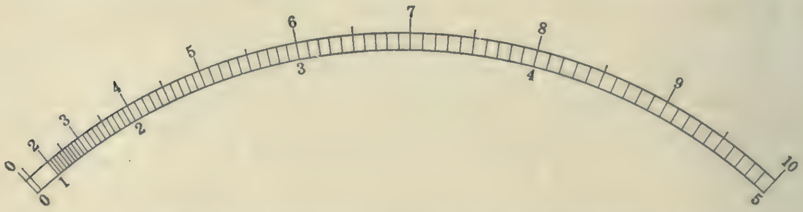


FIG. 170.—Scale of a dynamometer ammeter, to show unequal divisions.

If the alternating-current meter is to operate by means of a moving coil placed in a magnetic field, it is evident that the magnetic field in which the coil is lying must reverse at the same instant that the current in the coil reverses; if such is the case the coil will continually deflect in the same direction. The magnetic field, then, in which the moving coil turns, must be generated by the current flowing through the coil; permanent magnets are of no avail. From this it follows at once that the weight of the a.c. voltmeter or ammeter may be much less than that of a corresponding c.c. meter, and also that the a.c. meter will use more power than the c.c. type. In general, the a.c. meter (voltmeter or ammeter) will use from five to ten times as much power as the equivalent c.c. meter. A 150-volt a.c. voltmeter will use from ten to twenty watts at full-scale deflection.

117. Kind of Actuating Force for Voltmeter and Ammeter.—We have seen from previous discussion that it is the effective value of the alternating quantity in which we are generally interested, not the average value; the meter must be so designed, therefore, that its deflection is proportional to the average square of the quantity measured, rather than to the average of that quantity itself, as is the case for the ordinary c.c. meter. The force tending to make the meter move must then vary as the square of the current flowing through it; this requirement must evidently be satisfied if the polarity of the alternating current is not to reverse its reading. The square of a quantity remains positive even though the quantity itself may become negative.

A meter of the *dynamometer type* would give the right kind of actuating force, as the current through the moving coil also goes through the stationary coils, in the field of which the moving coil lies; if the current through the moving coil doubles, the force quadruples, as not only the current through the coil, but also the magnetic field in which the coil lies, has doubled.

The *iron-vane* meter should operate properly also; the vane, being of soft, thin iron, reverses its magnetism when the field in which it is lying reverses, so that the force is always in the same direction and proportional to the square of the current flowing through it. In the most common form of this meter, the force is exerted between two iron vanes, placed close together in the same magnetizing field. One vane being fixed and one movable, the force on the movable one will evidently increase with the square of the current.

Another type of meter, *the hot-wire type*, utilizes the expansion of a piece of metal through which the current to be measured is flowing; as the heat produced is proportional to the square of the current flowing through the meter, and as the expansion is proportional to the heat produced, this type of meter will also make a record proportional to the square of the current.

In some meters employing the heating effect of the current, the indicating meter is actuated, not by the expansion of the heated part but by the continuous e.m.f. of a *thermocouple*, which itself utilizes the heat from the wire or ribbon through which the alternating current is flowing. In this case the meter used is a sensitive millivolt meter (calibrated in amperes, however) of the D'Arsonval type.

In another type of meter, the current to be measured is passed through a coil which sets up some kind of a rotating magnetic field; this type is known as the *induction meter*. In this rotating magnetic field, the strength of which is proportional to the current to be measured, is an aluminum disc which has eddy currents induced in it by the rotating field. These eddy currents react with the magnetic field to drag the disc around; and as the disc has to turn against the action of a spring, its position will be proportional to the force set up by the eddy currents. The strength of the eddy currents is proportional to the strength of the magnetic field and this in turn is proportional to the current through the meter. Hence the force acting on the disc (proportional to the product of eddy currents and magnetic field intensity) varies as the square of the current through the meter.

118. Damping of A.C. Meters.—The two schemes used for damping the oscillations of the moving part may be the same for a.c. meters as for c.c. meters. The reaction of eddy currents in a metallic part of the moving coil on the field of a permanent magnet is used to a considerable extent, but the actual arrangement of the device is somewhat different. For the c.c. instrument, the aluminum frame upon which the moving coil is wound serves to carry the damping eddy currents, but for the a.c. instrument, a small aluminum disc is mounted on the shaft of the moving part, and turns between the poles of a small horseshoe magnet.

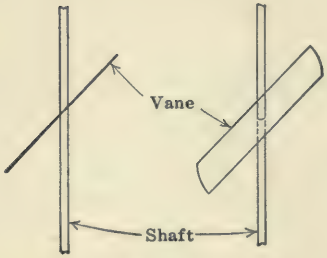


Fig. 171.

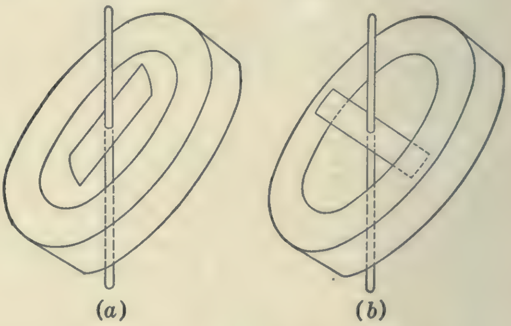


Fig. 172.

Fig. 171.—A thin vane of soft iron mounted on a shaft at 45°, constituted the moving element of the earlier types of iron vane meters.

Fig. 172.—The iron vane of Fig. 171 was mounted in a coil as shown here; (*a*) shows the vane at rest, and (*b*) shows it when deflected as far as possible.

A vane moving in a closed air cell is used more than any other device for damping a.c. instruments. Some of the older types use a vane moving in free air, but this damping is very ineffective. The vane should move in an inclosed air cell, nearly touching the walls of the cell, if effective damping is to be secured.

119. The Iron-vane Meter.—In its earlier form, the iron-vane meter consisted of one or two thin, soft iron plates, fastened to a shaft reaching through a small coil which carried the current to be measured. It was known as the inclined-coil type, because the coil was so mounted that the shaft of the moving vane made an angle of about 45° through the axis of the coil. Furthermore, the vane (or vanes) was mounted on the shaft in such a way that its length made an angle of 45° with the shaft. This idea is illustrated in Fig. 171, and the position of the vane and shaft in the coil is shown in Fig. 172. It can be seen from this figure that if the shaft turns counter-clockwise from position *a* to position *b* of the diagram, the vane has moved itself into the axis of the coil, from a position at right angles to this. As a piece of iron (in the form of a vane) tends always to pull itself into line with the magnetic field, it is seen that such an instrument as that indicated in Fig. 172 would operate. Suitable spiral springs tend to hold the vane in position *a*, and the vane pulls from this position towards position *b* in proportion to the current flowing through the coil.

A much more effective type of iron-vane meter utilizes the repulsive effect of like magnetic poles; it has two thin iron vanes which can reverse their magnetism readily in an alternating magnetic field. Although the vanes are actually cylindrical in form, we shall simplify the diagram at first by considering them flat. In Fig. 173 are shown two thin strips of iron, one rectangular (1), and the other trapezoidal in form (2). They are placed close together and parallel, as indicated in sketch *b* of Fig. 173. Now assume an alternating magnetic field to exist in the plane of the paper. The top sides of both vanes become north poles and the lower sides south poles; therefore the discs will repel each other if possible. Vane 1 is fixed, but vane 2 can move to the right in the plane of the paper. Because of its trapezoidal form, it will move to the right; and if it is retarded by spring action, its position will vary with the strength of the magnetic field in which the vanes are lying.

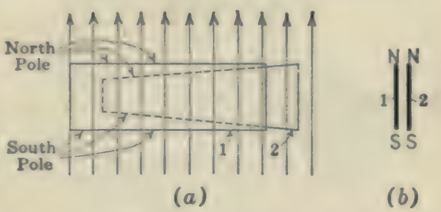


Fig. 173.—Showing conventionally the two iron vanes used in the modern iron-vane meter.

In the actual meter, both vanes (of the actual shape shown in Fig. 173) are bent into cylindrical form, and placed inside a small coil which is carrying the current to be measured. An attempt to picture the shape and arrangement of the cylindrical vanes is given in Fig. 174; the vane 2 of Fig. 173 is bent into a cylinder of smaller diameter than is vane 1, and is fastened by small spokes to the shaft of the meter, as shown in Fig. 174. The two vanes are properly mounted and placed in a coil, as is shown in Fig. 175, the shaft on which vane 2 is mounted being in the axis of the coil. In Fig. 176 is shown a picture of the movement of one of these iron-vane meters; it can be seen that it is very small, compact, and rugged. This type is used probably more than any other, especially for portable laboratory instruments. For a voltmeter, the coil is wound of many turns of fine wire, and for the ammeter a few turns of heavy wire; the iron-vane part of the meter is the same for either ammeter or voltmeter.

Voltmeters of this type generally have about 12 ohms resistance per volt of scale in the higher voltages. A 150-volt meter might have 1800 ohms resistance; this gives a power consumption at full-scale deflection of about 12 watts in such a meter. The corresponding continuous-current voltmeter would use less than 2 watts; the difference is really due to the fact that the permanent magnet is applicable to the c.c. instrument but cannot be used with the a.c. instrument. In the lower voltage ranges, the current required to operate one of these voltmeters is much higher than the value given above; a 15-volt voltmeter, for example, requires a current of about 0.2 ampere to give full-scale readings.

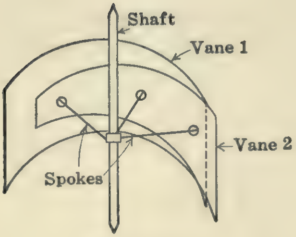


FIG. 174.

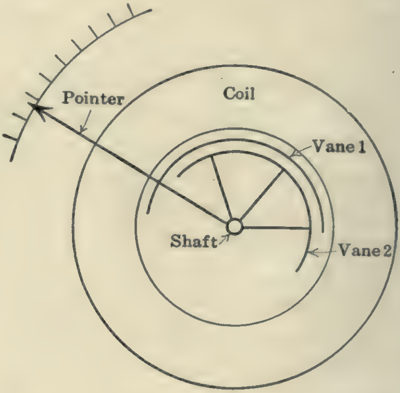


FIG. 175.

FIG. 174.—The vanes of Fig. 173 are actually cylindrical in form in the meter; the inner vane is bent on a smaller radius than the outer, so as to rotate freely.

FIG. 175.—Showing how the two vanes of Fig. 174 are mounted with respect to the coil, the current of which is to be measured.

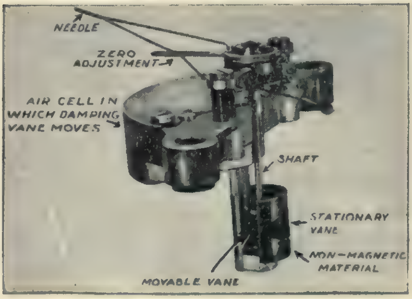


FIG. 176.—Showing the movement of a modern iron-vane meter; the movable vane in this one is rectangular and the stationary one trapezoidal.

The scales of these iron-vane meters are more uniform than those of the dynamometer type and the hot-wire type; the scale can be changed somewhat by properly shaping the trapezoidal disc used in the instrument.

120. Dynamometer Type of Meter.—The dynamometer type of meter uses no iron at all. There are two sets of coils in series; one set, consisting of two co-axial coils, some distance apart, is stationary, and inside this is pivoted the second set, generally consisting of only one coil, which can rotate. This type of instrument is illustrated by the Siemens dynamometer, the Kelvin balance, and a certain line of modern precision meters. Obviously, the moving coil cannot carry very heavy currents unless special means are used for getting the current in and out.

The meter is used to a considerable extent as a transfer instrument, that is, as an instrument which can be calibrated on continuous current and then used as a secondary standard for calibrating other alternating-current instruments. For voltmeters no difficulty of construction is encountered, because the current required for a voltmeter is not more than 0.1 ampere, and this can be led in and out of the moving coil through the springs which tend to keep it in its zero position. For ammeters a special construction must be used, because large currents cannot be carried into the moving coil. The arrangement of coils used is indicated in Fig. 177, the two parts of which show the moving coil in its extreme positions, and the circuit arrangement is given in Fig. 178. Diagram *a* of this figure is for a dynamometer voltmeter, and diagram *b* is for an ammeter. For the voltmeter many turns of fine wire are used in both stationary and moving coils, and a high resistance is inserted in series with them to hold the current to a reasonably low value.

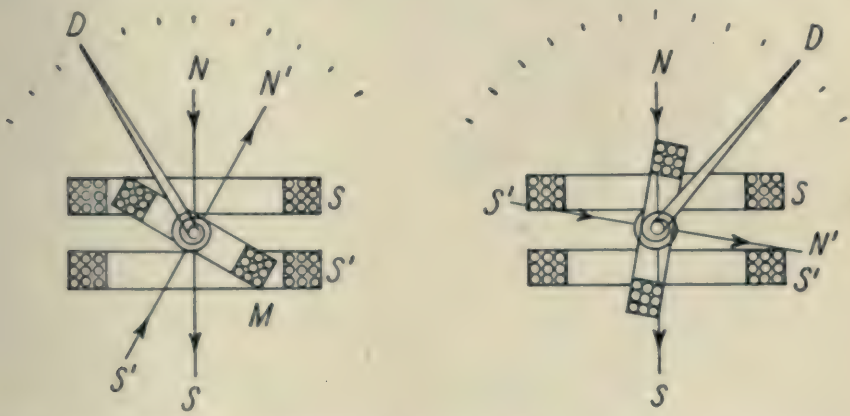
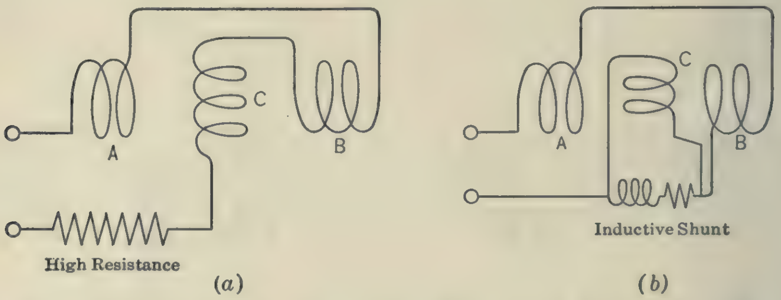


FIG. 177.—Showing the arrangement of stationary and moving coils in a dynamometer type of ammeter, for two positions of the moving coil.

In the ammeter the stationary coils, *A* and *B*, are generally built to carry 5 amperes, and the moving coil, *C*, carries but a small fraction of this. In series with the stationary coils is an inductive shunt, having the same ratio of reactance to resistance as has the moving coil which connects across its terminals. Only a small part of the 5 amperes is thus carried through the moving coil (as must be the case if light flexible connections are to be used to lead the current in and out of this coil) but this moving-coil current is directly proportional to the current through the meter. Because the power factor of the shunt is the same as that of the moving coil, the division of current between them will be independent of the frequency, and therefore the instrument will be reasonably correct at all commercial frequencies.

The dynamometer meter may be used with a deflecting pointer, as are other types of meters, but in its more sensitive form, used for a secondary standard with which to calibrate other a.c. meters, it is intended as a zero-reading instrument. A spiral spring, connected with the moving coil, is so arranged that its other end may be turned to bring the deflected coil back to its zero position. This is done, and the number of divisions the spring has to be turned varies directly as the square of the current being measured. This type of meter may be depended upon for an accuracy of about $\frac{1}{4}$ of one per cent if handled with care. It may be calibrated on continuous current by potentiometer and standard cell, and then the meter may be used to calibrate other types of a.c. meters by direct comparison.



(*a*) (*b*)

FIG. 178.—Circuit arrangement of a modern dynamometer type of instrument; (*a*) is for a voltmeter and (*b*) for an ammeter.

121.—The Wattmeter.—It was shown in Chapter III that voltmeter and ammeter readings in an alternating circuit were not sufficient to obtain the power used in the circuit, because the amount of power used depends upon the phase difference of E and I, as well as upon their magnitude. It is necessary, then, to have, in addition to voltmeter and ammeter, some means of determining the phase angle between them if volts and amperes are to be used to determine power. Power may, however, be measured directly if a *wattmeter* is employed; if, in addition to the wattmeter, voltmeter and ammeter are used, the power factor (cosine ϕ) of the circuit can be obtained, because we know that watts $= EI \cos \phi$.

The wattmeter is always a dynamometer-type instrument, the stationary coils of which are in series with the load, and the moving coil of which is connected (in series with a suitable limiting resistance) across the line supplying the load; this is indicated in Fig. 179. As the inductance of coil C (the movable one) is only a few millihenries, and as the limiting resistance R may be 1000 ohms or more, it is seen that the reactance of the moving-coil circuit is negligible compared to its resistance; the current in the coil is therefore practically in phase with the line voltage. Fig. 180 shows the appearance of a well-known portable wattmeter and Fig. 181 shows the internal construction of such an instrument.

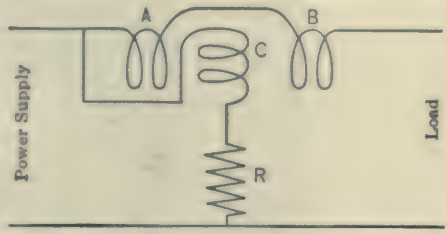


FIG. 179.—The coil arrangement of a wattmeter and its connections to a circuit.

The force tending to make the movable coil deflect is proportional to the current through this coil and the strength of the magnetic field in which the coil is lying; naturally, as both the current through coil C and the magnetic field set up by coils A and B vary sinusoidally, the torque acting to deflect C will be a pulsating one, varying from zero to a maximum in exactly the same way that the single-phase power fluctuates.

The torque at any instant may be written

$$t = Ki_cB, \quad \ldots \ldots \quad (113)$$

in which i_c is the current through coil C, B is the flux density set up by coils A and B, and K is a constant, depending upon the number of turns in coil C, its diameter, etc. As $i_c = \frac{e}{R+r}$, where e is the line voltage and r is the resistance of coil C (Fig. 179), and as the flux density B must vary with line current i, we may write for the instantaneous torque

$$t = K'ei, \quad \ldots \ldots \quad (114)$$

in which K' involves K of Eq. (113), the resistances R and r, number of turns in coils C, A and B, and other constant factors of the meter design.

Now as $e = E_m \sin \omega t$ and $i = I_m \sin (\omega t - \phi)$ in which ϕ may have any value between $-90°$ and $+90°$, depending upon the character of the load, we may write

$$t = K'E_mI_m \sin \omega t \sin (\omega t - \phi). \quad \ldots \ldots \quad (115)$$

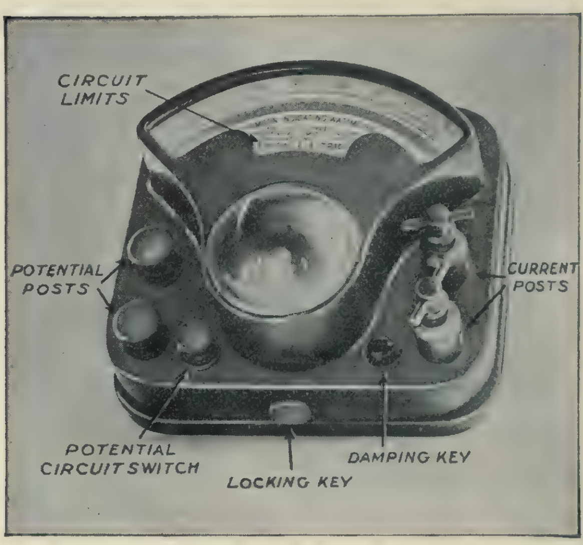


Fig. 180.—One type of portable wattmeter, suitable for laboratory use.

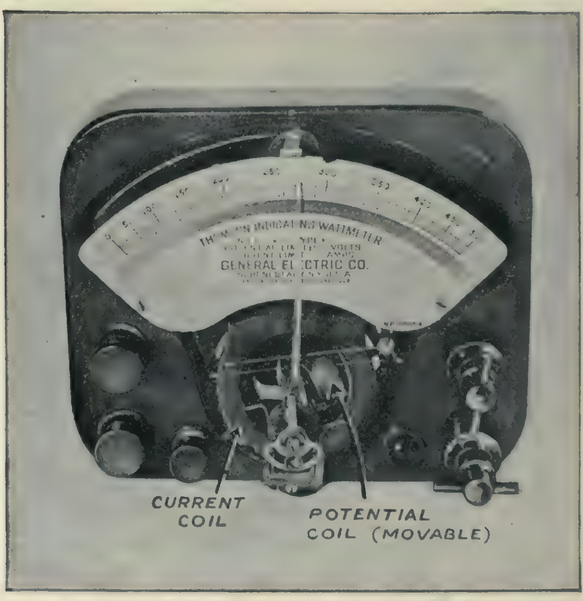


Fig. 181.—Showing the internal appearance of the meter pictured in Fig. 180; the potential coil has many turns of fine wire and imbedded in the instrument base are several spools of resistance wire, connected in series with the coil.

The average torque is the average value of this expression, and by reference to section 50 it is seen that the average torque will be given by

$$T_{av} = K''EI \cos \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

But this equation shows that the average torque (and hence the deflection of the moving coil against the control spring) will be proportional to $EI \cos \phi$, which, we know, is the power used in the circuit. Such a dynamometer meter can therefore be calibrated directly in watts, and will read the power used in any kind of load if it is connected with a circuit as shown in Fig. 179. The power will be correctly indicated no matter what the power factor of the load may be, provided this is not too low (say less than 0.1). By the use of ammeter, voltmeter, and wattmeter it follows then that the power factor of a load can be at once determined, and this is actually how the phase difference between the voltage and current in an ordinary circuit is determined.

It is evident that the current in the potential circuit of a wattmeter cannot be exactly in phase with the voltage impressed thereon; the moving coil must have some inductance, and so, of course, the current in the circuit must lag somewhat behind the voltage on the circuit. In the average wattmeter this lag is less than 1° at ordinary frequencies, so that no appreciable error results unless the wattmeter is being used for measuring the power in circuits of very low power factors; in such cases a large error may occur. For example, an ordinary wattmeter cannot be used for measuring the power loss in a condenser; the indication of the wattmeter cannot be relied upon at all in such a circuit, as the indicated loss (by wattmeter) may be only a small percentage of the actual loss, or perhaps may even be indicated as negative. This error, due to the inductance of the potential circuit, results in too small a reading of the wattmeter on condensive circuits, and too large a reading on inductive circuits.

122. Capacity of a Wattmeter.—The wattmeter consists of two circuits, either of which will be burnt out if too much current is sent through it; this fact is to be especially emphasized in the case of wattmeters, because the reading of the meter does not serve to tell the user that the coils are being overheated. In the case of an ammeter or voltmeter which is overloaded, the pointer will be indicating beyond the upper limit of its scale; in the case of the wattmeter, both the current and potential circuits may be carrying such an overload that their insulation is burning, and yet the pointer may be only part way up the scale. This is because the position of the pointer depends upon the power factor of the circuit as well as upon the voltage and current. Thus a low power-factor circuit will give a very low reading on the wattmeter even when the current and potential circuits are loaded to the maximum safe limit. This safe rating is generally plainly given on the face of the meter, as can be seen in the wattmeter dial shown in Fig. 182.

A wattmeter is always distinctly rated, not in watts but in volts and amperes, and care must be exercised to see that neither of these limits is passed. A certain wattmeter may be rated 150 volts and 5 amperes. The scale of such a meter, marked in watts, probably will not be more than 600; therefore, full rated voltage and current on resistive load would make the pointer move off the scale. As loads do not generally have unity power factor, and as the current and voltage limits are not reached at the same time, the scale will be more generally useful if the springs on the moving coil are designed to give full-scale reading at about 80 per cent of the safe volt-amperes of the meter.

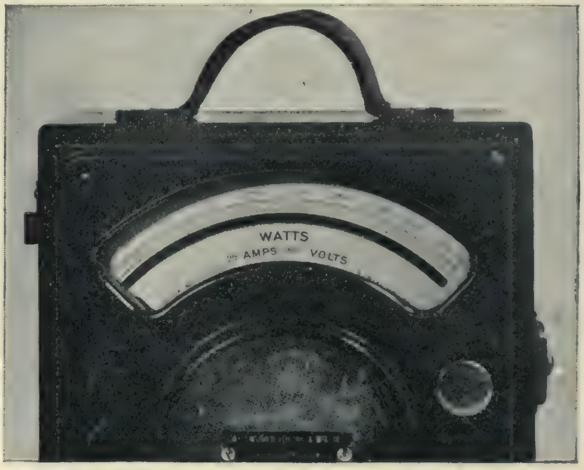


FIG. 182.—The limits of both current and potential circuits of a wattmeter are always clearly indicated; with the meter shown here not more than 25 amperes must be passed through the current coil, nor must a voltage in excess of 150 be impressed on the potential circuit.

Suppose that the above wattmeter (150 volts and 5 amperes) is being used to meter a 110-volt circuit carrying 5 amperes at 10 per cent power factor. The power used is $110 \times 5 \times .1 = 55$ watts, which will make the pointer occupy a position at less than 10 per cent of full-scale reading. Evidently, an accurate reading of this power cannot be obtained (even if the error due to the reactance of the potential circuit is neglected). For such circuits, wattmeters are available which are made with much lighter springs on the moving coil and which will indicate full-scale reading when perhaps 20 per cent of the rated volt-amperes of the meter are passing through it. Such a low power-factor wattmeter, for 150 volts and

5 amperes, would have a full-scale reading of 150 watts; it would be built the same as is the ordinary wattmeter, but its springs would be much weaker. The ordinary portable wattmeter uses from 10 to 20 watts in its circuits when carrying full load.

123. The Induction Type of Meter.—By an ingenious construction, it is possible to utilize, in the design of alternating-current instruments, a principle which is not available for continuous-current meters, as will be explained in detail in Chapter VIII, which describes the operation of an induction motor. It is possible, by suitable alternating currents in coils, to set up a so-called rotating magnetic field. This idea is indicated in Fig. 183. Suppose that four poles are wound in pairs, 1 and 2 having their coils in series, and 3 and 4 having their coils in series, and that alternating currents of different phases are sent through these coils. Suppose that two-phase currents are sent through them; at a certain instant there is a magnetic field from 1 to 2, and there is at this instant no field across from 3 to 4 (because when the current in 1–2 is a maximum, the current in 3–4 is zero). An instant later the current in 1–2 has dropped to zero, and that in 3–4 has built up to a maximum; hence there will be a magnetic field across 3–4. Halfway between the two instants just considered, there will be field across 1–2, as well as across 3–4, so that the resultant field will be between these two positions.

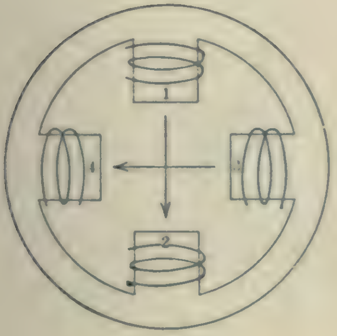


FIG. 183.

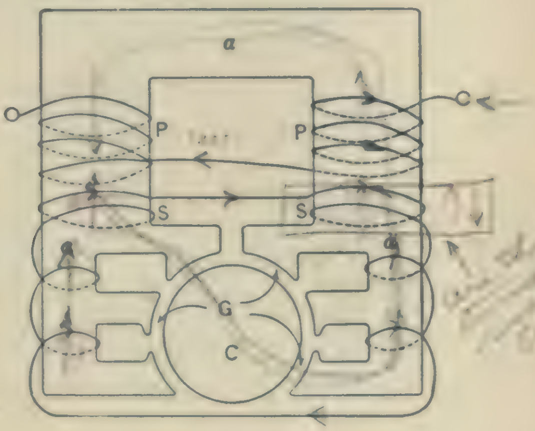


FIG. 184.

FIG. 183.—A conventional diagram to illustrate the action of two sets of coils, carrying currents of different phases, setting up a rotating magnetic field.

FIG. 184.—Such a magnetic structure, supplied with single-phase current in coils *P-P*, will produce a rotating magnetic field in core *C*.

If we consider successive instants of time, it will be found that the magnetic field seems to rotate (clockwise for the condition assumed in Fig. 183) at a uniform rate. Any piece of metal located between the poles will, of course, be dragged around in the direction of the field, because of the force between the eddy currents set up in the metal and the rotating field.

Evidently, if two-phase currents were available, the scheme of Fig. 183 would be applicable, but, in general, meters are required for single-phase operation. An instrument constructed as shown in Fig. 184 will, however, act as though it were supplied by two-phase current, in so far as the rotating field is concerned, and the construction illustrated in this figure brings out the fundamental principle of all induction-type meters. The horseshoe-shaped part of the magnetic circuit, *a–a–a* is, made up of punchings, as is the cylindrical core, *C*. An air gap, *G*, between the core and the pole pieces, is sufficiently wide to permit the turning of an aluminum cylinder which serves as the moving part of the meter. The laminations are wound with two coils, *P* and *S*, one of which, *P*, is connected directly in series with the circuit in which the current is to be measured (if the meter is designed as an ammeter). The second coil, *S*, is short-circuited on itself, and further differs from the first in that it has some turns in the slots of the pole faces. If this second circuit, *S*, has been properly designed, the current set up in it by its mutual induction with circuit *P* will lag behind the current in circuit *P* by such an amount that the sequence of fluxes produced by the two coils as shown in Fig. 185. The result of these two coils, both magnetizing the pole pieces, is then as shown in Fig. 185, *a* and *b*. At the instant when the current in *P* is a maximum, the flux has the direction shown in sketch *a*; coil *S* is ineffective at this moment as the current through it is zero. A quarter of a cycle later,

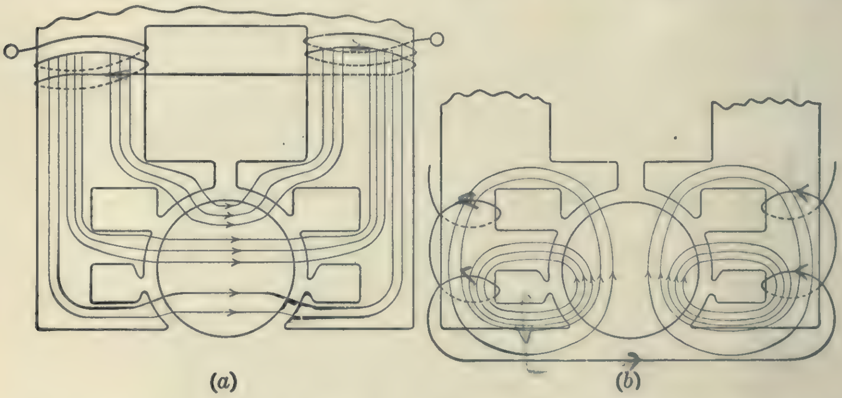


Fig. 185.—Current in coil *P* (Fig. 184) will produce a magnetic field as delineated in (*a*), whereas current in coil *S* will tend to give a field as shown in (*b*).

however, the current in coil P has dropped to zero and that in coil S is a maximum, and the flux set up in the pole face has somewhat the distribution shown in sketch b. It might seem that coil S would also give some flux in the direction shown in sketch a, because of the turns it has on the leg of the horseshoe, but as this is wound in opposite directions on the two poles, there is no tendency to send flux all around the magnetic circuit; at this instant, therefore, the flux would have the distribution shown in sketch b of Fig. 185. Ninety degrees (electrical) later the flux will have come back to the position shown in Fig. 185*a*, but it will be in the reverse direction, from right to left, instead of from left to right. And still 90° later the flux will have come back to the position shown in Fig. 185*b*, but in the reverse direction, that is, down instead of up. Hence such an arrangement of coils and iron core is able to produce from a single-phase circuit a rotating magnetic field.

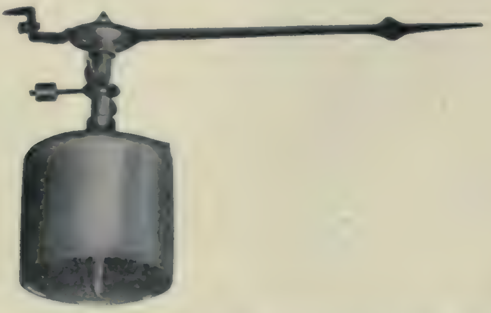


Fig. 186.—The moving element of a meter operating on the induction principle.

In the air gap of the magnetic structure is placed an aluminum cylinder properly fitted with shaft and bearings so that it can turn; such is shown in Fig. 186. The rotating magnetic field will set up eddy currents in the cylinder, tending to make it revolve in the same direction as the magnetic field. It is held from turning by a spiral spring, and as the tendency to turn increases, the pointer is twisted farther and farther around. The scale of such an instrument is very long, generally reaching over nearly 360° of arc. This induction type of meter may be made for ammeter (as described above), for voltmeter by making coil P of many turns of fine wire, and for wattmeter by using coil P of many turns of fine wire for the potential circuit and coil S of few turns for the current circuit of the wattmeter. There are certain compensating actions occuring in this type of meter which makes them nearly free of errors due to variation of frequency and temperature. Because of the large, open scale, they are especially good for switchboard installation, and are much used in such places.

The actual construction of one of these rotating-field induction meters is shown in Fig. 187.

124. High-frequency Meters.—None of the meters so far described is good if used with frequencies above a few hundred cycles per second; even here they have somewhat large errors which must be cared for. At 100,000 cycles per second, or possibly a million or more, all these types of meters will be inoperative. In the case of an ammeter, the impedance of the coil would be so great that it would require hundreds of volts to force the current through; in the case of a voltmeter practically no current would flow, because of the high reactance of the circuit. Thus a voltmeter intended for use at 60 cycles might have 1200 ohms resistance and 15 millihenries inductance. At 60 cycles the impedance would be 1200 ohms within the precision of ordinary measurement. At a million cycles its reactance would be nearly 100,000 ohms.

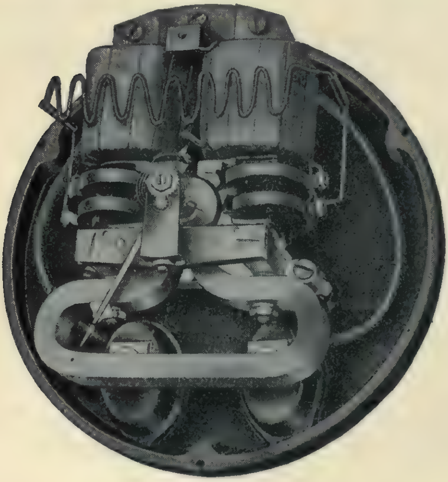


Fig. 187.

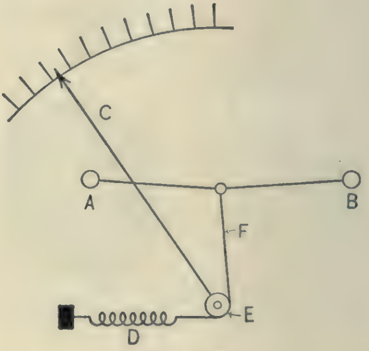


Fig. 188.

Fig. 187.—Showing the assembly of an induction type of meter. The four coils fitted in the pole faces are easily seen as well as the resistance wire through which they are short-circuited. The large permanent magnet is for damping the motion of the moving element.

Fig. 188.—The general form of low current-capacity, hot-wire ammeters. The current to be measured traverses the thin wire A–B, heats and expands it, and so permits the spring to move the pointer over the scale.

Iron-vane meters would not work at all, because of the excessive eddy-current and hysteresis losses. A high-frequency meter must therefore be so designed that it has no iron and no appreciable reactance; the two types of meter which have so far found application are the hot-wire meter and the thermocouple meter.

In the hot-wire type of meter, the current is made to traverse a thin, high-resistance wire which, because of its rising temperature, expands and allows a spring to turn the indicating pointer. In its simpler form, the current is passed through the wire between posts *A* and *B*, Fig. 188. A silk fiber, fastened to a glass bead threaded on the hot wire, winds around a drum on the shaft of the meter. The drum is continually tending to turn clockwise because of the action of spring *D*, and when wire *AB* expands, the drum turns and the pointer moves over the scale. To correct for room temperature, support *B* is adjustable, to change the initial tension of the wire. This enables the operator to make the zero correction which is necessary as the room temperature changes.

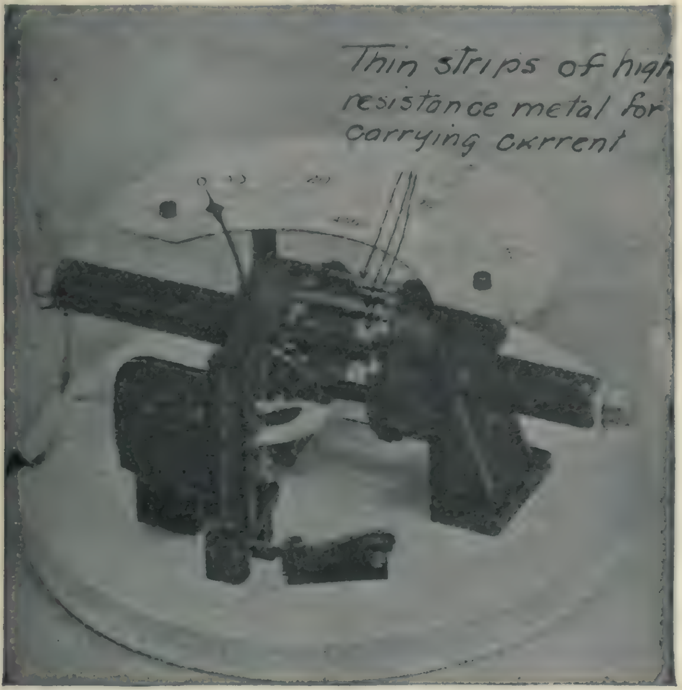


FIG. 189.—For high capacities, meters of the hot-wire type have barrel-like arrangements of thin, resistance-metal strips as the heating element.

In large capacities, a barrel-like arrangement of many thin platinum-iridium ribbons is used for the hot wire, as shown in Fig. 189. In this form the meter can be made in ranges as high as 200 amperes or more. The most sensitive meters of this type require about 50 milliamperes to give full-scale deflection; the wire for such meters is so fine as to be scarcely visible to the naked eye. In the barrel-like, high-range meters, the ribbon used must be very thin if errors are not to be introduced by large changes in frequency; actually the ribbons are only one or two thousandths of an inch in thickness.

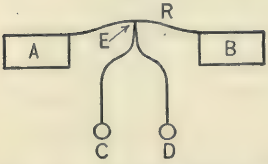


FIG. 190.—In the better type of thermocouple meters, the current to be measured passes through a small wire, *R*, across which is straddled the thermocouple. Sometimes the thermocouple is welded to the heater instead of merely resting on it.

The thermocouple meter depends upon the continuous e.m.f. developed at the junction of two dissimilar metals; if the junction is heated by the alternating current to be measured, a continuous-current millivoltmeter connected to the couple serves as a measure of the alternating current. Sometimes the alternating current flows through the junction itself, but as such a meter cannot be calibrated on continuous current it is seldom employed. Generally a heater is used, as shown in Fig. 190. A thin ribbon of high-resistance metal is fastened between the two terminals of the instrument, *A* and *B*; a sensitive thermocouple, *E*, is welded to the ribbon, and the other end of the couple is connected to posts *C* and *D*. To these posts a continuous-current millivoltmeter is connected. As the temperature of the heater varies with the square of the current, and as the thermal e.m.f. of the junction varies as the temperature of the hot junction (fastened to the heater), the reading of the millivoltmeter is proportional to the square of the current in the heater.

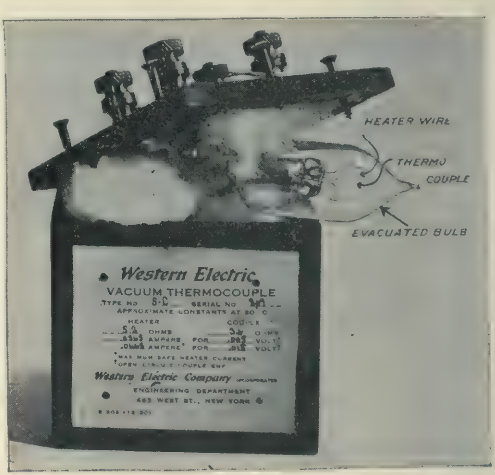


FIG. 191.—A typical vacuum thermocouple. The heater wire is stretched between the outer pair of prongs while the inner pair carries the thermocouple. This instrument gives 0.001 volt on the couple for about 15 milliamperes through the heater.

In the high ranges this thermocouple meter is built for many amperes, and in the more sensitive types is perhaps good for as low as 25 milliamperes full scale. The heater and the couple are inclosed in an evacuated glass bulb. For such couples (illustrated in Fig. 191) the heater may have as much as 1000 ohms resistance, being made of carbon filament or platinized glass capillary tubes; the higher this resistance, the more sensitive is the device as an ammeter.

125. Instrument Transformers.—If it is desired to extend the range of a continuous-current voltmeter, a suitable resistance is put in series with the instrument; if the range of an ammeter is to be increased a larger shunt is used. Both of these expedients might be used also for alternating-current instruments, but certain errors will probably occur (due to effects not present in c.c. circuits), and furthermore it is not desirable in high-voltage circuits that the meter itself should be electrically connected to the power line at all, even through a high resistance. It is evident that if an ammeter were connected directly in series with, say, a 100,000-volt line, the operator, coming near the meter for reading or adjustment, would receive a fatal shock. And in the case of a voltmeter, not only would the danger be present, but also a great deal of power would be unnecessarily used. Thus, if a voltmeter of the dynamometer type (taking 0.1 ampere) were used, the power consumed in a series resistance would be 10 kw., and this expenditure of energy is entirely unnecessary.

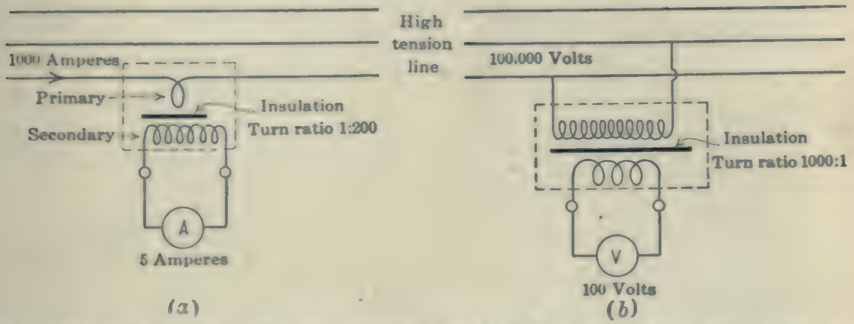


FIG. 192.—Instruments used for metering high-voltage lines are always operated through suitable transformers, which are insulated for high voltage.

Shunts are practically never used with alternating-current ammeters because of the difficulty of making such shunts accurate, so that changes in frequency would not introduce errors of calibration. If a shunt is to be used it must have exactly the same power factor as the meter to which it is to be connected, and no flexible connections are allowable; a change in their disposition would appreciably change the reactance of the circuit of which they formed a part. To obviate these difficulties, instrument transformers are always used when meters are to be connected to high-voltage circuits.

The general theory of a transformer is taken up in Chapter VII; it is sufficient here to say that a transformer consists of two separate electric circuits on the same iron core, the two windings being well insulated from each other, so that even if one winding is connected to a 100,000-volt transmission line, the other winding may be handled without danger of shock. In the case of the current transformer, it is customary to so proportion the number of turns in the windings that, for any specified value of current in the power line, the current in the secondary coil to which the ammeter is connected will not exceed 5 amperes. Thus, if the high-voltage line is to carry 1000 amperes at full load, the current transformer, by which this current will be metered, would be so proportioned that 1000 amperes in the primary winding (in series with the transmission line) would give 5 amperes in a secondary winding to which a 5-ampere meter would be connected. This transformer would be designated as a 200 : 1 current transformer, and the ammeter to be used in conjunction with it would really be a 5-ampere meter, but it would be marked with a 1000-ampere scale. The insulation between the primary winding and the core, as well as that between primary and secondary windings, would have to stand 100,000 volts and so would have to be tested at the factory to at least 200,000 volts. The connection of a current transformer to one wire of a three-phase transmission line is shown in Fig. 192*a*.

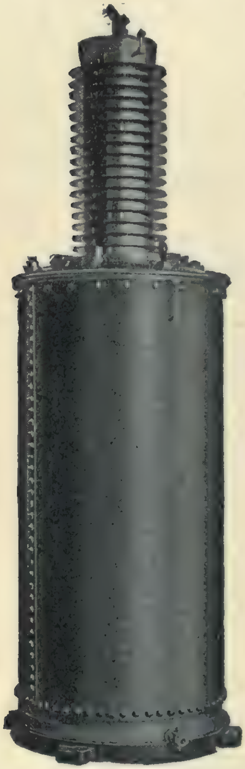


Fig. 193.—A current transformer for use in a high-voltage line; there is practically no voltage across the primary winding, both leads going down the same insulator. The highest grade of insulation is required between primary and secondary, the latter being at ground potential.

It is customary to so design voltage transformers that the secondary side generates no more than 150 volts, no matter what voltage the primary side has been designed for. Thus a voltage transformer to be used in conjunction with the transmission line of Fig. 192 would have a ratio of 1000 : 1, so that 100,000 volts on the line would give 100 volts on the voltmeter. This voltmeter would, however, have its scale calibrated in terms of transmission-line voltage; that is, when it was really reading 100 volts, the scale would indicate 100,000 volts or 100 kilovolts, as indicated in Fig. 192*b*. This transformer also would have to be highly insulated, as explained for the current transformer.

Although such transformers have to supply only a few watts (just enough to operate one or two meters) they are very bulky because of the space required for the insulation. Figure 193 shows a current transformer for use on a 150,000-volt transmission line.

For laboratory work these instruments are made in much smaller sizes; thus a common potential transformer has a ratio of 2200 : 110, so that a 150-volt voltmeter may be used in testing 2200-volt circuits. Current transformers are made in such a form that different ratios are obtainable; in one common type the number of turns in a primary may be changed at will so that any current ratio is obtainable. Thus a 5-ampere ammeter becomes suitable for testing machines and circuits no matter how much current they actually use. Whereas the laboratory transformers are built for a rated load of about 15 watts, the larger ones (Fig. 193) will deliver much more, enough to operate trip coils, relays, etc.

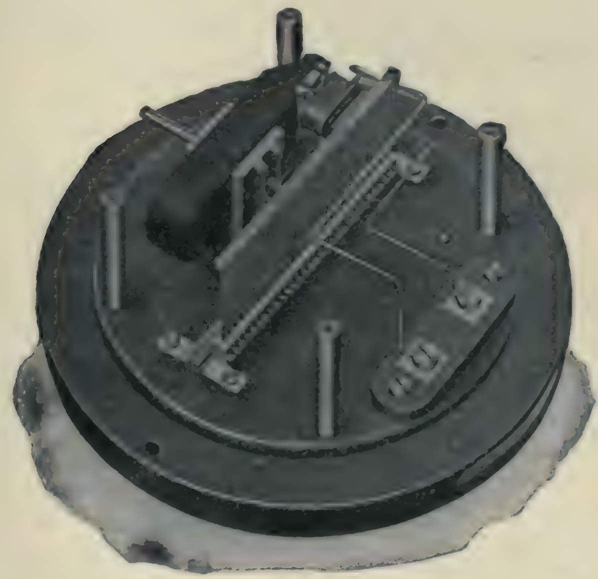


FIG. 194.—The reeds of this frequency meter are mounted on a flexible metal strip which is caused to vibrate by the electromagnet seen in the upper right part of the figure; that reed having a natural frequency equal to that caused by the electromagnet will vibrate most strongly.

126. Frequency Meters.—There are two general types of meters used for measuring the frequency of an alternating current. The simpler one is an electromagnet excited by the current to be tested, having close to it a series of thin metal reeds which are attached to a flexibly supported base bar; to this bar is attached a piece of soft iron which is attracted by a pair of electromagnets excited by the current, the frequency of which is to be determined. The vibration of the base block, thus brought about, causes all the metal reeds to vibrate a little, but the one that has a natural period equal to that of the alternating current is the only one that vibrates with a large amplitude. The reeds are so designed as to have natural frequencies of 59, 60, 61 vibrations per second, etc., this adjustment generally being accomplished by using various lengths and suitably loaded reeds.

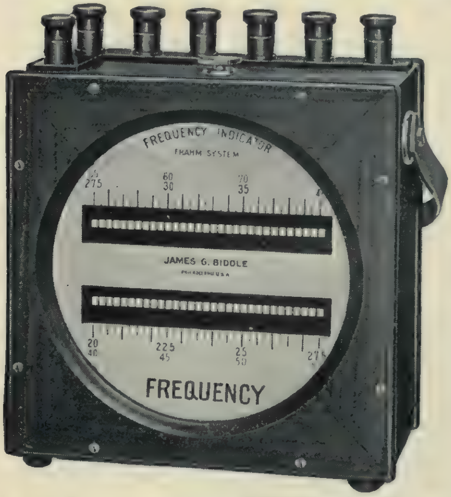


Fig 195.—Front view of a frequency meter intended for frequencies from 20 to 80 cycles per second; the doubling of the range (indicated by the double figures in the scale) is accomplished by using a permanent magnet in conjunction with the electromagnet, the various posts are for different voltages.

In frequency meters built on the principle of an electromagnet attracting a piece of soft iron it is comparatively simple to double the readings of frequency by the addition of a permanent magnet. As the soft iron armature is attracted to the electromagnet no matter which way the current flows through the magnet it is seen that a 60-cycle current would attract a piece of soft iron 120 times a second. Thus the natural mechanical frequency of the reed which is to respond to a 60-cycle current must be 120 vibrations per second. If now the field of a permanent magnet is superimposed on that of the electromagnet and the field strength of the permanent magnet is the same as that of the maximum strength of the electromagnet then with a 60-cycle current through the electromagnet the iron armature will be attracted only 60 times a second. Thus a reed with a natural mechanical frequency of 60 per second will respond when a 30-cycle current is flowing through the electromagnet with the permanent magnet moved away and to a 60-cycle current when the permanent magnet acts in conjunction with the electromagnet.

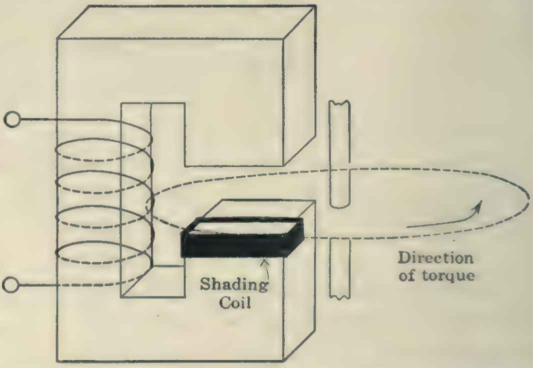


Fig. 196.—If one-half of the pole of an electromagnet is surrounded by a collar of good conductor, the magnetic field set up in the pole "moves across" its face, moving towards the collar, called a shading coil. If one-half of the pole is covered by a sheet of copper the same effect is obtained.

In the reed frequency meter a permanent magnet is so mounted that its field may be superimposed on that of the electromagnet or not, as desired; thus a meter which has a range of from 20 to 40 cycles, by the addition of the magnet has its range changed to from 40 to 80 cycles.

The general idea of this type of frequency meter is shown in Fig. 194, and Fig. 195 shows the face of such an instrument. Fractional cycles per second are readable on this type of meter even though the reeds themselves are separated by whole cycles. If, for example, the 59-cycle reed is vibrating quite strongly and the 60-cycle reed is showing small vibration, the frequency is about $59\frac{1}{4}$ cycles per second.

In the other type of frequency meter, the instrument works on some kind of a balancing effect between a reactive and a resistive circuit, the balance changing if the frequency changes. In one of these types, the so-called "shading coil" principle is used, a principle which has much application in other types of a.c. apparatus. The simple idea is illustrated in Fig. 196; around one-half of one pole of a laminated electromagnet, through which alternating current is being passed, is placed a copper ring. Sometimes the face of one-half of the pole is covered with a copper plate and the same effect is obtained. An alternating flux is set up by the winding and will naturally induce currents in the copper collar; the currents in this collar will so act as to make the field build up in this part of the magnetic path later than in the part around which there is no copper collar. The net effect is to give a magnetic field which "sweeps across" the pole face from the part without the collar to the part enclosed by the collar. A metal disc between the poles will therefore have eddy currents induced in it, and these currents will react with the magnetic

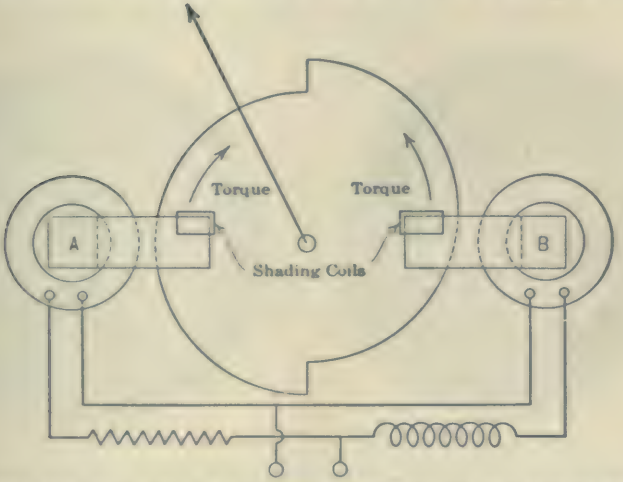


FIG. 197.—The induction type of frequency meter utilizes the idea of balancing the torque from a resistive circuit against that from a reactive circuit.

field so as to turn the disc in the direction in which the flux is moving. The disc will therefore tend to turn in the air gap from the uncovered part of the pole to that part which is covered or enclosed by the copper collar.

The torque exerted on the disc varies with the square of the voltage impressed on the electromagnet (with constant frequency), and directly with the frequency for a given current through the magnet; this principle is therefore available for making ammeters, voltmeters, and frequency meters. In the frequency meter two such shaded-pole magnets are employed in connection with a specially formed aluminum disc. The connection scheme and shape of disc are shown in Fig. 197. Both electromagnets are connected across the line, the frequency of which is to be determined, but one is in series with a reactance and the other in series with a resistance. Both magnets have their shading coils on the same side of their poles, so that the two magnets give oppositely directed torques to the disc. A circular disc would evidently maintain no position of equilibrium under such conditions; hence the disc is so cut that the side under the pole of magnet *A* is circular, and that under the pole of magnet *B* circular but eccentric. Owing to this shape, the area of disc under magnet *A* remains constant as the disc turns, while that under magnet *B* varies, decreasing as the disc turns counter-clockwise.

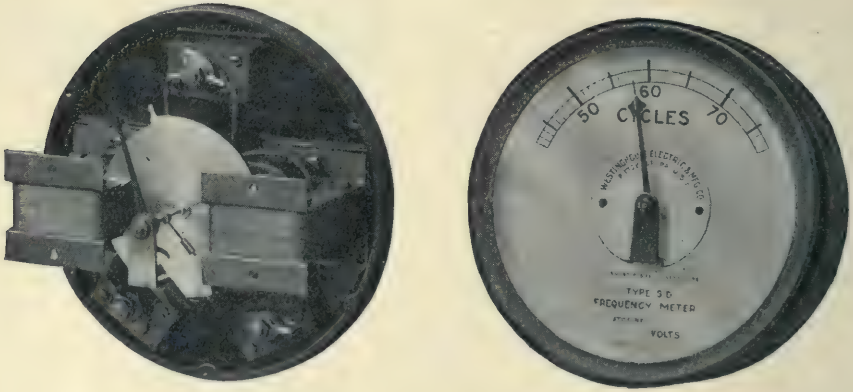


Fig. 198.—Internal and external appearance of a frequency meter built on the principle shown in Fig. 197.

Such being the case, at any given frequency the disc will have a point of stable equilibrium when the amount of metal under the two poles is such that the two torques balance. If the disc is displaced clockwise, more area is exposed to pole *B*, so that its torque overbalances that of *A* and the disc turns in the direction of the torque of B, that is, counter-clockwise. Now, if the frequency stays constant but the voltage of the line increases, both torques will increase in the same proportion and the position of the disc will remain the same; if, however, voltage being constant, the frequency decreases, the flux in magnet B will increase, and because of this increase in flux the torque of B will become greater and the disc will turn counter-clockwise until, as a result of the decreased area under magnet B, its torque is again balanced by that of A. As the torque on the disc varies with the square of the flux density in the disc and with the frequency, it is seen that the torque of B is increased and that of A is decreased by decreasing frequency. This increase in torque in B occurs in spite of the decreased frequency, because of the greater effect of the increased flux. The construction and external appearance of a frequency meter which operates on the principles just outlined is shown in Fig. 198.

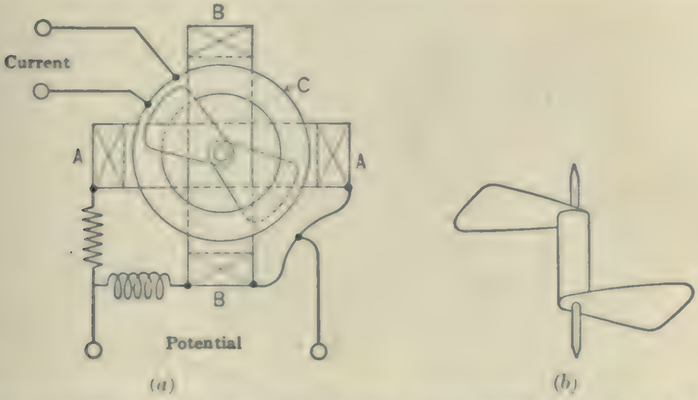


Fig. 199.—Showing the coil arrangement and circuit connections of a single-phase power-factor meter in diagram a and the form of the rotating iron vane in b.

127. Power-factor Meters.—A power-factor meter must evidently work on some principle which will place the moving element (carrying the indicating pointer) in a position which depends upon the phase of the current in the current coil with respect to that of the current in the potential coil, which latter should be in phase with the voltage of the line. Furthermore, the moving element must take the same position (for a given phase displacement between these two), irrespective of the magnitude of either the voltage or current. None of the movements so far described is suitable to perform such a task; the power-factor meter involves another principle. One type of single-phase power-factor meter works on the principle suggested in Figs. 199 and 200. From Fig. 199 it is seen that there are three coils and an iron vane used in the essential part of the meter.

The two outer coils, *A* and *B*, are similar in construction and are placed at right angles to one another. One is connected to the line through a high resistance and the other through a high reactance, so that their currents will be 90° out of phase with each other. Such a combination of currents and coils will produce a rotating magnetic field, as was shown in connection with Fig. 183.

Coil *C* has its axis at right angles to the axes of coils *A* and *B*, and is placed in series with the line, so that the line current flows through it. It will have an alternating magnetic field of the same frequency as that of coils *A* and *B*. Pivoted on the axis of coil *C* is a light iron vane of peculiar shape, as shown in diagram *b* of Fig. 199; in diagram *a* of this figure is shown the disposition of the parts of this form of power-factor meter. This iron vane (of thin, soft iron) can reverse its magnetization with the reversing current in coil *C*. The force exerted between this magnetized iron vane and the rotating magnetic field of coils *A* and *B* is such that the vane tends to pull into such a position that when the magnetization of the vane is a maximum, the vane lines up with the position of the rotating magnetic field at that instant. As the phase of the current changes with respect to that of the voltage, the iron vane will have its maximum magnetization when the rotating field has different positions, so that a pointer attached to the free iron vane (it has no springs or other restoring force) can move over a scale graduated in power factor.

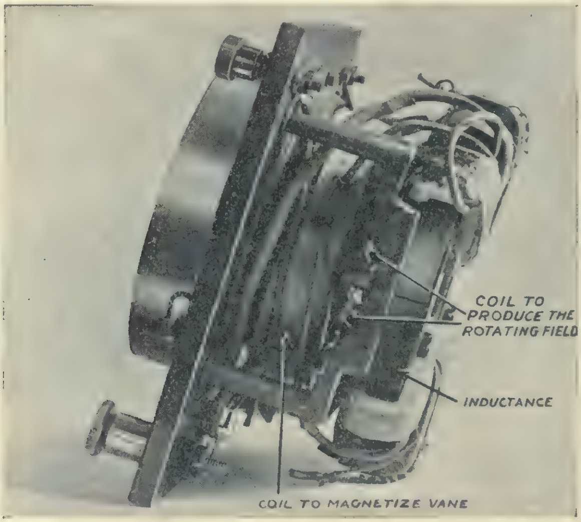


FIG. 200.—Internal construction of a single-phase power-factor meter, the principle of operation of which is given in Fig. 199.

If the potential circuit of such a meter is left closed, with no current (or only a very small current) in the current coil, the iron vane will rotate rapidly, trying to keep up with the rotating magnetic field; if the current coil is closed and the potential circuit is open, the vane will occupy no definite position unless there is some stray magnetic field getting into the meter. To prevent the entrance of such stray fields, the working part of the meter is surrounded with a laminated iron ring which acts as an effective shield against all outside magnetic fields. An aluminum disc carried on the shaft rotates in the field of a strong permanent magnet to damp the oscillatory movement of the pointer. To give accurate readings, there should be at least 2 amperes in the current coil of the meter, this coil being normally built for a full-load capacity of 5 amperes.

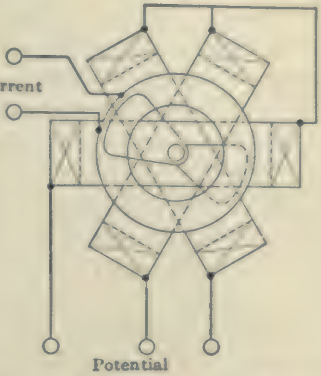


Fig. 201.—In the three-phase power-factor meter it is not necessary to use the split-phase scheme of Fig. 199; three coils, 120° apart, are connected (through suitable resistances) to the three wires of the power supply line.

For three-phase circuits, it is not necessary to use the "split-phase" scheme of Fig. 199; instead of two coils at right angles, three coils at 120° are used, and these coils are connected to the three wires of the three-phase system, the coils being connected in Y. This scheme is shown in Fig. 201; the current coil and iron vane are disposed exactly as they are for the single-phase power-factor meter. The single-phase power-factor meter is subject to a considerabe error if other than rated frequency is

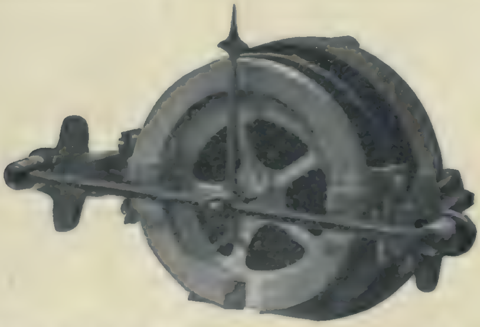


Fig. 202.—Power-factor meter with cover removed; the nearly circular magnet and aluminum ring (supported on five spokes) are for damping the motion of the moving iron vane.

used, but the three-phase meter is independent of frequency. In Fig. 202 is shown the general appearance of one of these power-factor meters; on the top can be seen the nearly circular magnet used for damping purposes, and directly under it can be seen the aluminum disc (having five spokes) which moves in a field of the permanent magnet to give the damping. Surrounding the working part of the meter can be seen the laminated iron ring which serves to shield the meter against stray fields.

The total power used in a power-factor meter is about 15 volt-amperes at full load; the coils to produce the rotating field use about 10 volt-amperes at rated voltage.

128. Synchronoscope.—This instrument (generally called a synchroscope), as its name suggests, is used to show whether or not two alternating voltages are maintaining the same relative phase, that is, whether or not they are in synchronism. In the manipulation of alternating-current generators, it becomes necessary to know whether or not the two machines are generating voltages of the same frequency and phase, and the synchroscope is the instrument commonly used to give this indication. To show the relative phase of two voltages is practically the same problem as to show the phase between a current and a voltage. Hence, the idea of the power-factor meter should work for a synchroscope, and in fact, it does so. If the current coil of the power-factor meter is rewound so that it has many turns of fine wire, and sufficient resistance is put in series with it to properly limit its current, the power-factor meter becomes a synchroscope. The instrument is generally used as indicated in Fig. 203; alternator *A* is already running and connected to the bus-bars. As the load on the station increases, it becomes necessary to connect in another alternator, but the switch of this incoming machine may not be closed until it is ascertained that its voltage is equal and opposite to that of the bus-bars, and that its frequency is practically the same as that of the machine already operating. When coil *C* of Fig. 199 (after it has been rewound for high resistance) has been connected to machine *B*, and coils *A* and *B* to the corresponding terminals of the bus-bars, the power-factor meter, now being used as a synchroscope, would indicate the relative phase of the two voltages. As the two machines, *A* and *B*, will never be running at exactly the same speed, their relative phase will slowly

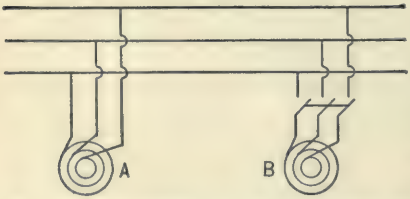


FIG. 203.—In connecting an alternator to a set of bus-bars already alive, the relative phase of the bus voltage and that of the incoming machine must be known. A synchroscope must be connected across the open switch of machine *B*.

change and the finger of the synchroscope will correspondingly slowly rotate. A mark on the dial of the instrument shows when the two voltages are just opposite in phase, and this shows the time when the switch of machine B may be properly closed.

The synchroscope may have the construction of the single-phase power-factor meter or of the three-phase one; in the latter case, the three-coil combination is sometimes put on the rotating part, thus resembling a small 3-coil armature for an alternator.

If the incoming machine is running too fast the pointer of the synchroscope turns in one direction, and if too slowly it turns in the other. In case the machines are running at the same electrical speed, the pointer of the synchroscope remains stationary, showing by its position the relative phase of the two voltages.

In another type of synchroscope, the indicating finger oscillates back and forth over a scale instead of having the rotating motion of the type just described. In the oscillatory type of instrument, the difference in frequency is shown by the rapidity with which the finger moves over the scale, and the condition of exact phase opposition (when the switch may be closed) is marked on the scale just as it is for the rotating synchroscope.

These two types of synchroscope are shown in Figs. 283 and 284.

129. The Oscillograph.—There are many problems in alternating-current engineering the answers to which can only be given when we have an exact picture of the voltages and currents existing during the occurrence of the phenomenon in which we are interested. What happens when a generator short-circuit occurs, when an arcing ground occurs on a transmission line, when a transformer is first connected to the line, when bad interference is caused to neighboring telephone lines, when a synchronous machine hunts? These questions, and innumerable others, can only be answered when we have a photographic record of the various voltages and currents in their proper relative phases and magnitudes. The oscillograph is the instrument that gives us these records. Whereas a few years ago the oscillograph found application only in laboratories, at

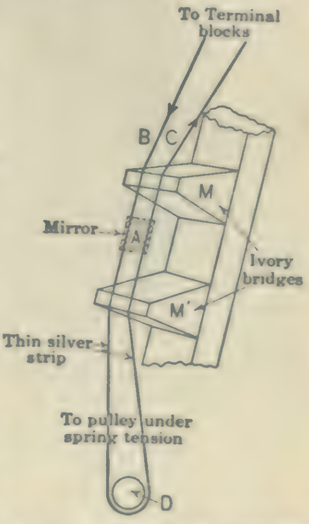


Fig. 204.—A simple diagram of the essential parts of the oscillograph, vibrator. The vibrating portion of the bifilar system, between bridges M-M', has very little moment of inertia; its natural frequency is about 5000 vibrations per second.

present every large electric company has at least one, generally arranged on a truck for transportation, thus making it easy and simple to carry out oscillograph tests on any part of a distribution system. Essentially the oscillograph is a bifilar suspension of fine silver ribbon, cemented to which is a very small, light mirror about as shown in sketch of Fig. 204. The silver ribbon *B–C* passes over the pulley *D*, which is held down by a spring, thus serving to keep the ribbon stretched tight. Two slotted ivory bridges *M–M′* serve to hold the wires suitably spaced, and between them is cemented the mirror *A*. This mirror is 0.02 inch wide and 0.06 inch high. Under a normal tension of about 8 ounces the natural frequency of the oscillatory system is several thousand cycles per second, thus permitting an accurate reproduction of wave forms having a frequency as high as about 1000 cycles per second. A photograph of an actual vibrator from the oscillograph is shown in Fig. 205.

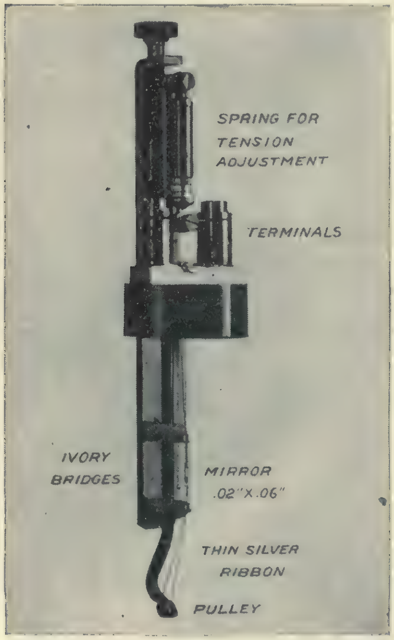


Fig. 205.—Photograph of the vibrator of an oscillograph. The oscillating part of the bifilar system is that between the two ivory bridges.

The poles of a powerful electromagnet reach close to the suspension on either side, so that a strong magnetic field surrounds the silver ribbons, the field being parallel to the plane of the mirror. Current, passing down one wire and up the other, will twist this suspension and with it the mirror. A beam of light reflected from the mirror is thus made to vibrate, and if the reflected beam is allowed to fall on a moving sensitive film, an accurate record of the wave form through the suspension is obtained.

The maximum deflection of the mirror is only about 3° on either side of its zero position, yet the most minute ripples on current and e.m.f. waves are shown on its records, called oscillograms. The maximum current capacity of the suspension (which operates in oil to damp out natural oscillations) is 1/10 ampere, and as the resistance of the vibrator is 1 ohm it requires 0.01 watt to operate the vibrator to full scale. However, gold leaf fuses are generally put in series with the vibrator to give some protection against burn-out, and as the resistance of one of these fuses is

9 ohms, the resistance of a vibrator as installed is about 10 ohms and it therefore requires 1 volt to give full-scale deflection.

By decreasing the tension on the vibrator its sensitiveness can be increased, but it is not then useful at so high a frequency because its natural frequency has been lowered. If care is used to keep the optical system clean and good mirrors are used, it is possible to get good photographic records at such a film speed that the beam of light travels about 50 feet per second.

A view of a three-element oscillograph, properly mounted on a table with all necessary rheostats, motors, speed-reduction pulleys, arc lamp, etc., is shown in Fig. 206.

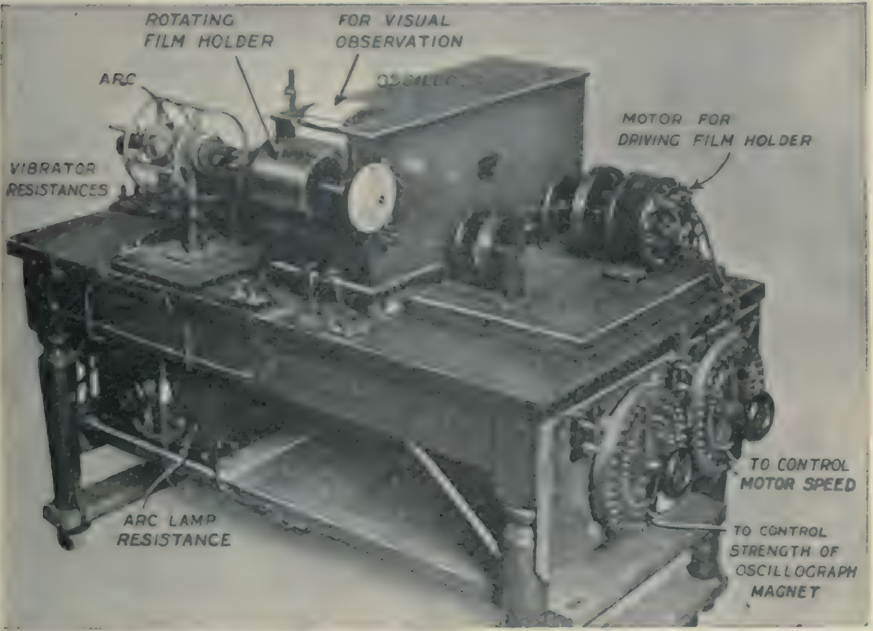


Fig. 206.—A three-element oscillograph portably mounted, with all the requisite accessories for convenient operation.

130. Recording Meters.—Meters intended to keep a record of instantaneous values are called recording meters; recording ammeters, voltmeters, and wattmeters are much used in central stations. The moving element of a recording meter carries a pen which draws a curve on a moving sheet of paper, the paper being moved past the pen at an adjustable speed by suitable clock work. In the type of recording meter shown in Fig. 207, the moving element is a kind of magnetic balance which carries the glass-tube pen back and forth over the paper strip, as the current through the meter, or voltage impressed on the meter, varies.

The records from such meters are evidently valuable in showing the station manager how the load on his station varies, when peak load occurs and for how long, etc. The records of a recording voltmeter show how faithfully the voltage of the station is maintained at its rated value, and in case of litigation, involving the operation of the station, they are the best kind of evidence as to the facts in the case.

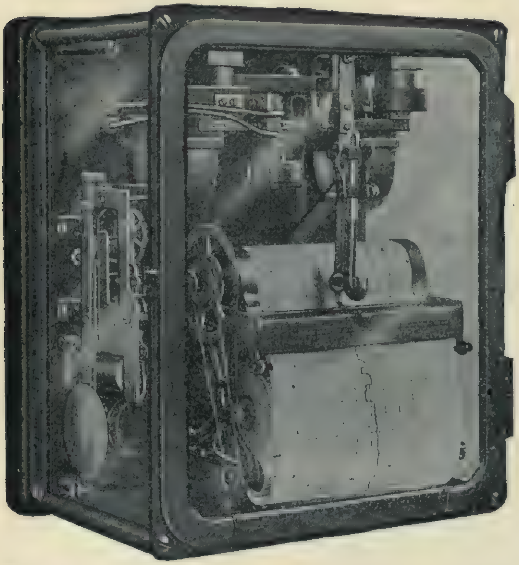


Fig. 207.—A recording meter in which the moving element is very similar to a Kelvin balance.

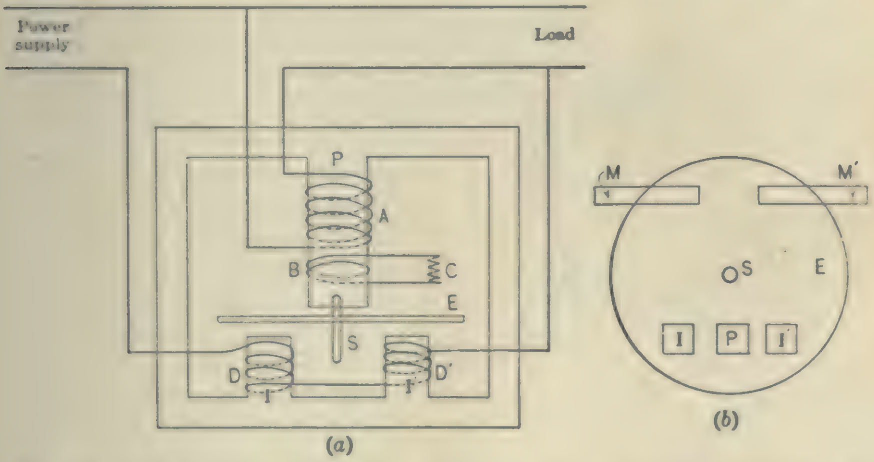
131. The Watt-hour Meter.—To record the total energy output of a station, and to measure the energy the customer uses, the watt-hour meter is always used. It always consists of a small motor so designed that its speed is proportional to the power supplied through the meter; its total revolutions, therefore, show the integration of (power × time) which is energy. Evidently such a meter is really an electric energy meter.

The Thomson watt-hour meter, described in Chapter VI of Vol. I, will run on alternating-current circuits, but as the induction type of watt-hour meter is applicable to alternating-current service, and as this type is superior in maintenance and accuracy to the Thomson type, it is universally employed. Two-and three-phase watt-hour meters are frequently used on station switch-boards, but practically all customers' meters are single-phase.

There is no piece of apparatus in the electrical system of more importance to the station manager than this little induction-motor meter; it is from the records of such meters that the whole income of the station is obtained. By comparing the monthly total of the readings of customers' meters with the indication of the station's output meter, the manager gets accurate knowledge of the efficiency of his distribution network; it shows what percentage of the energy output of the station is sold, and how much is wasted in transformers, auxiliary apparatus, leakage, etc.

The operation of the induction type of meter may be understood by the help of the diagrams in Figs. 208 and 209. In Fig. 208*a* is shown the approximate arrangement of the magnetic circuit and connection of the coils in this meter, and in Fig. 208*b* is shown a plan of the disc and poles. *P* is the potential pole of the meter, *I* and *I′* are the current poles of the meter, *E* is the aluminum disc in which eddy currents are induced, *S* is the shaft and *M*-*M′* are the permanent horseshoe magnets which serve to control the speed of the disc.

The meter develops a torque because of the interaction of the eddy currents set up in its disc and the magnetic field of the potential and current poles. In order that the meter may indicate properly, it is necessary that the flux in the potential pole should lag exactly 90° behind the voltage of the line, and that the flux of the current poles should be in phase with the current in the line. Evidently this latter condition is at once fulfilled if the poles are suitably laminated and the coils *DD′* are connected in series with the line. The current in coil *A* will not lag 90° behind the line e.m.f., however, because of the losses occurring in coil *A*. To make the flux in the pole *P* lag just 90° behind the line voltage, a second coil, *B*, is wound on this pole and is short-circuited through a suitable resistance, *R*. When pole *P* is magnetized by coil *A*, a current will be set up in coil *B*, and if this is properly proportioned and *C* has the right resistance, the vector sum of the m.m.fs. of coils *A* and *B* will produce a flux which is just 90° behind the line voltage impressed on coil *A*. Coil *B* is generally called the "lag coil" of the meter.



(*a*) (*b*)

FIG. 208.—The essential parts of a watt-hour meter movement, employing the induction-motor principle.

The interaction of the eddy currents in the disc and the magnetic poles is shown in the four diagrams of Fig. 209; only that portion of the disc in which eddy currents are set up is shown. It is supposed that a non-inductive load is being supplied through the meter, so that the line current and the line voltage have the same phase.

In diagram *a* the conditions are shown for the instant when the voltage and current of the line are zero. When the voltage is zero the flux in *P* is a maximum (flux lags 90° behind voltage) and the flux in *I*–*I*′ is changing at a maximum rate. The eddy currents set up around their poles will then be a maximum, and as they are wound in opposite directions the eddy currents will be distributed as shown. (In checking the direction of currents and magnetic poles it is to be remembered that, in the figures, poles *I* and *I*′ are below the disc and pole *P* is above the disc.) The torque generated by the eddy currents surrounding the current poles is, as shown by the arrow, from left to right.

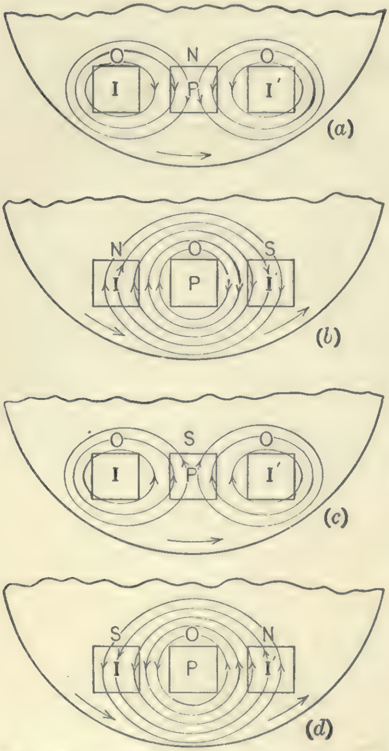


FIG. 209.—At successive quarter-cycle intervals the eddy currents in the disc, and polarity of the magnets, are as shown here.

A quarter of a cycle later, the conditions are as shown in Fig. 209*b*; the flux through *P* will be zero, but as it is changing at a maximum rate the eddy currents flowing in the disc, due to its field, will be a maximum and distributed as shown. The flux through the current poles is now a maximum with polarity as shown, and the torque due to the current over both poles *I* and *I*′ is again from left to right. Another quarter of a cycle later, conditions are as shown in Fig. 209*c*, torque again being from left to right, as both current and polarity have reversed from the values they had in the first diagram. During the last quarter of the cycle, the conditions are as shown in Fig. 209*d*, torque again being from left to right.

If we consider a load of power factor less than unity, the average torque will still be in the same direction as that developed by resistive load (Fig. 209) but the average value of the torque will grow less and less with decreasing power factor. For the limiting condition (cos ϕ of load $= 0$), the average torque will be zero, as may be found by constructing a series of diagrams similar to those given in Fig. 209. Accurate analysis shows that the torque developed in the induction type of meter is directly proportional to the power factor of the load, at least for those power factors encountered in practice.

To overcome the static friction in the watt-hour meter a small disc of copper is arranged to slide over part of the face of the potential pole; it acts exactly like the shading coil of the frequency meter (Section 126). By shifting the shading plate, as it is called, the static friction of the meter is just neutralized. The speed of the motor is adjusted by shifting the magnets M-M', Fig. 208, toward or away from the center of the disc, away from the center to slow the disc down, and *vice versa*.

The average induction meter develops a somewhat greater ratio of torque to weight of the rotating element than does the Thomson commutator meter, and it is, therefore, more permanent in its adjustment. The induction meter has a torque of about 0.3 gram-centimeter per gram weight of the moving element. On the average, the potential circuit of the induction meter uses about 2 watts (at 110 volts), and in the current circuit the meter uses somewhat less than 1 watt when carrying rated current.

CHAPTER VI

THE ALTERNATING-CURRENT GENERATOR

132. General Construction.—The alternating-current generator, or *alternator*, as it is frequently called, does not differ in principle from the continuous-current generator; it consists of a field structure, magnetized by electromagnets, and an armature, in the winding of which the electromotive force is generated.

In three respects, however, the alternator generally differs from the continuous-current generator: it has no commutator; it is practically never self-exciting, and the field structure rotates, whereas, in continuous-current machines, the armature rotates. Small alternators sometimes have a stationary field and a rotating armature, but all large alternators are of the revolving-field type. The armature and the field of a revolving-field type alternator are frequently called the *stator* and the *rotor*, respectively.

133. Slip Rings instead of Commutator.—In case the armature rotates, the ends of its winding are connected to slip rings instead of to a commutator; the number of rings used depends upon the type of armature winding, but generally there are either two, three, or four, depending upon whether the winding is to generate single-phase, three-phase, or two-phase power. Brushes (generally made of blocks of some special metal—mixtures principally of graphite and copper) bear on these rings and serve to carry the current to the external circuit. For machines in which the field revolves, the continuous current for its excitation is led into the field winding by means of two suitable slip rings and brushes.

134. Alternator not Self-exciting.—The current for the field winding must be continuous, and as the armature of the alternator furnishes only alternating current it is evident that some additional source of power must be available for obtaining the field excitation.

135. Reasons for a Stationary Armature.—As the armature of an alternator is not equipped with a commutator, it is not at all necessary to have the armature rotate and, as a matter of fact it is nearly always stationary.

There are several reasons why the field of an alternator generally rotates instead of the armature. The armature winding ordinarily generates quite a high voltage, perhaps 2300 volts or even 13,000 volts. Such voltages require that the armature winding be very carefully insulated, and it is much easier to insulate a stationary winding than it is to insulate one designed to run at a high speed. In the latter case the winding has to be designed to stand properly the mechanical stresses due to centrifugal forces and, as a result, the insulation is likely to be inferior.

As a stationary armature requires no slip rings, the leads from the armature can be continuously insulated from the armature winding to the switchboard, leaving no bare conductor with the consequent danger of fatal shock. With a rotating armature, the slip rings required to carry the current from the armature to the external circuit are exposed. If the armature generates a high voltage, these rings are difficult to insulate and become a source of trouble due to short-circuits, etc. As the voltage applied to the field of an alternator seldom exceeds 250 volts, the slip rings required to carry the current to the field winding, as well as the entire field structure, are easy to insulate.

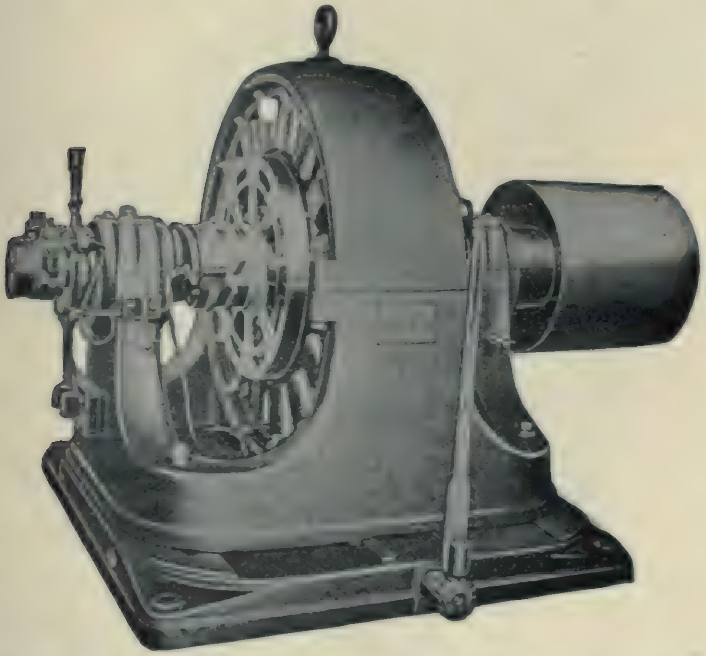


Fig. 210.—A small revolving-armature alternator; this type is practically never built to-day.

136. Field Structure.—The field structure of small alternators, having rotating armatures, does not differ materially from that of continuous-current generators, as may be seen from Fig. 210.

The rotating field of a slow-speed, engine-driven alternator is shown in Fig. 211. It consists of a number of laminated poles, keyed or bolted to a steel spider, which in turn is keyed to the shaft. A typical pole is shown in Fig. 212; the two projecting lugs serve to hold the field spools in place. The winding for such a pole piece is shown in Fig. 213; it will be seen to be of copper strap, wound edgewise, the turns being separated by paper slipped between them.

137. Special Field Structure for Turbo-alternators.—The steam turbine is used very often as the driver for alternators, and a turbine must necessarily run at high speed to use the steam economically.

With the high speeds used for turbo-alternators, it would be difficult to build a rotating field, with projecting poles, sufficiently strong to withstand the centrifugal forces. Moreover, projecting poles would cause excessive windage losses, and would make the operation of the alternator extremely noisy.

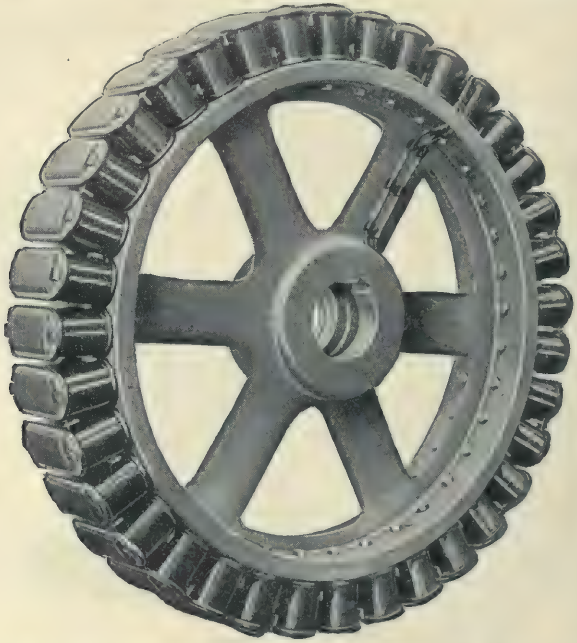


Fig. 211.—The field structure of a revolving-field, slow-speed alternator, for reciprocating-engine drive.

The field structure of a turbo-alternator is cylindrical in form, and is either a solid steel forging, or is built up of thick discs; slots are then cut in the periphery to accommodate the field winding. A view of the rotor of a turbo-alternator in process of being wound with copper-strap field coils is given in Fig. 214. The shaft does not generally pass through the core, but is in two pieces, fastened to end plates, which are in turn bolted to the core. To allow the shaft to pass through the core, there would have to be a large hole through the core, seriously lessening its mechanical strength. (Because of the high rotational speeds used in these large field structures, the centrifugal forces developed are very high.) A completed rotating field for a turbo-alternator is shown in Fig. 215; the metal covers at the ends of the core are to protect the ends of the field coils and hold them in place.

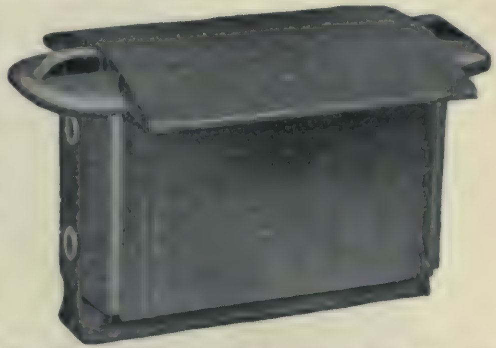


FIG. 212.—The poles of alternators are generally laminated and properly formed to key firmly into the spider on which they are carried.

138. Air Gap.—The length of the air gap in alternators and synchronous motors is fixed by the armature reaction, as in the case of continuous-current machines. In order that the effect of armature reaction may not be exaggerated, the air gap must be made considerably longer than is necessary for mechanical clearance; in turbo-alternators the air-gap length may be an inch or more.

As the revolving field of a turbo-alternator is a perfectly smooth structure, the air gap is of uniform length. In water-wheel-driven and other slow-speed alternators the pole pieces are *chamfered*, metal being removed from the pole tips to improve the wave-form. The air gap is then longer under the pole tips than under the middle of a pole.

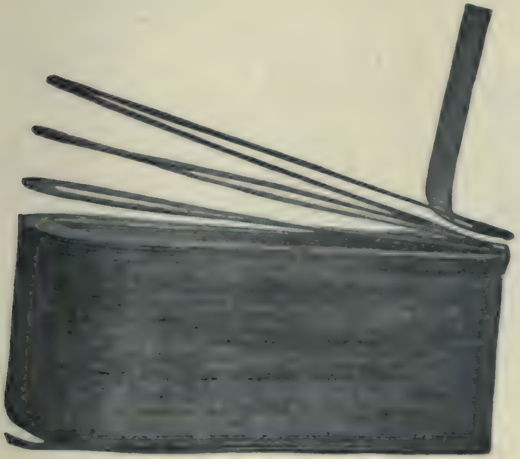


FIG. 213.—Generally the field poles are wound with copper strap; this makes for very efficient cooling, as each turn is directly in contact with the air.

139. Armature Core.—The armature cores of alternators and synchronous motors are built up of laminations, as are those of continuous-current machines, in order to reduce the losses due to eddy currents.

In Fig. 216 are shown sections of armature laminations, that in (*a*) being for a rotating armature, and that in (*b*) for a stationary armature.

It will be seen in (*a*) that as the slot becomes deeper, the width of a tooth becomes narrower; with a stationary armature the teeth become wider as the slot is deepened. This is an additional advantage in favor of stationary armatures, it being possible to put more copper on a stationary armature, by widening and deepening the slots, than on a rotating one.

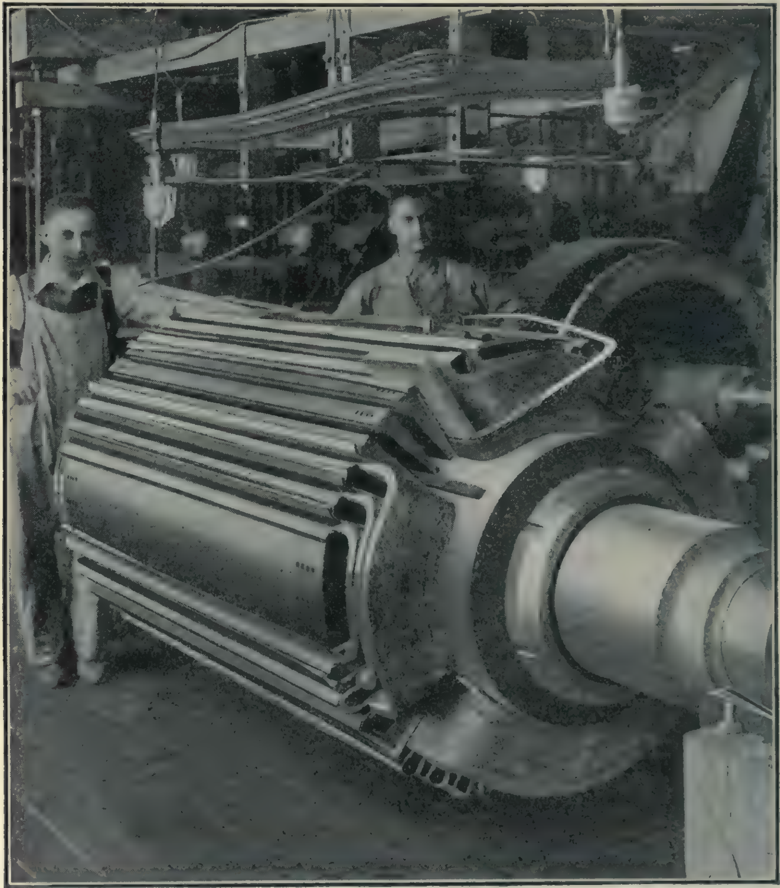


FIG. 214.—The rotor of a turbo-alternator being fitted with its strap-wound field coils. This is a four-pole rotor having a distributed winding of four coils per pole.

The armature of a slow-speed revolving-field type alternator is shown in Fig. 217; its field was shown in Fig. 211.

The form of armature laminations for a turbo-alternator may be seen in Fig. 218, which shows the armature core assembled.

140. Ventilation.—Small alternators with revolving armatures, and also slow-speed alternators with revolving fields, are ventilated in the same manner as continuous-current machines. With revolving armatures the air is drawn, by a fan, into the interior of the core, and is thrown out through air ducts in the core, the air ducts being formed by spacers introduced between laminations every two or three inches of core. In stationary armatures the same procedure is followed, the air being forced by the rotor (assisted at times by fans attached to the rotor) through the air ducts in the stator and out through openings in the frame.

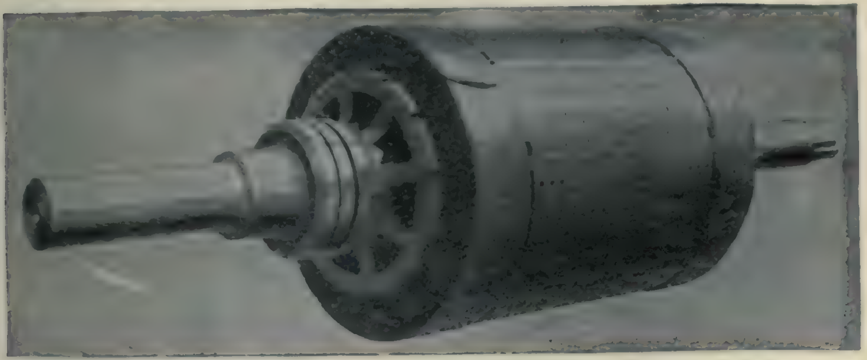


FIG. 215.—A completed field structure, wound for a turbo-alternator. The field windings are completely imbedded in slots, and there are no salient poles.

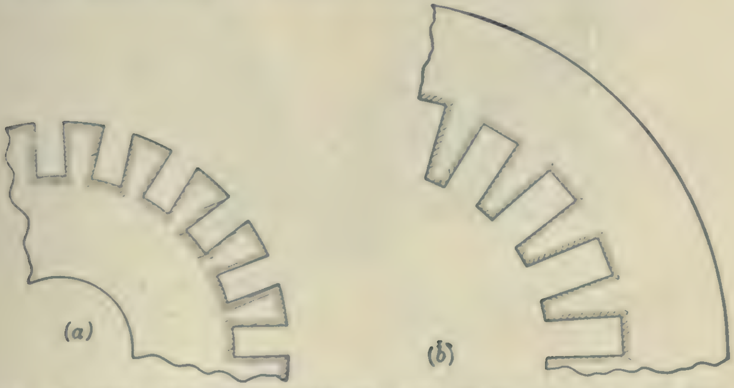


FIG. 216.—In case the armature revolves, the laminations are as shown in (*a*), whereas for machines in which the armature is stationary (the general case), the laminations are formed as indicated in (*b*).

Conditions are very different in the case of high-speed turbo-alternators, the output per unit volume of which is high, and in which the quantity of heat to be dissipated per unit area of cooling surface is large. Forced ventilation is therefore necessary with such machines, and there must be provided: (1) Sufficient air to carry off the heat generated with reasonable rise of temperature of both machine and air; (2) Ample duct capacity to prevent too high velocity of the air, so that the windage losses are not too high; (3) Proper spacing of the ducts, so that no part of the core is very far from a duct; (4) Precautions to prevent the air from carrying dirt and moisture into the machine.

There are two methods of ventilation used with turbo-alternators, the "radial" and the "axial." In the radial method, air enters the air gap at both ends of the machine and passes out radially through air ducts between laminations of the armature, as indicated in Fig. 219. In the axial method the core contains many ducts or holes parallel to the shaft. Air may enter from one end of the core, and after passing through the ducts, leave at the other end; or, the air may enter at both ends of the core and pass out through large radial ducts between laminations at the middle of the core, as is indicated in Fig. 220. With axial ventilation there is also a flow of air along the air gap, the air passing out through the large radial ducts at the middle of the core. Generally both radial

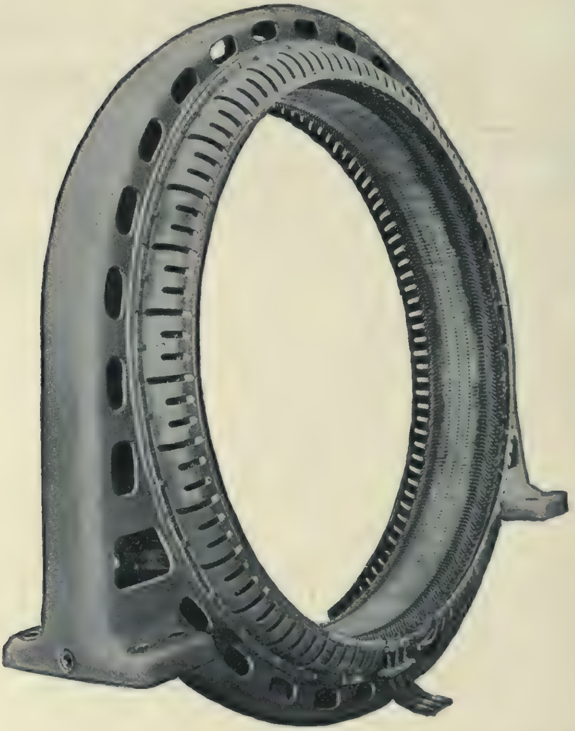


Fig. 217.—Completed armature for a slow-speed revolving-field alternator. Its field structure was shown in Fig. 211.

and axial ventilation are employed and the air is introduced at a definite entrance and expelled at a definite exit.

The rotor is cooled by drawing in air through holes at the ends of the core, and discharging it into the air gap through radial ducts, the air then passing through the armature.

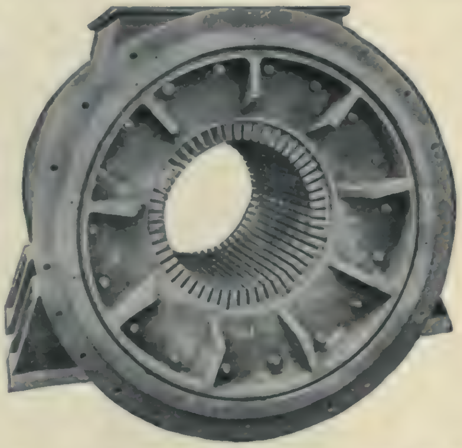


Fig. 218.—Assembled armature core for a high-speed turbo-alternator.

141. Typical Forms of Alternators.—The actual shape and appearance of an alternator depends largely upon what type of driver it is to be equipped with. In the turbo-alternator nothing apparently electrical can be seen; the machine is completely enclosed and its casing much resembles that of the turbine to which it is attached. Such a machine is shown in Fig. 221.

For reciprocating drive, the field structure must have many poles to permit the generation of commercial frequencies; the poles and armature windings are generally visible and the whole machine is of large diameter compared to the turbo-alternator, as may be seen by comparing Fig. 222, an engine-driven alternator, with that given in Fig. 221.

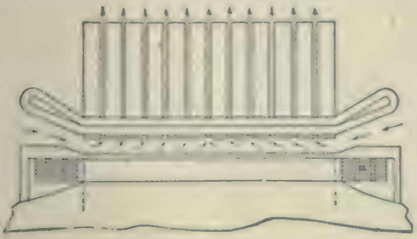


Fig. 219.

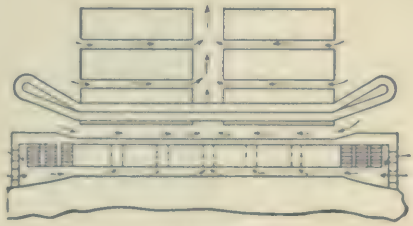


Fig. 220.

Fig. 219.—In the radial method of ventilation the air enters the air gaps at both ends and leaves through the slots in the laminated armature core.

Fig. 220.—More effective ventilation requires that axial slots be used as well as radial slots. The axial slots are made by holes in the punchings used to build up the laminated armature core.

For water-wheel drive the generators are generally built with a vertical shaft; the earlier machines were generally slow speed, but more recently the speed of water-wheel driven alternators has increased. A typical alternator intended for water-turbine drive is shown in Fig. 223.

142. Frequency.—Alternators have been designed for many different frequencies, but the standardization of electrical apparatus has practically eliminated all but two, so that all modern machines are designed for either 60 cycles or 25 cycles per second, the former used to supply lighting loads, and the latter used for power loads. In a few installations in which the lighting and power loads are of about equal magnitude, frequencies of

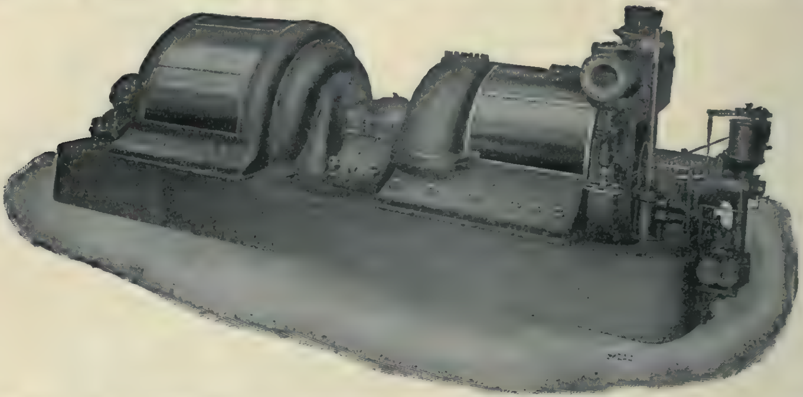


Fig. 221.—A turbine-driven alternator is generally completely enclosed, to accomplish more effective ventilation.

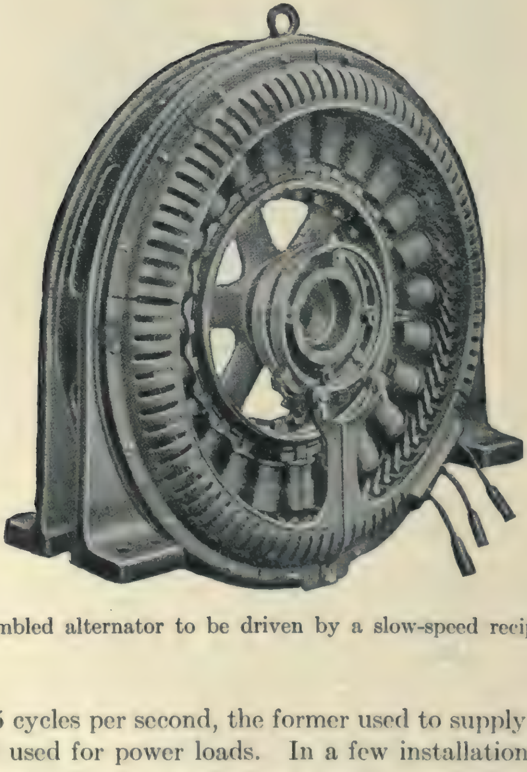


Fig. 222.—Assembled alternator to be driven by a slow-speed reciprocating engine.

40 and 50 cycles are used. Earlier machines were built to give 133 cycles per second and there has been much talk of machines for 15 cycles, but the first two named above are the only standard frequencies used in American practice. In Europe other frequencies are much used.

The frequency of the e.m.f. which an alternator generates depends upon its *speed and upon the number of poles.* A complete cycle is generated in any conductor on the armature when a pair of poles has moved past it, *hence to find the frequency of an alternator it is only necessary to calculate the number of pairs of poles passing any armature conductor in one second*, i.e., frequency = r.p.s. × number of pairs of poles. Take, for example, a four-pole machine turning 1800 r.p.m.; there are two pairs of poles, so that every time the field makes a complete revolution there are two cycles generated in any armature conductor. Now this machine is turning 30 r.p.s. and hence the frequency is equal to $30 \times 2 = 60$ cycles per second. Suppose a ten-pole machine turns 300 r.p.m.; it generates 5 cycles per revolution and makes 5 r.p.s.; therefore it generates a frequency of $5 \times 5 = 25$ cycles per second. A certain turbo-alternator has two poles and turns 1500 r.p.m.; it therefore generates 25 cycles per second.

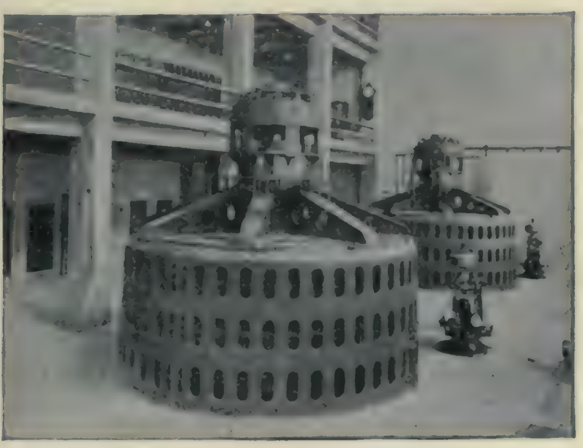


Fig. 223.—Modern water-wheel generators with vertical shafts; the turbines are below the station proper in the wheel-pit. The direct-connected, continuous-current exciters may be seen at the upper ends of the shafts. These alternators have a rating of 12,500 kv.a., 11,500 volts, 180 r.p.m., the wheels being designed for a head of 130 feet.

143. Excitation.—Attempts have been made to build self-exciting alternators, but they have been failures as evidenced by the fact that there are practically none in service, except an occasional machine of very small capacity.

All alternating-current stations have two or more small, self-exciting, shunt-wound (or compound-wound), continuous-current machines whose sole purpose is to furnish the field current for the alternators. As the amount of power used in the field coils of an alternator is generally less than 5 per cent of its capacity, the exciters may be comparatively small machines. An alternator having a capacity of 1000 kw., for example, would require an exciter of about 25 kw. capacity.

144. Arrangement of Exciters in a Station.—Sometimes each alternator is furnished with its own exciter (either belted or direct-connected, the exciter in the latter case often built into the core of the alternator), but in large stations there is generally a set of exciter bus-bars on the switchboard, and two or more exciters (each equipped with its own driver) furnish power to these buses. The field circuits of all alternators are connected to this set of exciter bus-bars. The exciters are sometimes driven by alternating-current motors and at other times by small steam turbines or engines. In all stations at least one of the exciters is driven by a steam turbine or engine, or by a separate water turbine, to insure excitation for the alternators if the entire station has shut down because of an accident. If the exciters are driven by motors obtaining their power from the alternators, their motors cannot be started until the alternators are excited.

The field winding of an alternator is generally designed for 125 or 250 volts and these voltages are, of course, independent of the voltage generated in the armature winding, which may be several thousand volts.

145. Armature Winding.—In general, the armature winding of an alternator resembles that of a continuous-current machine but is much simpler; it has comparatively few coils and all of them are generally connected in series. It was seen in Vol. I that the winding of a c.c. armature might be rather complex, using fractional pitch and being either of the wave or lap type. But not so many complexities are encountered in alternator armatures; in the smaller sizes of machines there are generally not more than six coils per pair of poles and seldom more than twelve, and the arrangement and connections of these coils are very simple. In the continuous-current armature it is necessary to use many coils of few turns each in order that the commutation may take place without sparking, but the alternator has no commutator and therefore this limitation on its winding does not exist.

Sometimes all the coils on an armature are connected in series with one another, so that there is only one set of coils on the armature and only two ends to the armature winding; this is said to be a *single-phase* winding and the machine is called a *single-phase generator*. Sometimes the coils of the armature are divided in a certain way, to form two equal independent groups, and this is called a *two-phase winding*. More often the coils are divided into three equal groups and form a *three-phase winding*. Probably 95 per cent of the alternators in use to-day are three-phase machines.

146. Different Types of Winding.—The development of these three styles of winding and the meaning of the names will now be taken up. For simplicity we shall consider alternators with revolving armatures, although what we shall say applies equally well to the armature windings of alternators with revolving fields.

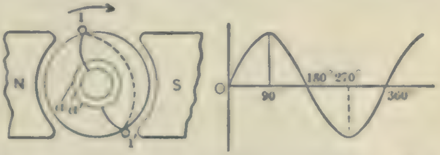


FIG. 224.—A simple rotating-armature alternator having one coil only; supposedly it generates a sine wave of voltage.

Consider an elementary generator, having only one coil and two slip rings, as shown in Fig. 224. The e.m.f. wave which such an armature would be supposed to generate is also shown in the figure; it is a simple sine wave.

Now suppose another coil is wound on the same armature, insulated from the first coil and connected to two more slip rings. This armature is shown in Fig. 225; the four rings are shown one outside the other,

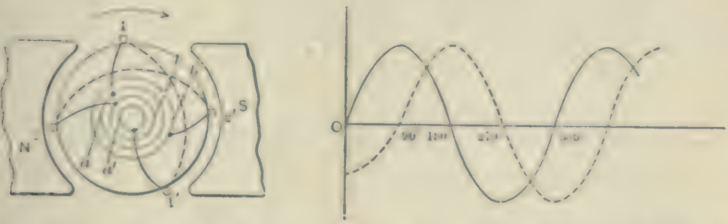


FIG. 225.—A two-coil armature, with four slip rings, furnishes two voltages differing in phase by an amount depending upon the relative placing of the coils on the armature.

whereas really they would be all of the same size and placed beside each other on the armature shaft.

The e.m.f. generated by coil 2–2′ will evidently be of the same shape and magnitude as that generated by coil 1–1′, but its maximum value will occur 90 electrical degrees later in time than the maximum value of that generated in coil 1–1′. The e.m.f. waves of the two coils are shown in Fig. 225, that of coil 1–1′ being given by the full-line curve and that of coil 2–2′ by the dotted curve.

This armature would, therefore, feed power to two separate circuits, one connected to the brushes *a–a′* and the other to the brushes *b–b′*.

The e.m.fs. on these two circuits would be equal in magnitude, but the phase of one would be 90° behind that of the other. The generator is said to be a *two-phase generator*, and the two circuits to which it supplies two-phase power are said to constitute a *two-phase system.*

Now instead of placing two coils on the armature, 90° apart, we might place on it three coils 120° apart, as shown in Fig. 226. The three e.m.f. waves generated by such a winding would all be of the same magnitude but would differ in phase from each other by 120°; this winding is called a *three-phase winding*, and the system of wires to which such an armature supplies power is called a *three-phase system.*

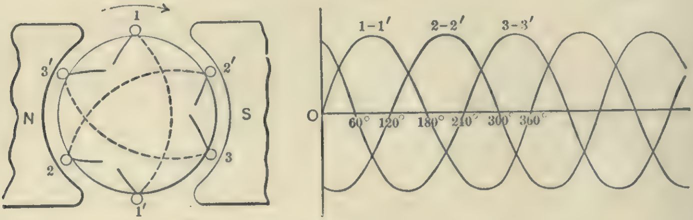
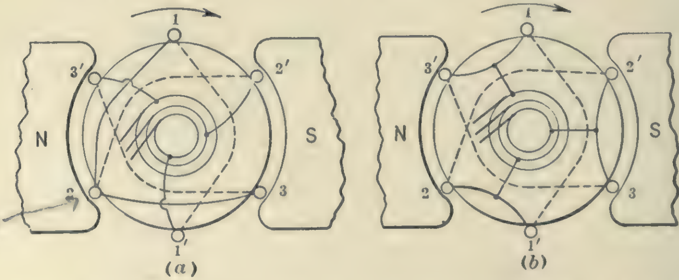


Fig. 226.—An armature having three coils suitably placed, gives three equal voltages 120° apart; it is a three-phase armature.

We have already seen (pp. 69–79) that the six ends of the three phases may be inter-connected so that only three leads need be brought out from the winding, and therefore only three slip rings are required.



(*a*) (*b*)

Neutral

Fig. 227.—An elementary three-phase armature, connected Y in (*a*) and Δ in (*b*).

There are two ways of making this inter-connection of the coils; sometimes one is used and sometimes the other, but in either case only three rings are used on the armature and only three wires are used in the outside circuit. Figures 227 (*a*) and (*b*) show the two methods of connecting a three-phase armature. In (*a*) all of the beginnings of the coils are connected together and the ends of the coils are connected to the three slip rings. The junction of the beginnings of the coils is not connected to any ring. An armature so connected is said to be *Y-connected* because a schematic diagram of the winding resembles the capital letter Y.

The other method of connecting the coils is called the *delta connection*, so named because its schematic diagram resembles the Greek letter Δ. In this connection the end of coil 1 is connected to the beginning of coil 2, the end of coil 2 connected to the beginning of coil 3, and the end of coil 3 connected to the beginning of coil 1. Then the three junction points are connected to the three slip rings, as shown in diagram (*b*), Fig. 227. The Δ-connection is not used as frequently as the Y-connection, about 90 per cent of the alternators in use being Y-connected.

147. Capacity of a Three-phase Armature Winding.—We have seen that the three-phase system is peculiar in that *the voltage measured between any pair of wires is the same*, and this is true no matter whether the generator armature is Δ- or Y-connected. Having a given three-phase winding on an armature, the output capacity will be just the same whether the Δ- or Y-connection is employed, but the Y-connected machine will give a higher voltage between lines than the Δ-connected machine. The possible current output of the latter machine is greater than that of the Y-connected machine in the same ratio as its voltage is smaller, so that the two outputs are equal.

If the safe current per phase is I and the voltage generated per phase is E, then the voltage between the terminals of the Y-connected armature is $E\sqrt{3}$ and the safe current per terminal is I; if Δ-connected, the voltage between terminals is E and the safe current per terminal is $I\sqrt{3}$. In either case the kv.a. rating of the machine is $EI\sqrt{3}$. The factor $\sqrt{3}$ appears in both cases because of the subtraction of two equal vectors 120° apart. These relations were discussed in Section 39.

148. Open-circuit and Closed-circuit Armature Windings.—All a.c. armature windings may be divided into two general groups:

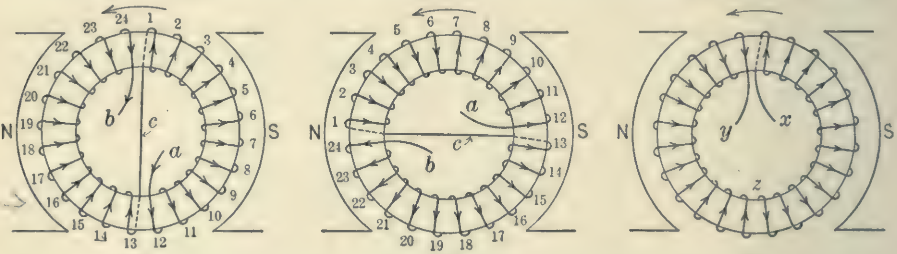
1. Open-circuit windings.
2. Closed-circuit windings.

An open-circuit winding is one that is not closed upon itself, the open ends forming the terminals of the winding. A closed-circuit winding is one in which all the inductors are placed in series, forming by themselves a closed circuit.

We have seen that all c.c. armature windings are closed-circuit, and that in such windings there are at least two paths in parallel. The windings for alternators may be either open- or closed-circuit, but as the open-circuit type of winding is generally more suitable, it is almost universally used.

149. Single-phase Open-circuit Winding.—In Fig. 228*a*, a single-phase, open-circuit winding is shown, a ring winding having been chosen, and the slip rings omitted, for the sake of simplicity in the diagram. It will be seen that the inductors under the south pole generate voltages in a direction from *a* to *c*, while those under the north pole generate voltages from *c* to *b*. The connection *c* thus places the two halves of the armature winding in series in such a way that their respective voltages are added together. In Fig. 228*b*, the same armature has rotated 90° further ahead; it will be seen that the group of inductors between *a* and *c* lies half under the north pole and half under the south pole, so that the net voltage in the group is zero at that particular instant. Similarly, the instantaneous voltage in the group between *c* and *b* is also zero.

When the armature is in the position, with respect to the poles, shown in Fig. 228*a*, it is generating the maximum instantaneous voltage of which it is capable; in any other position the instantaneous voltage generated by some of the inductors in a group (between *a* and *c*, or between *b* and *c*) will be in opposition to that generated by the remaining inductors in the same group. When the armature is in the position shown in Fig. 228*b*, the instantaneous voltage of the armature is zero, but practically all of the individual inductors are generating voltage.



(*a*) (*b*) (*c*)

Fig. 228.—A single-phase, single-circuit winding requires that the two halves be properly connected in series, as shown in (*a*) If connected as in (*c*) there will be no voltage across the terminals of the winding.

In Fig. 228*c* is shown another way in which the inductors used before might be connected. Consideration of this winding will disclose the fact that, although the individual inductors are all generating voltage, the resultant voltage of the armature winding will be zero for any position of the armature with respect to the field. In the position shown, the voltages generated by the inductors between *x* and *z* are in opposition to those generated by the inductors lying between *z* and *y*. The winding is therefore useless, but it is given to illustrate the fact that, in connecting the inductors in a given winding, care must be taken that the voltages of the various inductors, which are to be placed in series, do not nearly neutralize one another, that is, are not always in opposition. To obtain instantaneous values of voltage other than the maximum, there must be some such opposition of voltages in a given group; but when the armature is generating its maximum instantaneous value, the voltages generated in the given group must all be additive.

150. Single-phase Closed-circuit Winding.—In Fig. 229 a closed-circuit, single-phase winding with the same number of inductors, as used in Fig. 228, is shown; it is seen to be identical with that for a c.c. generator. In part *a* of the figure, each half of the winding is generating maximum voltage; in part *b*, the voltage of the machine is zero. As the two halves of the winding are in parallel, the voltage of the machine is that of either half. When this winding is compared with Fig. 228*a*, it will be seen that, when the winding is connected as a closed-circuit winding, the voltage which it is capable of generating is one-half of what it will generate as an open-circuit winding; because of the two paths in parallel the current capacity as a closed-circuit winding is, however, double that as an open-circuit winding, leaving the kw. capacity the same.

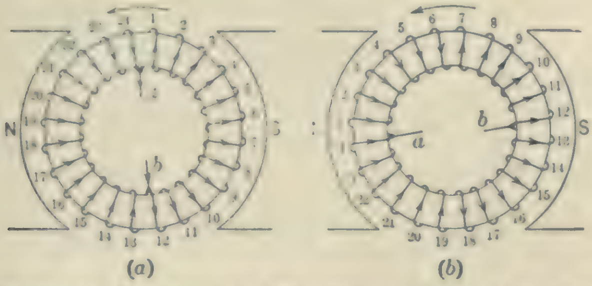


FIG. 229.—Another way of connecting a the armature of Fig. 228 to deliver single-phase power; in this scheme the voltage will be half as much as that of Fig. 228, but the current capacity will be doubled, as there are two paths in parallel.

This is in reality a 2 ϕ winding

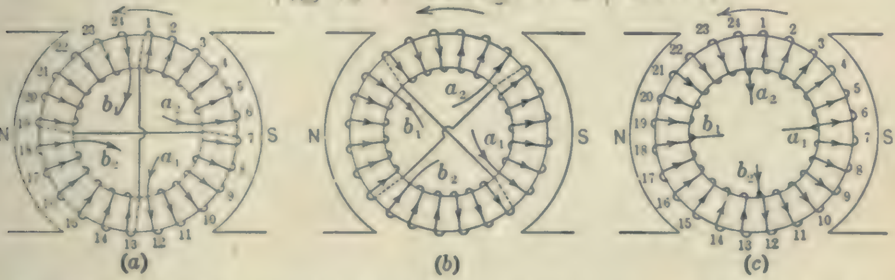
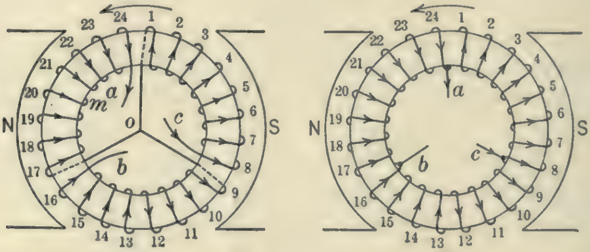


FIG. 230.—A winding connected to deliver two-phase power. In (*a*) and (*b*) the two phases are separate and distinct, whereas in (*c*) the two phases are interconnected, this generally being called a mesh winding.

151. Two-phase Open- and Closed-circuit Windings.—An open-circuit, two-phase winding is represented in Fig. 230*a*; the entire winding is divided into four groups. Two of the groups lying 180° apart are connected in series to form one phase of the winding, and the two remaining groups form the second phase. It will be seen that, for the position shown in this diagram, the instantaneous voltages of the two phases are equal, but that the voltage of phase a_2b_2 is decreasing while that of phase a_1b_1 is increasing; the voltage of phase a_2b_2 is at its 135° value, and that of phase a_1b_1 at its 45° value. In Fig. 230*b*, the armature has moved 45° further ahead; the voltage of phase a_2b_2 is now zero, while that of phase a_1b_1 is at its maximum value.

In Fig. 230*c*, the same inductors, arranged as a closed-circuit winding, are shown. In the position indicated, the winding is generating maximum voltage between terminals a_2 and b_2, while the voltage between terminals a_1 and b_1 is zero. When the armature moves 90° further ahead, the voltage between a_1 and b_1 will be a maximum, and the voltage between terminals a_2 and b_2 will be zero.



(*a*) (*b*)

FIG. 231.—A winding connected for delivering three-phase power. In (*a*) the Y-connection is used and in (*b*) the Δ-connection is used.

Half coil

152. Three-phase Open- and Closed-circuit Windings.—In Fig. 231 the same inductors used before are connected to generate three-phase voltages, being arranged as an open-circuit winding in part *a*, and as a closed-circuit winding in part *b*.

Any phase generates its maximum voltage when the middle of that group of coils passes the center of a pole, i.e., when the point *m*, for example, lies in the horizontal. Thus in Fig. 231*a* phase *a* is generating its 60° value of voltage, phase *b* its 180° value (i.e., its instantaneous voltage is zero), and phase *c* is generating its 300° value of voltage. In Fig. 231*b*, the voltages *b* to *a*, *c* to *b*, and *a* to *c* have values corresponding to those of phases *a*, *b* and *c* of Fig. 231*a*.

It will be seen that the open-circuit, three-phase winding is what we have previously considered as a Y-connected winding, while the closed-circuit winding is what we previously called a Δ-connected winding.

153. Resultant Voltages of Various Groupings of Armature Inductors.—In each of the winding schemes shown in Figs. 228 to 231, there were

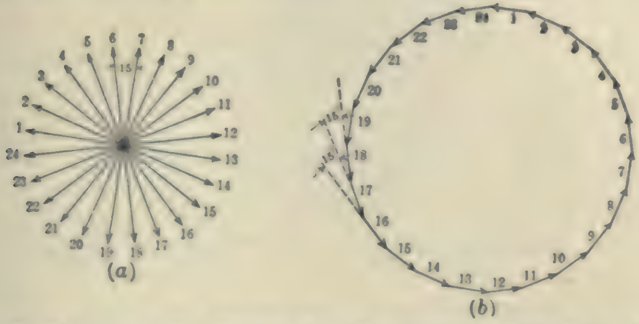


FIG. 232.—On the assumption that each coil generates a sine wave of e.m.f., the voltage of a 24-coil winding (2-pole machine) may be represented by either of the methods used here.

twenty-four inductors, equal distances apart, upon the armature surface. A bipolar field having been chosen, one revolution of the armature completes one cycle in every inductor. The twenty-four inductors being separated from one another by one twenty-fourth of the periphery of the armature, it follows that the e.m.f. generated in each successive inductor lags 15 electrical degrees behind the e.m.f. in the adjacent inductor. We may represent this fact by a radial vector diagram, as in Fig. 232*a*, or by a topographical vector diagram, as in Fig. 232*b*.

When the inductors are arranged as in Fig. 228*a*, one-half of the inductors (12 to 1) are placed in series to form one group; the remaining inductors (13 to 24) form a second group. The voltage generated by either group will then be equal to the vector sum of 12 e.m.fs., each differing in phase from the next by 15°; evidently the resultant voltages of the two groups will be in phase. Plotting the separate inductor e.m.fs. as a topographic diagram, this vector sum is readily found, as is shown in Fig. 233, by the vector E_{12-1}.

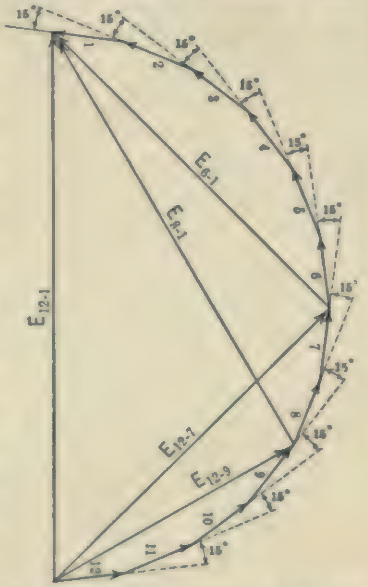


FIG. 233.—If the winding of Fig. 232 is tapped for a two-phase connection, the voltage between taps may be obtained by the construction shown here.

The value of the resultant voltage E_{12-1} may be calculated by adding the vertical projections of the vectors of the various inductors. Thus, if each inductor is considered as generating one volt (effective), the value of the resultant voltage (effective) of group 12–1 will be

$$
\begin{aligned}
E_{12-1} = {} & \sin 7.5° + \sin 22.5° + \sin 37.5° + \sin 52.5° \\
& + \ldots \sin 157.5° + \sin 172.5°
\end{aligned}
$$

which by combining similar terms becomes

$$
\begin{aligned}
E_{12-1} = {} & 2(0.1305 + 0.3827 + 0.6088 + 0.7934 \\
& + 0.9239 + 0.9914) = 7.6614 \text{ volts.}
\end{aligned}
$$

Since there are two groups of inductors in series, the voltage of the machine with its winding arranged open-circuit will be $2 \times 7.6614 = 15.3228$ volts. The closed-circuit winding shown in Fig. 229 will generate a voltage of 7.6614 volts, but has double the current-carrying capacity of the open-circuit winding.

The above calculation indicates that inductors 1 and 12 are contributing very little towards the voltage of the machine; if they were omitted the voltage of a group would drop from 7.6614 to 7.4004 volts, a change of 3.4 per cent; the resistance of a group would decrease in the ratio of 12 to 10, or 16.7 per cent. If inductors 2 and 11 were also left off, the voltage would drop to 6.6350 volts, a decrease of only 13.4 per cent; the resistance of the group would drop 33.3 per cent. Of the total voltage generated by twelve inductors in series, it will be seen that inductors 5, 6, 7, and 8 generate by themselves 3.8306 volts and that, in the winding scheme shown, the addition of eight more inductors (that is, increasing the number of inductors by 200 per cent) results in an increase in voltage of the group of only 100 per cent.

In the two-phase, open-circuit arrangement of Fig. 230*a*, each phase consists of two groups of six inductors in series. With the same voltage generated per inductor, as given in Fig. 232, the voltage of the group 6–1 may be obtained from the vector diagram of Fig. 233 by drawing a chord from the tail of vector 6 to the head of vector 1. This is the voltage E_{6-1}; the voltage E_{12-7} is similarly obtained.

The numerical value of the voltage E_{6-1} is obtained by adding the vectors 6, 5, 4, 3, 2, 1, which gives the chord 6–1. Thus

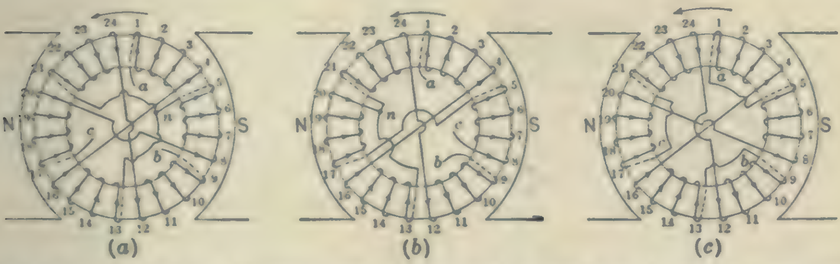
$$E_{6-1} = 2(0.7934 + 0.9239 + 0.9914) = 5.4154 \text{ volts}$$

and since the voltages of groups 6–1 and 13–18 are in phase, the voltage per phase of the machine when connected as in Fig. 230*a* is 10.8308 volts.

The two-phase, closed-circuit winding of Fig. 230*c* gives the same voltage per phase (i.e., across terminals a_1b_1 and a_2b_2), as the single-phase, closed-circuit arrangement, or 7.6614 volts. This is less than the two-phase, open-circuit arrangement yielded, but we shall see later that the current capacity of the closed-circuit winding is greater than that of the open-circuit winding, so that the kw. capacity of both windings is the same.

In both of the three-phase windings of Fig. 231 there are eight inductors in series per phase. Thus the voltage of phase *a* is obtained by joining the tail of vector 8 to the head of vector 1; this gives the voltage E_{8-1}. We have already calculated the voltage generated by a group of eight inductors in series and found it to be 6.6350 volts. We find then that the open-circuit or Y-connected winding of Fig. 231*a*, gives a terminal voltage of $\sqrt{3} \times 6.6350 = 11.4785$ volts; the closed-circuit, or Δ-connected winding, yields 6.6350 volts between terminals.

In Fig. 234 two other arrangements of the twenty-four inductors, to yield three-phase voltages, are shown, those in parts *a* and *b* being open-circuit, or Y-connected windings, and that in part *c*, a closed-circuit, or Δ-connected winding. In each of these arrangements the entire winding is divided into six groups of four inductors; each phase consists of two of these groups properly connected in series, the groups selected being 180° apart. Thus in Fig. 234*a*, starting at end *a*, we pick up inductors 1, 2, 3, and 4, and then pass directly across the armature and pick up inductors 16, 15, 14, and 13, the last inductor being connected to the neutral. The resultant voltage generated by a group of four inductors in series is obtained as was done for the voltage E_{12-1}, on page 228; this gives us



Whl
coil

FIG. 234.—In the actual three-phase winding each phase is made up by connecting two suitably selected groups in series, instead of using only one group as in Fig. 231. This change in coil connection appreciably increases the capacity of the winding.

$$E_{12-9} = 2(0.9239 + 0.9914) = 3.8307 \text{ volts.}$$

Since the total voltage of phase *b* is made up of group voltages E_{12-9} and E_{24-21}, and as these group voltages are in phase, the total voltage of phase *b* is equal to $2 \times 3.8307 = 7.6614$ volts. The voltage between terminals of the winding is $\sqrt{3} \times 7.6614 = 13.2542$ volts.

Comparing this winding with that of Fig. 231*a*, we find that by simply rearranging the winding we have increased the voltage of the armature from 11.4785 volts to 13.2542 volts, a gain of $\frac{1.7757}{11.4785} = 15.5$ per cent; and this with no reduction in the current-carrying capacity of the machine. Thus a little knowledge of phase relations has indicated the best way to connect the inductors.

In Fig. 234*b*, the same winding scheme is used as in Fig. 234*a*, with the exception of a change in the connections of phase *c*. We have seen that if the connections of a coil are reversed, the e.m.f. generated by that coil is also reversed, or moved through 180°. In Fig. 234*a*, phase *c* starts with inductor 17, with inductors in series in the order 17, 18, 19, 20, 8, 7, 6, 5, the last inductor being connected to neutral. In Fig. 234*b*, both the terminals and the order of connection of the inductors is reversed. We have now connected inductor 20 to the neutral and pick up in order inductors 20, 19, 18, 17, 5, 6, 7, 8, the last becoming now the terminal of the phase. This manner of picking up phase *c* in reverse order leaves its voltage in the same phase relationship with the other phases, as it had before. The gain in using the scheme of Fig. 234*b* is that the end connections of the phases are shortened; the terminals *a*, *b*, *c*, are closer together, as are also the three neutral ends.

In Fig. 234*c*, the closed-circuit, or Δ-connected winding corresponding to the open-circuit, or Y-connected winding of Fig. 234*a*, is indicated. Compared with the winding shown in Fig. 231*b*, the same gain of 15.5 per cent in voltage is obtained by rearrangement of the inductors. The winding might be further arranged by reversing the terminals of phase *c* and the order of connection of its inductors; this would, however, result in little gain in so far as the length of the phase connections is concerned.

154. Relative Power Capacity of Various Groupings of Armature Inductors.—If we now assume that each of the twenty-four inductors of the windings just discussed is capable of carrying safely a current of 5 amperes, then the power capacity of the entire single-phase, open-circuit winding of Fig. 228*a*, is $5 \times 15.3228 = 76.6$ watts, provided the load has unity power factor.

The total current-carrying capacity of the single-phase, closed-circuit winding is 10 amperes, since each of the two parallel paths can carry 5 amperes; the power capacity of the winding is therefore $10 \times 7.6614 = 76.6$ watts. Both single-phase groupings therefore have the same power capacity.

The two-phase, open-circuit winding has a power capacity of $5 \times 10.8308 = 54.15$ watts per phase, or a total power capacity of 108.3 watts, with a balanced loading.

To determine the capacity of the two-phase, closed-circuit winding, consider that a current of I_1 amperes is supplied from terminals a_1b_1, and an equal current of I_2 amperes is supplied from terminals a_2b_2, as in Fig. 235. Each of these phase currents will divide equally between the two parallel paths of the armature, so that each inductor carries a current of $\frac{I_1}{2}$ from phase No. 1 and an equal current of $\frac{I_2}{2}$ amperes from phase No. 2. These currents being 90° apart, the current per inductor is

$$I_c=\sqrt{\left(\frac{I_1}{2}\right)^2+\left(\frac{I_2}{2}\right)^2}=\frac{1}{2}\sqrt{I_1^2+I_2^2}=\frac{I}{2}\sqrt{2}=0.707I.$$

Since each inductor is capable of carrying 5 amperes, we may draw a current of $\frac{5}{0.707}=7.07$ amperes from the terminals of each phase. The capacity of the winding is then $7.07\times 7.6614=54.15$ watts per phase, or a total capacity of 108.3 watts with a balanced load.

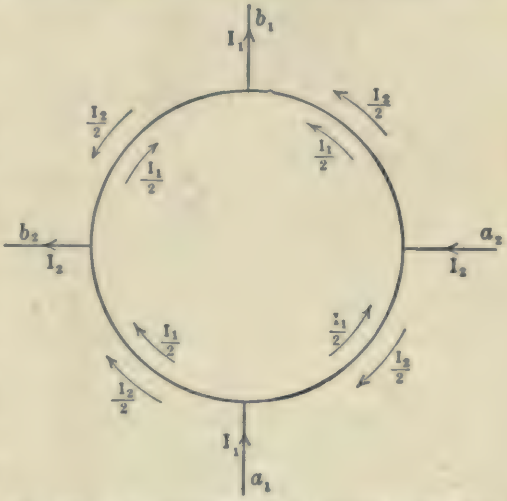


FIG. 235.—In determining the current capacity of a two-phase closed-circuit winding, the currents supplied to the different loads must be suitably combined to get the resultant current in the various coils

The three-phase, open-circuit, or Y-connected winding of Fig. 231*a*, using one group of eight inductors in series per phase, has a power capacity of $5\times 11.4785\times\sqrt{3}=99.4$ watts. The closed-circuit, or Δ-connected winding of Fig. 231*b*, can carry 5 amperes per phase; we may then draw a current of $5\times\sqrt{3}=8.66$ amperes from each terminal, so that the power capacity of the winding is $8.66\times 6.6350\times\sqrt{3}=99.4$ watts.

In the three-phase windings of Fig. 234*a* and *b*, we have two groups, each with four inductors, in series. The current-carrying capacities of the windings are the same as those of the three-phase windings of Fig. 231*a* and *b*, respectively, so that the power capacities are $5\times 13.2542\times\sqrt{3}=114.8$ watts for the Y-connected winding, and $8.66\times 7.6614\times\sqrt{3}=114.8$ watts for the Δ-connected winding.

In Table III a summary of the voltage, current, and power capacities of the different groupings is given, as well as the per cent capacity of each grouping in terms of the single-phase capacity. It will be seen that the proper two-phase and three-phase groupings have capacities 41 and 50 per cent greater than the single-phase, respectively, indicating clearly one reason why most a.c. machinery is three-phase rather than two-phase or single-phase.

TABLE III

Grouping of Winding	Figure Number	Terminal Voltage	Current per Terminal in Amperes	Capacity in Watts	Per Cent of Single-phase Capacity
Single-phase, open-circuit..	228*a*	15.3228	5	76.6	100
Single-phase, closed-circuit.	229*a*	7.6614	10	76.6	100
Two-phase, open-circuit	230*a*	10.8308	5	108.3	141
Two-phase, closed-circuit..	230*c*	7.6614	7.07	108.3	141
Three-phase, Y, or open-circuit (1 group per phase)................	231*a*	11.4785	5	99.4	130
Three-phase, Δ, or closed-circuit (1 group per phase)................	231*b*	6.6350	8.66	99.4	130
Three-phase, Y, or open-circuit (2 groups per phase)................	234*a*	13.2542	5	114.8	150
Three-phase, Δ, or closed-circuit (2 groups per phase)................	234*c*	7.6614	8.66	114.8	150

155. Forms of Armature Windings.—While there are many forms of armature windings which may be used in alternators, there are practically only two that are in general use in the United States. These are the *lap* winding and the *chain* and *basket* windings, the basket being a further development of the chain winding.

156. Lap Winding.—The lap winding is the most common type of armature winding for alternators, synchronous motors, and the stators of induction motors. It is similar in appearance to the lap, or multiple-circuit winding, of c.c. machines; in fact, any c.c. lap winding may be utilized as the armature winding of an alternator, either by tapping it at suitable places, with connections to slip rings, as is done in the synchronous converter, or by suitable rearrangement of the end-connections of the coils.

In the lap winding there are generally as many coils as there are slots; there will then be two coil-sides per slot, one above the other, making the winding a *two-layer* winding.

In Figs. 236, 237, and 238, developed lap windings, for generating single-, two-, and three-phase voltages, are shown, the armature having twenty-four slots, and the field four poles, in each case; there are therefore six slots per pole. In the single-phase winding only four of the six slots per pole are used; to have used all six slots would have increased the voltage which the winding could generate by only a few per cent, while the amount of copper used, and the armature resistance, would have increased 50 per cent. In the two- and three-phase windings all of the slots are utilized. Full-pitch coils are used in all three windings.

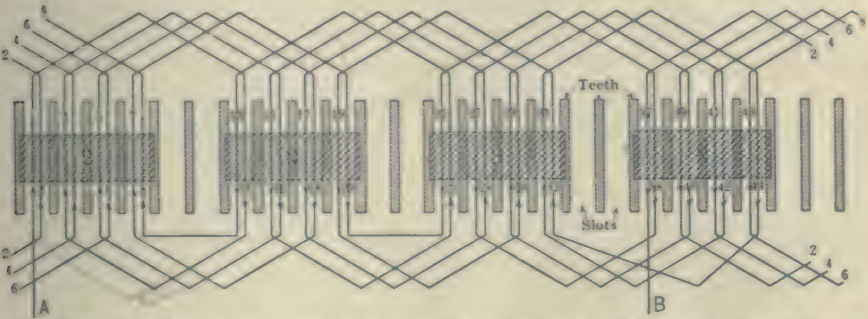


FIG. 236.—A developed single-phase winding using only two-thirds of the slots of the armature.

In the single-phase winding of Fig. 236, beginning at terminal *A*, we pick up in order inductors 1, 14, 3, 16, 5, 18, 7, 20, and then pass with a short end-connection to inductor 25, picking up inductors 25, 38, 27, 40, 29, 42, 31, 44. From inductor 44 we move back with a longer end-connection to inductor 32, picking up inductors 32, 19, 30, 17, 28, 15, 26, 13, and then pass to inductor 8 over a short end-connection and pick up inductors 8, 43, 6, 41, 4, 39, 2, 37; from inductor 37 we bring out terminal *B*. It will be seen that all inductors are properly connected in series, as may be seen by following the arrows, the winding having been drawn in

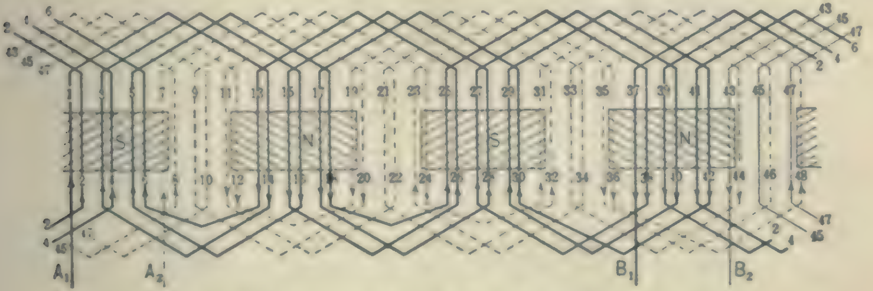


FIG. 237.—A developed lap winding, for two-phase power.

such relative position, with respect to the poles, that the winding is generating a maximum instantaneous voltage.

It is to be pointed out that after picking up inductor 44 we had a choice of passing either to inductor 32, or to inductor 8; the latter connection, however, would have required an end-connection with a slot-pitch of 7, whereas the connection used has a slot-pitch of only 6.

A two-phase lap winding, also using twenty-four slots for four poles, is shown in Fig. 237; as all the slots are utilized there will be three slots per pole per phase. Each phase is separate from the other, the terminals of phase No. 1 (shown in full lines) being A_1–B_1, and those of phase No. 2 (shown in dotted lines) being A_2–B_2. The scheme of connecting the coils in series is the same as used in the single-phase winding of Fig. 236, except that there are three coils in series per group in the two-phase winding, whereas in the single-phase winding there are four. In the position of the armature with respect to the field, as shown, phase No. 1 is generating maximum voltage, while that of phase No. 2 is zero.

hole
coil

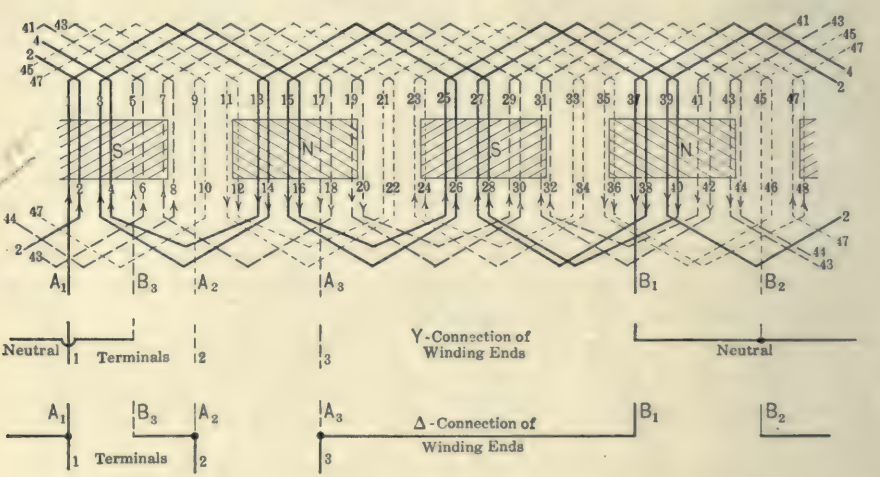


FIG. 238.—A developed lap winding for three-phase power. In the lower part of the diagram the proper end-connections for making either a Y- or Δ-winding are indicated.

A three-phase lap winding with twenty-four slots and four poles is shown in Fig. 238; there are now two slots per pole per phase. The beginning ends of the phases are A_1, A_2, and A_3 and the finish ends are B_1, B_2, and B_3; it will be seen that like terminals are 120° apart. Directly below the winding, the end-connections necessary to connect the winding in Y or in Δ are shown.

Since all the coils used in a lap winding are identical, only one size

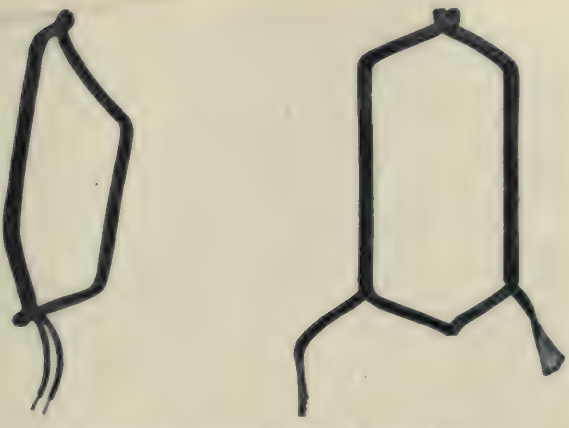


Fig. 239.—Typical lap coils of several turns.

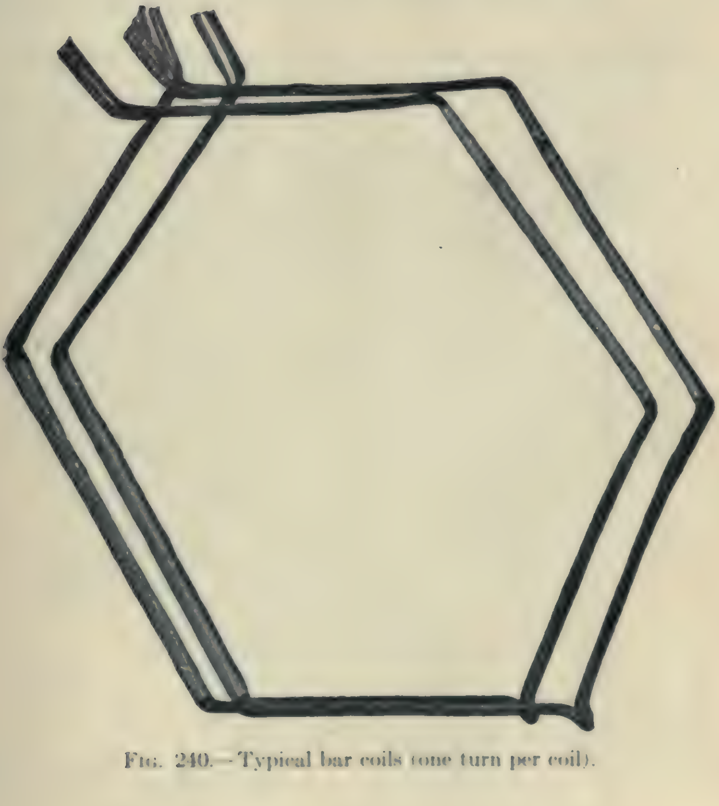


Fig. 240.—Typical bar coils (one turn per coil).

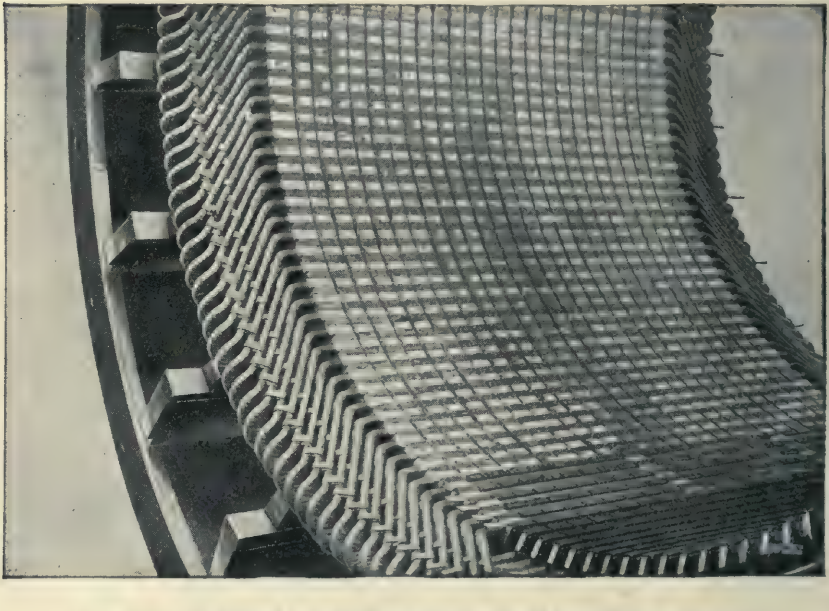


FIG. 241.—A typical lap winding, using coils of several turns per coil.

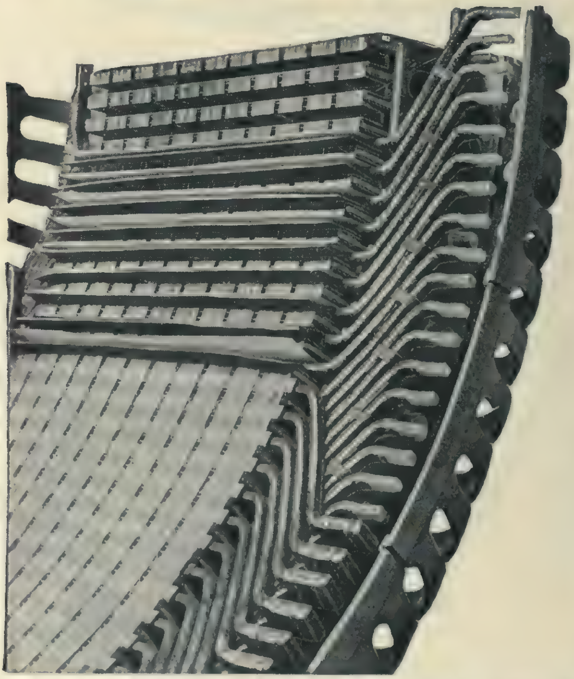


FIG. 242.—A typical lap winding using bar coils. Notice the heavy insulating material put into the slot before the coil-side is forced in.

and form of coil is necessary; in practice, formed coils, with the necessary twists in the end-connections to enable the coils to pass each other, are used. The coils are suitably insulated before insertion into the slots and are held in place by wedges, or similar means. Depending upon the speed and voltage to be generated, there may be more than one turn per coil; when only one turn per coil is necessary, the winding is often spoken of as a *bar* winding. Typical coils and completed windings are shown in Figs. 239, 240, 241, and 242.

Half Coil
One
[illegible]

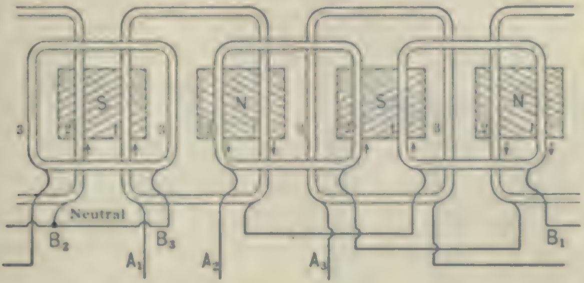


FIG. 243.—A three-phase chain winding; notice that in each phase, coils of both forms are connected in series, so that the resistance and reactance of the different phases may be the same.

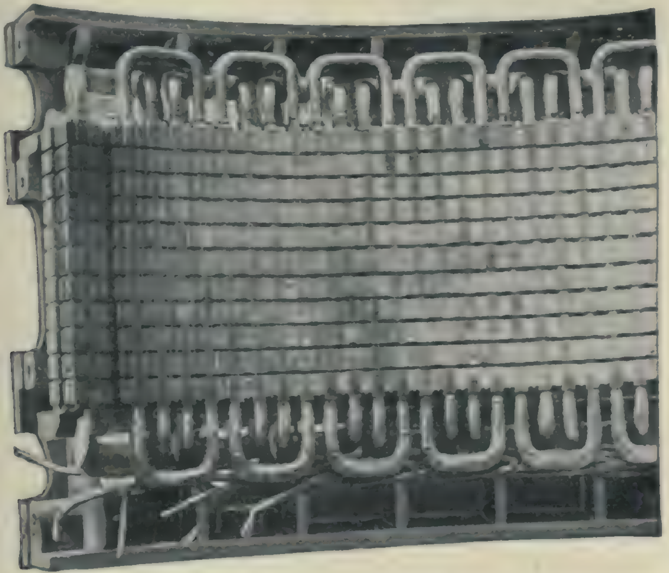


FIG. 244.—Section of a completed chain winding.

157. Chain Winding.—A chain winding is one having one slot per pole per phase, and one-half as many coils as there are slots; it derives its name from the appearance of the completed winding, as may be seen from Fig. 243, in which a developed three-phase, chain winding is shown, or from Fig. 244 which shows a completed chain winding. There being only one coil-side per slot, a chain winding is a single-layer winding.

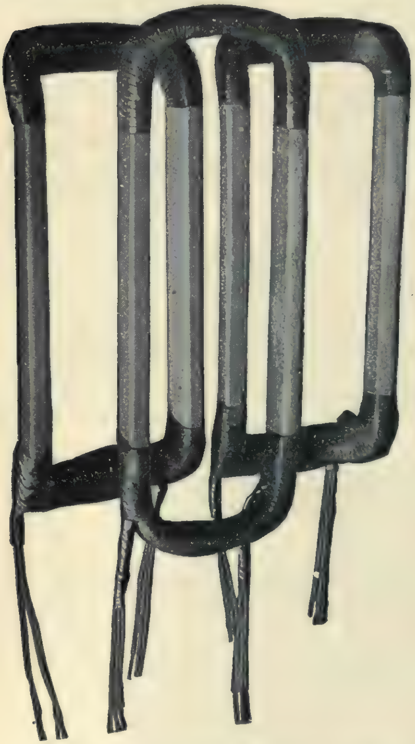


Fig. 245.—Formed coils for a chain winding.

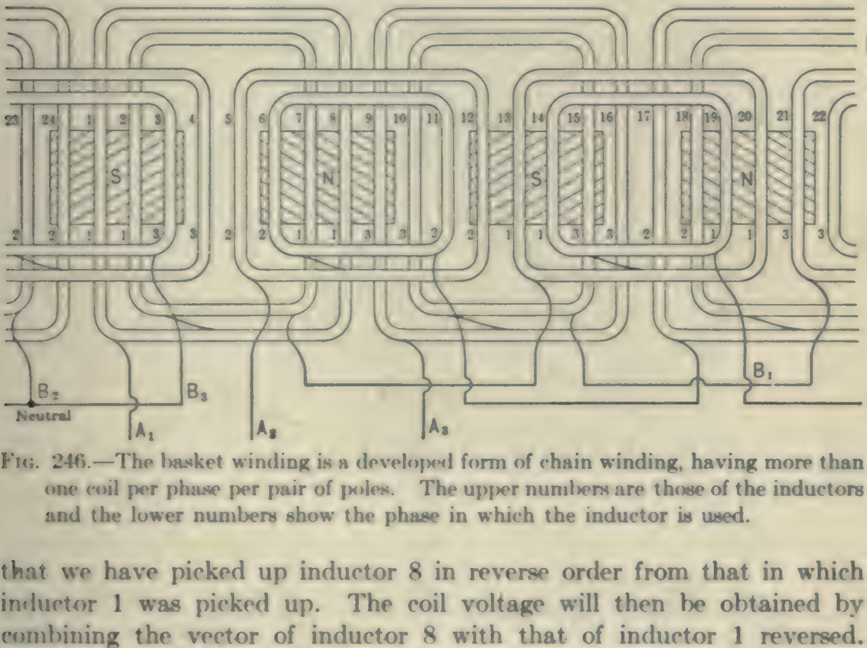
Coils of two shapes or sizes are required in order that the end-connections of the various coils may pass each other. Figure 245 shows formed coils for a chain winding; it will be seen that the ends of one size or shape of coil are left straight while those of the other size or shape are suitably bent.

The advantage of the chain winding is that crossings of the end-connections are reduced to a minimum; in a lap winding there must obviously be many such crossings. If the end-connections of the coils are bent away from each other, there is little opportunity for the insulation to break down at these points. Chain windings were extensively used for the windings of small, high-voltage, slow-speed, engine-driven alternators. Because of the increasing use of the turbine as the steam-driven prime mover, the chain winding now has but little application.

158. Basket Winding.—If it is necessary to use more than one slot per pole per phase, then the chain winding develops into the basket winding. A developed, three-phase, basket winding, using two slots per pole per phase, is shown in Fig. 246, and a completed basket winding in Fig. 247. The principle of the basket winding is the same as that of the chain winding, a second coil being added concentrically with each coil of the chain winding, as one basket is inserted into a second; a basket winding is also known as a *spiral* or *concentric* winding. Four sizes or shapes of coils are required for a basket winding with two slots per pole per phase, six sizes or shapes with three slots per pole per phase, etc. In

Fig. 248 a group of two coils for a basket winding is shown; a second shape of coils with bent end-connections must also be provided.

The inner coil of a group of concentric coils will obviously be of fractional pitch, while the outer coil will have a pitch greater than full-pitch; it follows from this that the several concentric coils of a group may generate different voltages. There being twelve slots per pair of poles in the winding of Fig. 246, the voltage in each inductor will be 30° out of phase with that of the adjacent inductor. In Fig. 249*a*, the e.m.f. vectors of inductors 1 to 8 are shown, the armature being considered as moving to the left in Fig. 246. In connecting inductors 1 and 8 to form a coil, we have joined them so that they will add their voltages, or we may say that we have picked up inductor 8 in reverse order from that in which inductor 1 was picked up. The coil voltage will then be obtained by combining the vector of inductor 8 with that of inductor 1 reversed. Similarly, the voltage of the inner coil is obtained by combining the vector of inductor 7 with that of inductor 2 reversed.



Half C

FIG. 246.—The basket winding is a developed form of chain winding, having more than one coil per phase per pair of poles. The upper numbers are those of the inductors and the lower numbers show the phase in which the inductor is used.

If the voltage induced per inductor is one volt, the voltage induced by the outer coil will be (Fig. 249*b*)

$$E_{8-1} = 2 \times e_1 \times \cos 15° = 2 \times 1 \times 0.9659 = 1.9318 \text{ volts}$$

and that induced by the inner coil will be (Fig. 249*c*)

$$E_{7-2} = 2 \times e_2 \times \cos 15° = 2 \times 1 \times 0{,}9659 = 1.9318 \text{ volts.}$$

The vector diagram shows these two voltages to be in phase, so that the voltage of a group is 2 × 1.9318 = 3.8636 volts.

Basket windings were extensively used for large, high-voltage, slow-speed alternators; but as the turbine has superseded the steam engine, and as the tendency is to operate water-wheel alternators at high speeds, basket windings are being used but little for alternators. They are still

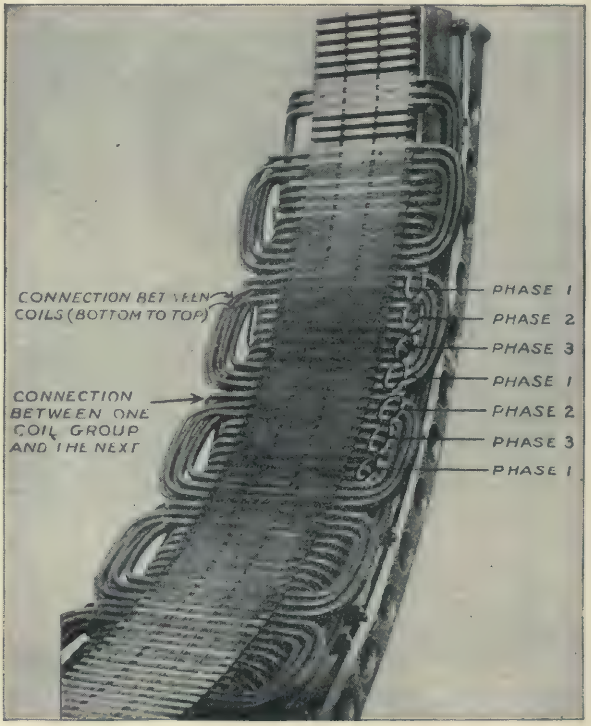

CONNECTION BETWEEN COILS (BOTTOM TO TOP)
CONNECTION BETWEEN ONE COIL GROUP AND THE NEXT
PHASE 1
PHASE 2
PHASE 3
PHASE 1
PHASE 2
PHASE 3
PHASE 1

Fig. 247.—A completed basket winding; the groups of coils used in each phase have been indicated and it is seen that short as well as long coils are connected in series for each phase, to keep the resistance and reactance of the different phases the same.

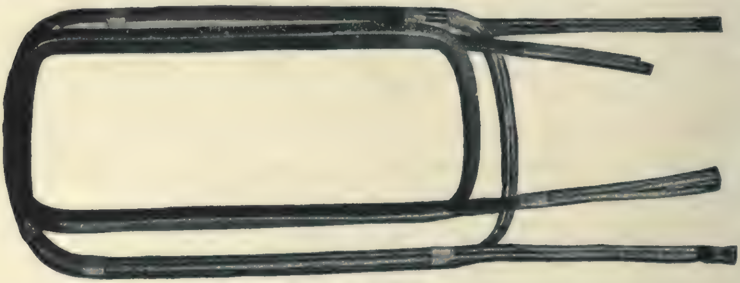


Fig. 248.—Two coils to be used in a basket winding; the smaller coil is shown taped, while the longer has not yet been served with its insulating cover.

used in small synchronous motors which are to be operated on voltages above 2200; in such small high-voltage machines, the insulation of the armature is a problem which is most easily solved by using a basket winding.

159. Number of Poles and Speeds of Commercial Alternators.—It is present standard practice to build 25-cycle turbo-alternators with two poles for all sizes, including those as large as 35,000 kv.a., the machines operating at 1500 r.p.m. For 60 cycles, turbo-alternators are built with two poles, operating at 3600 r.p.m. for sizes up to 6000 kv.a.; four poles and 1800 r.p.m. are used up to 30,000 kv.a., and six poles and 1200 r.p.m. for larger sizes.

In water-wheel-driven alternators the number of poles varies from about eight to about fifty-four, depending upon the size and frequency, the present tendency being to operate such alternators at higher speeds than heretofore. A typical modern high-speed 12,000-volt, 25-cycle, water-wheel-driven alternator of 45,000 kv.a. capacity, for example, has sixteen poles and operates at 187.5 r.p.m. This machine has a vertical shaft; the stator frame is 24.5 feet in diameter and 10 feet long (parallel to the shaft).

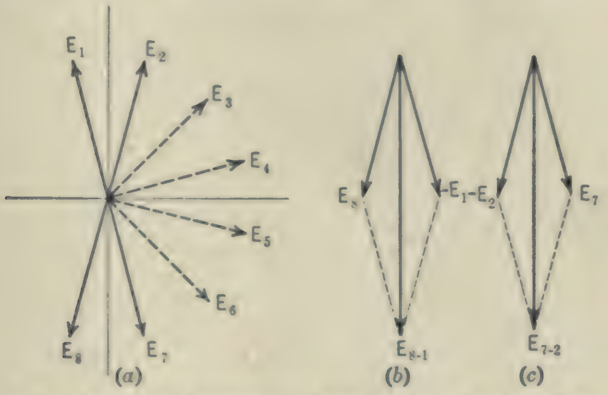


FIG. 249.—Showing the effect of connecting in series the coils of a basket winding, the different coils being of different pitch.

160. Commercial Windings.—The armature windings of 25-cycle turbo-alternators use from eight to eighteen slots per pole per phase, or a total of from forty-eight to one hundred and eight slots in all on the armature, depending upon the size and voltage of the machine. The windings are always of the lap type, the pitch of the coils being a little over 50 per cent of the pole-pitch. Such a low value of coil-pitch is used to keep the end-connections of the coils from becoming unduly long. With a two-pole field, if full-pitch coils were used the end-connections of

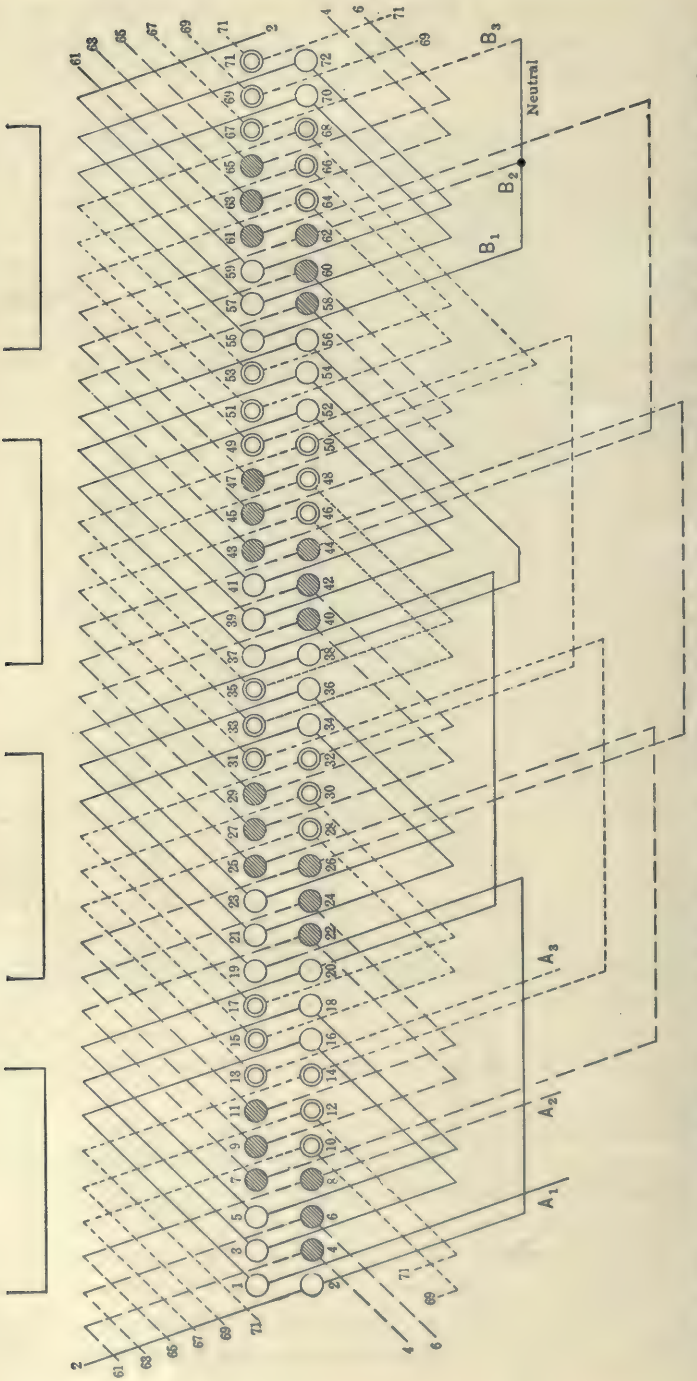


FIG. 250.—Commercially coils of fractional pitch are generally used on alternators. This is a typical winding for a three-phase machine having $\frac{7}{9}$ pitch.

the coils would reach halfway around the armature. The use of fractional coil-pitch also improves the wave form of the voltage generated by the alternator.

In the armature windings of 60-cycle turbo-alternators, from five to eight slots per pole per phase are used, the winding being always of the lap type. A coil-pitch of about 80 per cent of the pole-pitch is used, this value, compared to full pitch, cutting down the length of the end-connections of the coils materially with a loss of but a few per cent in voltage.

In water-wheel-driven alternators, from three to thirty slots per pole per phase are used, depending upon the frequency and voltage of the machine; the coil-pitch used is generally about 80 per cent when lap windings are used.

In Fig. 250 a typical winding is shown, with three slots per pole per phase; the coil-pitch is 7/9 of the pole-pitch. For the sake of simplicity, a winding for four poles only is shown and but one turn per coil is indicated.

The inductors of the three phases are differently marked, and it will be seen that, as a result of the fractional pitch used, in only one-third of the slots are both inductors of the same phase. The use of fractional pitch, however, still permits successive groups of inductors of the same phase to be 180° apart; thus inductors 4, 6, 7, 8, 9, 11 occupy the same relative position under one pole that inductors 22, 24, 25, 26, 27, 29, do under the next pole.

In Fig. 250 it will be seen that phases No. 1 and No. 3 are picked up in the same order; phase No. 2, however, is picked up in reverse order, with the result that the three start and the three finish ends of the three phases are close together. It is to be noted that some of the end-connections are represented as quite long (see connection between inductors 2 and 19). This was done to avoid confusion in the diagram; in the actual winding these connections would pass directly from one to the other of the inductors involved.

161. Armature Insulation.—Alternators are usually designed to generate their power at moderately high voltages, 6000 to 15,000 volts. With such values of voltage, the armature inductors must be carefully insulated to guard against short-circuits between coils, and grounds to the armature iron.

The material most generally used for insulating high-voltage armature coils is mica, in the form of tape composed of layers of mica between layers of thin, tough paper. After the copper wire or strap, of which a coil is to consist, has been properly formed, mica tape is applied by hand, first around each of the different turns of the coil. The coils are then placed in a tank, and after they have been subjected to a vacuum to remove the moisture, an insulating compound is forced into the coils under pressure to entirely fill all the spaces. The external insulation of the coil is then applied by hand; it consists of more mica tape with a special compound sticker (which also has high insulating properties) between layers. After this the coil may again be subjected to vacuum and filled under pressure.

The whole treatment results in an insulation for the coils which, when warm, is sufficiently plastic to allow the copper to expand and contract with changes in temperature with a minimum disintegrating effect on the mica insulation.

As the thickness of the insulation about a coil increases with the voltage of the machine, it becomes evident that the ratio of the cross-section of the copper in a slot to the cross-section of the slot itself will decrease as more and more insulation is added to the coils. The ratio just mentioned is called the *slot* factor; for machines designed for voltages above 10,000 volts the slot factor is about 0.3, which means that 70 per cent of the cross-section of a slot is used for insulation.

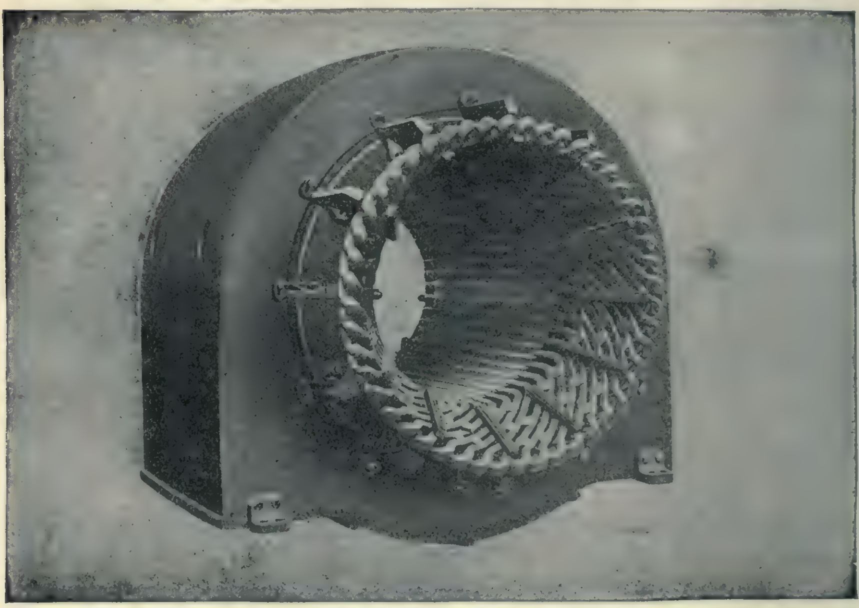


FIG. 251.—The armature for a turbo-alternator; the field is four-pole, so the pitch of the armature coils must be nearly 90 mechanical degrees. Notice how the end-connections are lashed solidly together, for mechanical strength.

Alternators are now built to generate up to 13,000 and 15,000 volts between lines; above these voltages the extra cost of insulation and the necessarily poor slot factors make it less expensive to install transformers, with a low-voltage alternator, than to use a very high-voltage alternator.

162. Proper Bracing for Armature Coils.—When the winding is for the armature of a slow-speed, multiple-pole generator, the end-connections are short and rigid and require no extra bracing to prevent bending in case of excessive current, in the armature winding. In the case of a turbo-generator, designed for high speed, and hence having a field frame of only two or four poles, the end-connections are very long; if a short-circuit occurs on such a machine the forces of attraction or repulsion between the various end-connections will twist them all out of shape unless special precautions are taken. The end-connections of such generators are, therefore, *well braced;* where the ends of the various coils cross they are rigidly fastened together by wooden blocks and twine, or perhaps lashed to a wooden frame designed especially for bracing the end-connections.

Figure 251 shows the end-connections of a four-pole turbo-generator; the length of the end-connections is apparent, and some of the bracing may be seen. Such a machine, if it has insufficient bracing at the end-connections, may have its coils so twisted and bent in case of a short-circuit that the whole winding is ruined.

163. Armature Reaction.—The armature reaction of alternators is more difficult to understand than that of continuous-current generators. The conductors of a continuous-current machine carry a current of constant value during one-half a revolution (180 electrical degrees) and the current has the same value during the next half revolution but is reversed in direction. It has been shown that this results in a constant magnetizing or demagnetizing effect, combined with a constant cross-magnetizing effect on the field.

Now the conductors of an alternating-current armature carry a current which is continually varying, so that each coil by itself produces a periodic effect on the strength of the field. In the case of a single-phase machine the entire armature winding produces a pulsating effect on the main field, while in the case of a polyphase machine the instantaneous effects of all the separate coils so combine as to produce on the main field an effect which is constant in magnitude and direction, like that produced in the continuous-current armature.

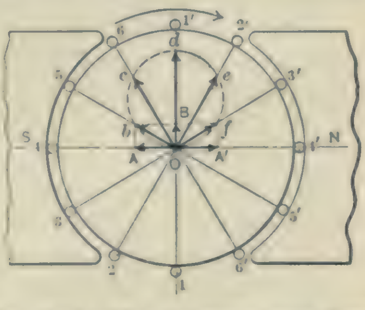


Fig. 252.—The magnetomotive-force vector of a single coil, carrying sine-wave current to a resistive load, has for its locus the dashed circle shown here. For one revolution of the armature the m.m.f. vector goes twice around its locus.

164. Analysis of the Action of a Single-coil Alternator with a Non-inductive Load.—A fair idea of the action of a single-coil alternator may be obtained by studying Fig. 252.

A two-pole machine is considered and it is supposed that the current in the coil is *in phase with the generated e.m.f.* so that the current is a maximum when the coil is in the axis of the main field. This position of the coil is numbered 4–4′ in Fig. 252. The m.m.f. of the coil is given by the vector *Od*, at right angles to the plane of the coil.

When the coil is in the position 1–1′ the current is zero and hence there is no m.m.f. generated by the coil. At the position 2–2′ the current has started to build up and the m.m.f. generated by the coil is shown by the vector *Ob*, which is proportional to the value of the current when the coil is in the position 2–2′ and is at right angles to the coil in this position. The m.m.fs. for the other positions of the coil given in Fig. 252 are shown by *Oc*, *Oe* and *Of*.

The vectors, if carefully constructed, are found *to be on a circular locus*, which is given in Fig. 252 by the dotted circle. As the armature continues to rotate (only 180° of rotation is considered in Fig. 252) the m.m.f. vector of the armature starts again over *this same circular locus*.

165. Division of Armature M.M.F. into Components.—If we consider any m.m.f. vector (as for example *Ob*), it is seen that this may be divided into two components, *OA* which tends to directly *magnetize* the main field, and *OB* which tends to cross-magnetize the main field. For every vector *Ob* there will be one symmetrically placed with respect to *Od* as an axis of symmetry. In the case of *Ob* the symmetrical vector is *Of*, or the m.m.f vector corresponding to coil position 6–6′. This vector *Of* may be resolved into its two components, *OA′* and *OB*; the component *OA′* tends to directly *demagnetize* the main field and *OB* is a cross-magnetizing effect.

Now every other vector has its mate and the resultant direct magnetizing or demagnetizing effect of any such pair is zero, because the magnetizing component of one is just equal to the demagnetizing component of the other; over a half-cycle the net magnetizing or demagnetizing effect of the coil is zero.

The cross-magnetizing effect of the two vectors (*Ob* and *Of* for example) is in the same direction, however, so that the cross-magnetizing effect is not neutralized.

Hence we conclude that *in the single-phase alternator, the armature of which is carrying current in phase with its generated e.m.f., the armature reaction produces no resultant demagnetizing effect but does cross-magnetize the main field so that the flux is concentrated in the trailing pole tips.*

166. Armature M.M.F. Produces Pulsations in the Strength of the Main Field.—But although there is no *net* magnetizing effect in such a machine it is to be noticed that at some instants the main field is stronger than normal, and at others weaker (because of the magnetizing effect *OA* and demagnetizing effect of *OA′*, for example). As a result, the main field pulsates in strength, and a study of this effect shows that this pulsation takes place with a frequency just twice as great as that of the generated e.m.f. of the armature. It may also be seen that the cross-magnetizing action is also pulsating in its effect, being a maximum when the armature current is a maximum, and being zero 90° later.

167. Effects of Single-phase Armature Reaction.—The total effect of the armature reaction in a single-phase generator, carrying current in phase with its generated e.m.f., is, therefore, a periodic weakening and strengthening of the main field combined with a periodic oscillation of the field back and forth across the pole face. *As both of these effects produce eddy currents in the pole face,* a single-phase generator, unless special precautions have been taken in the design, is likely to develop a great amount of heat in the poles. In some cases this effect has been the factor that limited the output.

168. Single-phase Armature Reaction with Inductive Load.—Suppose that the single-phase generator is carrying an inductive load. The current in the armature will have the same m.m.f. locus as it had in Fig. 252, but this circular locus will be rotated toward the right as shown in Fig. 253. This represents the conditions for a lagging load of power factor $= 0.7 (\phi = 45°)$; that the circular locus is rotated to the right is due to the fact that the current in the coil does not reach its maximum value until the coil has rotated 45° beyond position 4–4′, which is the position of maximum voltage. It may be seen that the components OA and OA', of a pair of vectors (Oc and Oe for example), *no longer neutralize one another;* in fact with $\cos \phi = 0.7$ (Fig. 253), *both of these represent a demagnetizing action* on the main field. Only when the m.m.f. vector lies to the left of the vertical is there any magnetizing action; therefore, the sum of all the demagnetizing and magnetizing components shows *the net result to be a demagnetizing effect for a lagging load.* The cross-magnetizing effect is still present but not to such an extent as it was for $\cos \phi = 1$, because during part of the alternation the cross-magnetizing effect pushes the *main field into the leading pole tips.* This would be the effect of the cross-magnetizing component of Of, for example.

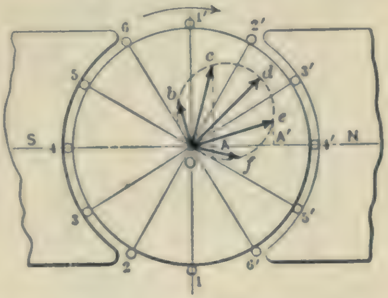


FIG. 253.—For an inductive load the m.m.f. locus of Fig. 252 is shifted in the direction of rotation, by an angle equal to that by which the armature current lags behind the generated voltage.

The result of these combined actions is that the armature reaction of a single-phase generator, furnishing current to an inductive load, not only makes the main field pulsate in strength and oscillate across the pole face, *but also produces a net weakening of the main field,* the amount of weakening depending upon the magnitude of the current in the armature and upon the power factor of the load, and being greater the less the power factor.

169. Single-phase Armature Reaction with Leading Armature Current.—In case the alternator is furnishing current, of the same magnitude as before, to a load having a leading power factor, the locus of the armature m.m.f. is the same circle but is rotated backward from the position it had for a non-inductive load; this condition is shown in Fig. 254. A study of this figure shows that for the single-coil alternator, furnishing a leading current, the armature reaction results in a periodic pulsation in the strength of the main field, an oscillation of the field across the pole face from the leading to the trailing pole tips and back, *and a net strengthening of the main field,* which is proportional to the armature current and dependent upon the power factor, being greater the less the power-factor.

170. Reaction of an Actual Multiple-coil Winding.—The previous analysis has been carried out for an armature having a single coil; however, any ordinary single-phase winding produces an effect nearly the same as though the armature had but one turn, as will be shown in the following analysis.

In Fig. 255 is shown a single-phase, full-pitch winding using four of the six slots per pole. The winding is shown as moving from left to right, so that in the inductors under a north pole the direction of the induced e.m.fs. is into the paper, and out of the paper under a south pole. Assuming that the winding is supplying current to a non-inductive load, and neglecting the reactance of the armature winding, the current in any inductor will be in phase with the e.m.f. induced in the inductor; the winding is therefore supplying maximum current at the instant shown.

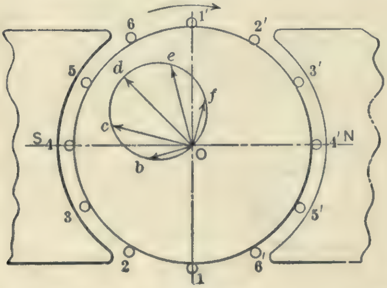


FIG. 254.—For capacitive loading the m.m.f. locus of Fig. 252 is shifted against the direction of rotation by an angle equal to the phase difference of armature current and generated voltage.

Coils 1–12 and 25–36 each set up an m.m.f., the direction of which is down, and coil 13–24 will set up an m.m.f., the direction of which is up; these are represented by the rectangles drawn to the dotted zero line *aa*. Similarly coils 3–14, 15–26 and 27–38 will set up m.m.fs., the same in value as the coils 1–12, 13–24 and 25–36; these are represented by the rectangles drawn to the zero line *bb*; these rectangles are naturally 15° behind the rectangles drawn to the zero line *aa*. The m.m.fs. of the

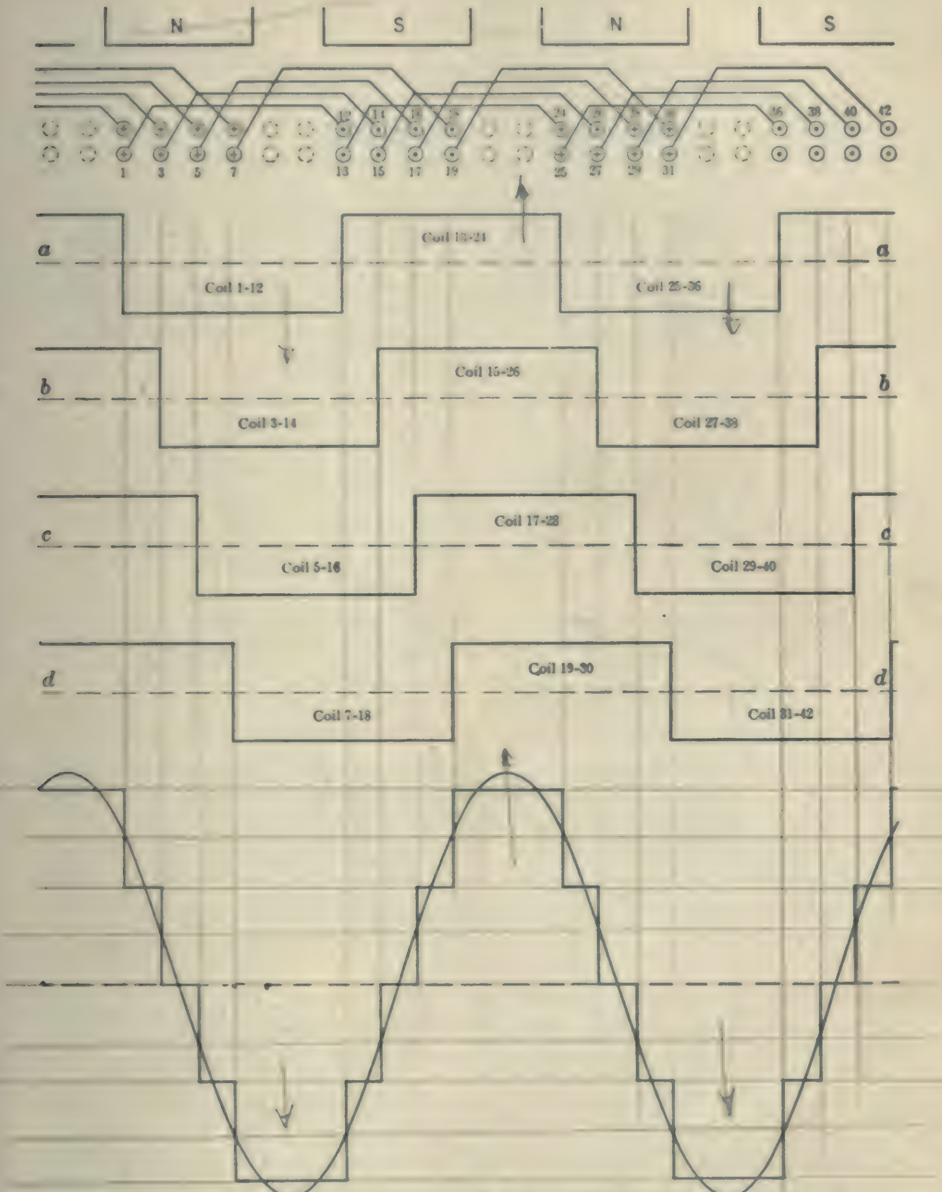


FIG. 255. By drawing the m.m.f. curve for each of the coils of the winding in its proper position, and then adding together the separate curves, the total m.m.f. of the winding is obtained. The stepped curve in the lower part of the diagram gives the total m.m.f. and it is readily seen that the stepped curve may be approximately represented by the sine curve shown.

remaining coils are represented by the rectangles drawn to the zero lines *cc* and *dd*.

The resultant m.m.f. of a group of coils may be obtained by combining

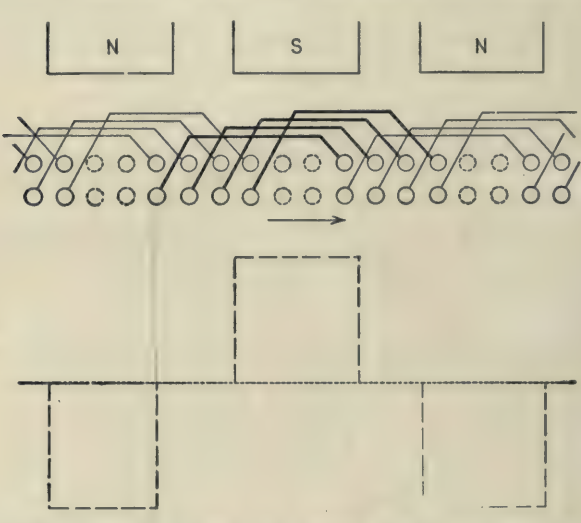


FIG. 256.—A single-phase winding at the instant when no current is flowing. The m.m.f. diagram of the field poles is given by the rectangular blocks.

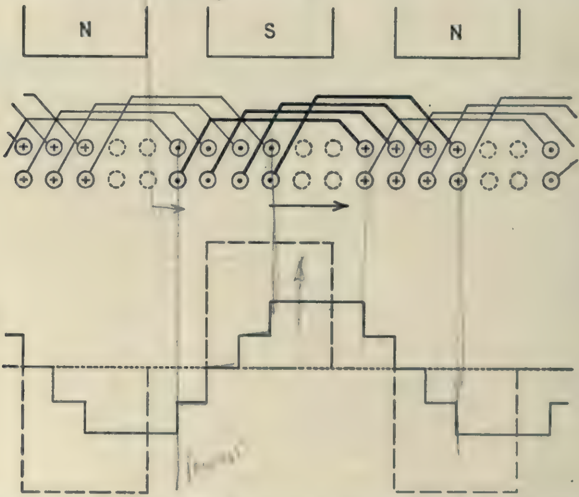


FIG. 257.—The winding of Fig. 256 has now moved along 45°, is carrying 0.707 of its maximum current value, and gives an m.m.f. distribution indicated by the stepped curve.

the m.m.fs. of the individual coils; the resultant so obtained is shown as the "stepped" curve at the bottom of the figure. This resultant "stepped" curve may, by rounding off the corners, be replaced by a sine curve, as shown.

In Figs. 256 to 259 the same winding is shown in four successive positions, the power factor of the load on the winding being again unity; in each figure one group of coils is drawn in heavy lines, so that its progress may be noted. In Fig. 256 the winding is generating zero voltage and carrying zero current, so that no m.m.f. is set up by the winding. This position of the winding corresponds to coil position 1–1′ of Fig. 252. The m.m.f. of the poles is shown by the broken-line rectangles; actually the latter would be smooth curves, derived by smoothing over the corners of the rectangles. No m.m.f. being set up by the windings, when in the position shown in Fig. 256, the field flux will be normally distributed over the air gap, as determined by the field m.m.f. only.

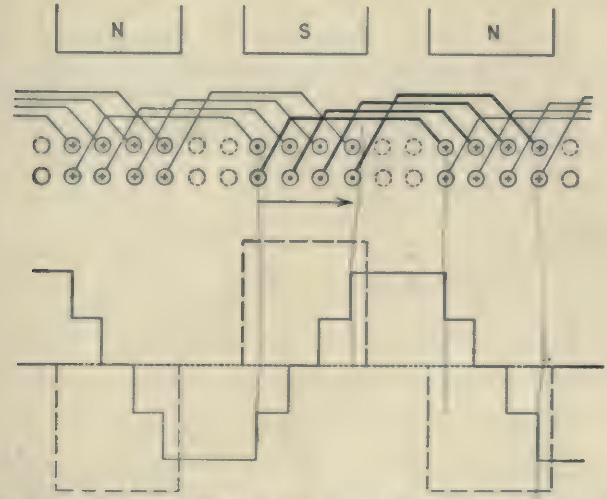


Fig. 258.—The winding of Fig. 256 has now moved along 90°, is carrying its maximum current, and gives the stepped m.m.f. curve shown here; the cross-magnetizing action of the armature is here a maximum.

In Fig. 257 the winding has moved 45° further in the direction of rotation; it is therefore generating 0.707 of its maximum voltage and carrying 0.707 of its maximum current. The m.m.f. set up by the winding is shown in full lines, and that of the field poles by broken lines, and it will be seen that the two m.m.f. curves overlap considerably, so that there is a distinct strengthening of the flux across the air gap. The fact that the middle of a peak of the armature m.m.f. curve lies under a trailing pole tip, indicates that the resultant m.m.f. under the trailing pole tips is greater than under the leading pole tips, so that the flux will be densest under a trailing pole tip; in other words, the flux under the poles has shifted somewhat from under the leading to the trailing pole tips.

In Fig. 258 the winding has advanced another 45° and is now 90° beyond its position in Fig. 256; the winding is now generating maximum voltage and carrying maximum current. It will be seen that the middle of a peak of the winding m.m.f. curve lies midway between poles, and that under the leading pole tips the m.m.f. of the winding opposes that of the poles, while under the trailing pole tips the m.m.fs. augment each other. The two m.m.fs. are, however, symmetrically placed, the winding m.m.f. opposing the field m.m.f. as much under one tip as it augments under another. Whereas the flux under a pole is therefore shifted to a greater degree than in Fig. 257, the total flux per pole is again the same as in Fig. 256, when the winding carried no current.[1] The position of the winding in Fig. 258 is relatively the same as position 4–4′ of the single coil of Fig. 252, the winding m.m.f. being at right angles to the field m.m.f. in both these cases.

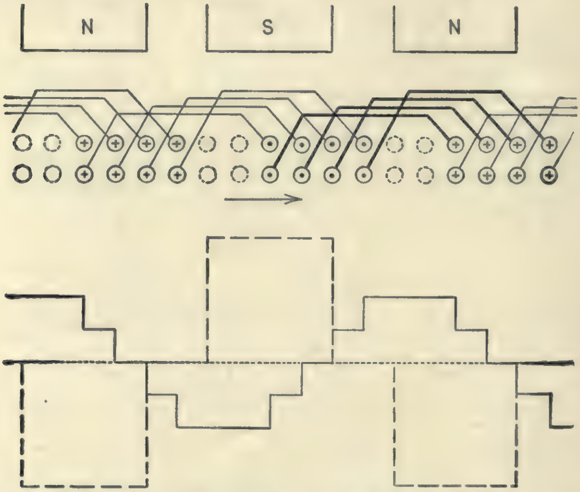


Fig. 259.—After the winding of Fig. 256 has moved along 135° its current has started to decrease, resulting in a lower m.m.f. value. What m.m.f. there is, however, is largely a demagnetizing one.

In Fig. 259 the winding has advanced another 45° and is now 135° beyond its position in Fig. 256; the winding is again generating 0.707 of its maximum voltage and is carrying 0.707 of its maximum current. The two m.m.f. curves are now seen to be much in opposition, the middle of a peak of the winding m.m.f. curve now being under the leading pole tip.

[1] This statement follows only if the magnetization curve of the machine is a straight line, that is, the iron of the field poles is far away from the condition of saturation.

There is therefore a considerable reduction in the flux in the air gap, and also some distortion of the field, the flux again being most dense under the trailing tips.

The conclusions to be drawn from Figs. 256 to 259 are the same as were drawn after study of Fig. 252, i.e., that there is a periodic strengthening and weakening of the main field, combined with a periodic oscillation of the field back and forth across the pole faces.

171. Reaction of an Actual Multiple-coil Winding with Lagging and Leading Current.—If diagrams similar to Figs. 256 to 259 were drawn with the winding supplying lagging current, the conclusions that would be drawn are the same as were reached after consideration of Fig. 253. Two such positions of the winding, carrying current lagging 45° behind the e.m.f., are shown in Figs. 260 and 261. In Fig. 260 the winding is in its 135° position; the e.m.f. generated is therefore equal to 0.707 of the maximum and decreasing, but the current, having been assumed as lagging 45° behind the voltage, is at its 90°, or maximum, value. It will be seen that the m.m.f. of the armature is much in opposition to that of the field, and that the field flux is distorted, being densest under the trailing pole tip. In Fig. 261 the winding has advanced 45° further so as to be at its 180° position; the e.m.f. generated is zero, but the current is equal to 0.707 of its maximum and decreasing. The two m.m.fs. at this instant are directly in opposition; the field is therefore reduced below normal, but is not distorted. When the power factor of the winding is unity, at

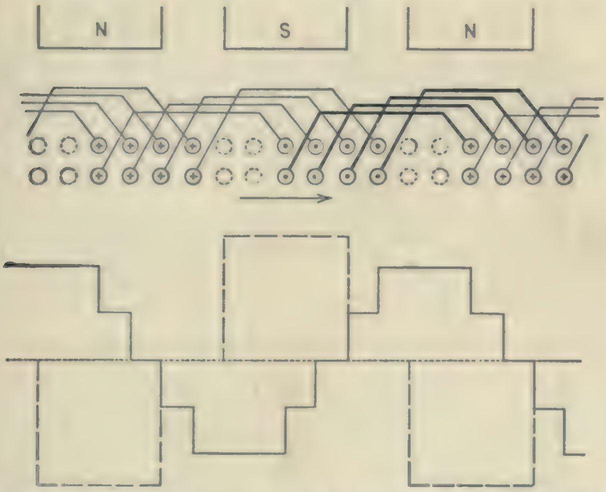


FIG. 260.—For lagging loads the current reaches its corresponding values for later positions of the armature winding. Here the current has its maximum value, although the position of the coils corresponds to that of Fig. 259.

no instant is there direct opposition of the winding and field m.m.fs.; the m.m.fs. only oppose or augment each other under one or the other of the pole tips.

If a figure were drawn with the armature in its 90° position, with the e.m.f. a maximum and the current equal to 0.707 of its maximum, the figure would be the same as Fig. 258, except that the corresponding m.m.f. values would be equal to 0.866 of the values in Fig. 258; at that instant there would be only distortion of the field, the leading pole tip being weakened as much as the trailing pole is strengthened.

With a zero power factor, lagging current, there will be some weakening of the field with every position of the armature, the armature m.m.f. reaching its maximum and being directly in opposition to the field m.m.f. when the e.m.f. is zero. A figure drawn for this instant would be the same as Fig. 261, except that the armature m.m.f. would be at its maximum value.

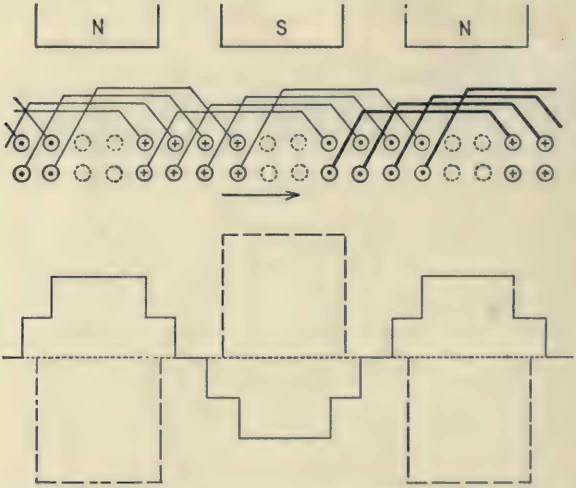


Fig. 261.—For a position 45° later than that of Fig. 260 the armature is in a position to directly demagnetize the main field and the current still has 0.707 of its maximum value.

With inductive load on the armature we again reach the conclusion that single-phase armature reaction causes the main field to pulsate in strength and oscillate across the pole face, and also that it produces a net weakening of the field. The amount of weakening depends upon the current in the armature and upon the power factor of the load, being greater the less the power factor.

In Figs. 262 and 263 are shown two instantaneous positions of the same winding supplying a current leading the e.m.f. by 45°. In Fig. 262

the e.m.f. generated is zero, so that the current is at its 45° value, i.e., equal to 0.707 of its maximum and increasing. It will be seen that the

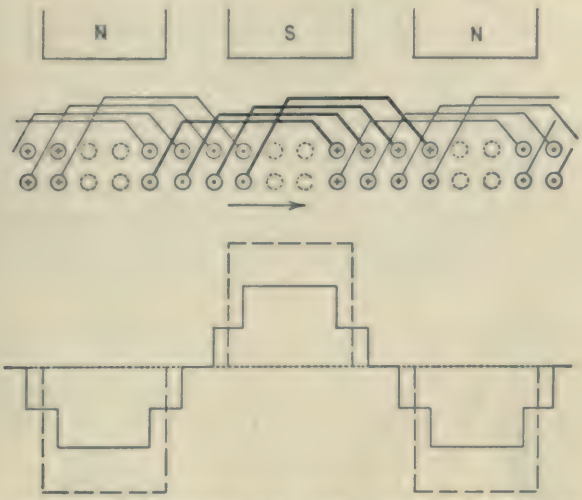


FIG. 262.—If the armature is supplying a leading current, the effect of the armature m.m.f. is to magnetize the main field; here the current has 0.707 of its maximum value and is increasing.

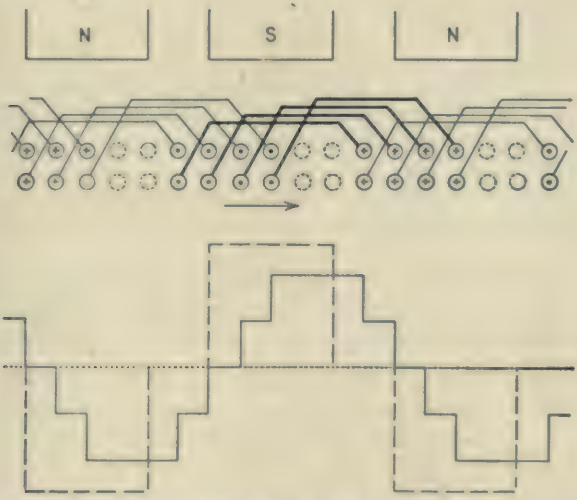
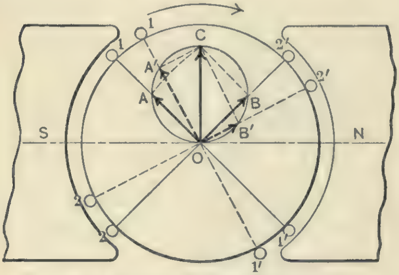


FIG. 263.—For a position 45° later than that given in Fig. 262, the armature m.m.f. is a maximum (with the leading current assumed). The armature reaction is here largely a magnetizing one.

armature m.m.f. now directly augments the field m.m.f., so that the field is strengthened; in Fig. 263 the winding has moved 45° further, the current now being at its maximum value. The two m.m.fs. are still acting together, although the field is strengthened more under the trailing pole tip than under the leading tip.

With a leading power factor we see then that single-phase armature reaction causes the field to pulsate in strength and oscillate across the pole faces, and also produces a net strengthening of the field. The amount of strengthening depends upon the current in the armature and upon the power factor of the load, being greater the less the power factor.

172. Analysis of Armature Reaction in a Two-phase Alternator.—The effect of armature reaction in a polyphase generator may easily be seen by drawing the vectors showing the armature m.m.f. for each phase separately and then combining them. Thus in the two-phase armature the total m.m.f. may be obtained by adding vectorially two chords of the circular locus at right angles to one another. They are added at right angles because the phase windings on the machine are 90° apart. Fig. 264 shows that the m.m.f. of the armature resulting from such a construction *is constant in magnitude and direction.* The m.m.f. of phase 1–1′ is shown at *OB* and at the same instant phase 2–2′ is producing the m.m.f. *OA*. The resultant of these is *OC*, and *no matter at what instant the armature reaction is considered, the resultant will always be OC, the maximum value of the m.m.f. of one phase.* Fig. 264 shows the conditions for a non-inductive load; the resultant *OC*, is seen to be entirely a cross-magnetizing effect. Now if the load were inductive the proper diagram would show that the armature reaction is of the same magnitude *OC*, but that it is advanced in the direction of rotation, as *OC* of Fig. 265,



Coils and vectors in dotted lines show conditions for a fraction of a revolution later than those shown by full lines.

FIG. 264.

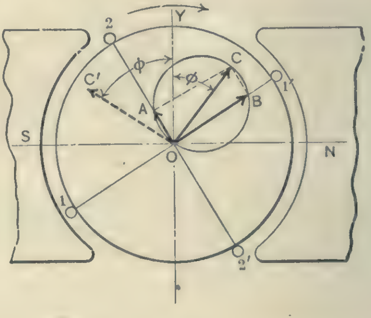


FIG. 265.

FIG. 264.—For two coils 90° apart, each connected to its own load (a four-wire, two-phase alternator) the combined m.m.f. is fixed in space and constant in magnitude, if the two-phase load is balanced. The value of the armature m.m.f. is equal to the maximum value of that of one coil.

FIG. 265.—For lagging or leading load currents, the armature m.m.f. vector is shifted with, or against, the direction of rotation.

the angle of advance being equal to ϕ, where $\cos\phi$ is the power factor of the load.

In the case of a load with a leading current, the armature m.m.f. would still be of a magnitude equal to OC but its direction would be behind that of OC, as shown at OC', of Fig. 265; again the angle YOC' is equal to ϕ, the angle of lead of the current.

173. Analysis of Armature Reaction in a Three-phase Alternator.—The effect of armature reaction in a three-phase alternator may be obtained by drawing the vectors showing the m.m.f. for each phase separately, and then combining them. In Fig. 266, OA represents the m.m.f. of phase 1–1′, OB, the m.m.f. of phase 2–2′, and OC, the m.m.f. of phase 3–3′, the power factor being unity. Since phase 1–1′ is generating maximum voltage at the instant shown, and therefore is carrying maximum current, its m.m.f. is a maximum; phases 2–2′ and 3–3′ are each carrying current equal to 0.5 maximum value, so that $OB = OC = \frac{OA}{2} = \frac{OM}{2}$, where OM is the maximum value of the m.m.f. per phase.

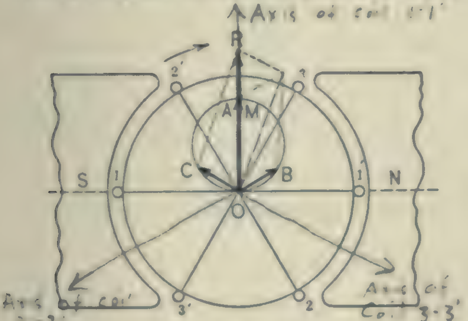


FIG. 266.

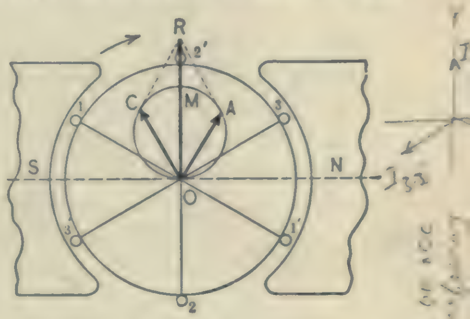


FIG. 267.

FIG. 266.—The armature m.m.f. for a three-phase armature is obtained by adding vectorially the m.m.fs. of the various phases. The magnitude of the resultant m.m.f., OR, is just 50 per cent greater than the maximum value of that for one phase.

FIG. 267.—By constructing the diagram of Fig. 266 for a time 30° later than that there shown, the total armature m.m.f. is again obtained as equal to OR, which is the same in magnitude and position as that obtained in Fig. 266. The position of the armature m.m.f. (for a given load) is fixed in position with respect to the field poles.

The resultant m.m.f. of the armature is the vector sum of the three phase m.m.f. vectors, i.e.,

$$OR = OA + OB\cos 60^\circ + OC\cos 60^\circ$$
$$= OM + OM\cos 60^\circ = OM + \frac{OM}{2} = 1.5\ OM.$$

In Fig. 267 the winding has advanced 30° further; the current in phase 2–2′ is zero, while phases 1–1′ and 3–3′ are carrying currents equal to

0.866 of their maximum values, i.e., $OA = OC = 0.866\ OM$. The resultant m.m.f. of the whole armature is now

$$\begin{aligned} OR &= OA \cos 30° + OC \cos 30° \\ &= 2 \times 0.866 \times OM \times \cos 30° \\ &= 2 \times 0.866 \times OM \times 0.866 = 2 \times 0.75 \times OM = 1.5\ OM. \end{aligned}$$

A similar result would be obtained for every instantaneous position of the armature, so that we find that the resultant m.m.f. of a three-phase armature is constant in magnitude, and fixed in position with respect to the field poles. The magnitude of the resultant m.m.f. depends upon the magnitude of the armature current and is equal to 1.5 times the magnitude of the maximum value of the m.m.f. of one phase of the winding.

It will be seen that if the three-phase winding is carrying a lagging current the resultant m.m.f., *OR*, of Figs. 266 and 267 will be rotated clockwise and for a leading current the resultant will be rotated counterclockwise, the cosine of the angle between the resultant and the vertical being in each case the power factor.

With a power factor equal to unity, the resultant m.m.f. vector, *OR*, is at right angles to the field, so that the field flux is merely distorted, becoming concentrated under the trailing pole tips. With a lagging power factor, the m.m.f. vector, *OR* (now inclined to the right) may be resolved into two components, as in Fig. 268, a cross-magnetizing component, *OC*, which distorts the field as before, and a demagnetizing component, *OW*, which weakens the main field. With a leading power factor, the m.m.f. vector, *OR′* (now inclined to the left), has a cross-magnetizing component and a magnetizing component *OM*, which strengthens the main field.

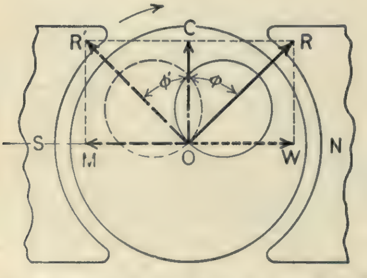


Fig. 268.—The armature m.m.f. of the three-phase machine, with balanced load, is constant in magnitude and position for a given load, but shifts its position (with respect to the field poles) as the power factor of the load changes.

The effect of power factor is thus to either weaken or strengthen the field, depending upon whether the current lags or leads the e.m.f., the amount of weakening or strengthening increasing as the power factor decreases.

Armature reaction in a polyphase alternator differs from that in a single-phase alternator in that, with a balanced load on the armature, *the m.m.f. of the armature of the polyphase alternator is constant in magni-*

tude and fixed in position with respect to the field, while in the single-phase alternator the m.m.f. is pulsating in magnitude and constantly changing in position with respect to the field. In the polyphase alternator, therefore, there are no pulsations of the field, nor does the field flux oscillate across the pole faces.

174. Reaction in an Actual Multiple-coil Winding.—In Fig. 269 a three-phase, full-pitch winding, with three slots per pole per phase, is shown. The resultant m.m.f. of coils 1–1′, 2–2′, 3–3′, may be represented by the curve *a b c d e f g h j k l m*, and that of coils 4–4′, 5–5′, 6–6′, by the curve *a′ b′ c′ d′ e′ f′ g′ h′ j′ k′ l′ m′*. The combined m.m.f. curve of phase No. 1 is then the curve drawn in heavy line; as before, this "stepped" curve may be approximately replaced by a sine curve. It is evident that the m.m.f. curves of the other two phases, when these phases carry their maximum currents, will be the same as that indicated for phase No. 1.

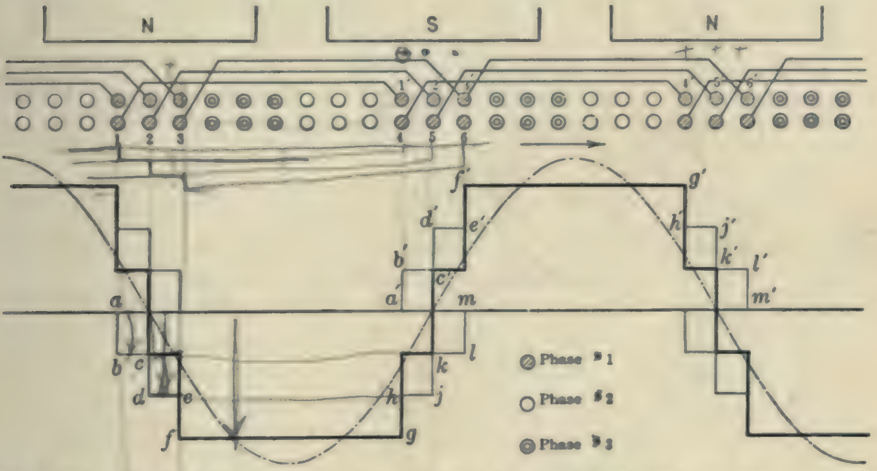


FIG. 269.—By depicting the armature winding in the developed form the armature m.m.f. of a multiple-coil winding may be easily shown. The form of the m.m.f. curve for phase 1 is shown as the heavy-broken line, which may be approximately replaced by the sine-wave curve.

In Fig. 270 the winding is shown in such position that the e.m.f. generated by phase No. 1 is at its maximum value; with a power factor equal to unity, this phase will be carrying maximum current. The other phases will generate voltages and carry currents equal to 0.5 maximum values. The m.m.f. of phase No. 1 is represented by the curve *a b c d e f g h j k l*, that of phase No. 2, by the dotted curve, and that of phase No. 3 by the broken-line curve. Combination of the three phase m.m.fs. yields the heavy-line resultant curve, which is very close to a sine wave. The maximum of the resultant sine wave is 1.5 times the maximum of the sine wave representing the maximum m.m.f. per phase; this checks the analysis given in section 173.

In Fig. 271 the winding has advanced 30° further; the e.m.f. and current of phase No. 3 are zero, and those of phases No. 1 and No. 2 are equal to 0.866 of their maximum values. The resultant heavy-line curve may be replaced by a sine wave of amplitude equal to that of Fig. 270; it is seen to be in exactly the same relative position with respect to the field poles in both Figs. 270 and 271.

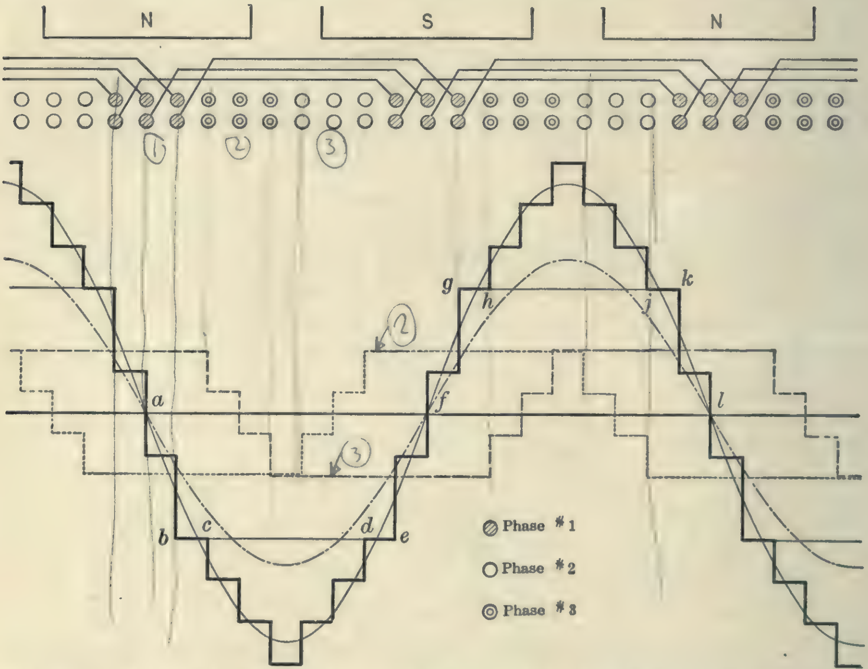


FIG. 270.—The m.m.f. curves for each of the phases, for this instant when phase 1 is carrying its maximum current. The other phases (m.m.fs. shown by dotted and dashed lines) are each carrying half as much current. The total m.m.f. of the three phases (solid, heavy, stepped line) may be approximately replaced by the sine curve (full light line) shown.

With a lagging power factor, the resultant armature m.m.f. curve would be shifted further to the right so as to come more into opposition with the field m.m.f.; with a leading power factor, the resultant would be shifted to the left, tending to strengthen the field.

175. Previous Analyses for Balanced Loads Only.—The previous analyses have been carried out on the assumption of a balanced polyphase load, i.e., all phases carrying currents of the same magnitude and power factor. With unbalanced loads there will be pulsations in the field strength, depending upon the amount of unbalance. Summing up the results for a polyphase machine, we see that, when the load is balanced, *the armature reaction is constant in magnitude and direction, the same as in a continuous current machine; the magnitude of the reaction depends upon the value of the armature current and the direction depends upon the power factor of the load.*

176. Magnitude of the Effect of Armature upon the Main Field.—The magnitude of the effect of the armature reaction depends upon the ratio of the armature ampere-turns to the field ampere-turns. A machine having a long air gap requires many ampere-turns on its field, and if it has but few ampere-turns on its armature the machine is said to have *a stiff field.* The field of such a machine is affected but little either in magnititude or position by armature reaction.

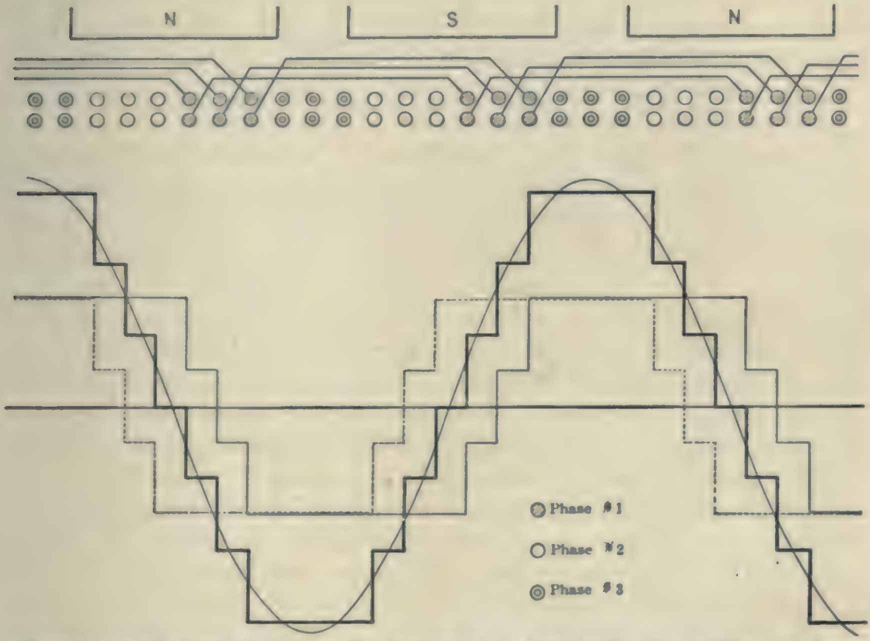


FIG. 271.—The armature m.m.f. for an instant 30° later than that shown in Fig. 270. The m.m.f. of phase 3 is now zero but the total armature m.m.f. (solid heavy stepped line) is the same in magnitude and position (with respect to field poles) as it was in Fig. 270. Although somewhat different in form, it may be nearly replaced by the same sine curve as used in Fig. 270.

It will be shown in later paragraphs that armature reaction affects the operating characteristics of alternators, particularly with the lagging power factors generally occurring in practice. In order to give alternators certain electrical characteristics, it is found desirable to make the ampere-turns of the armature at full load about equal to the ampere-turns expended in the air gap at no load and normal voltage. At full load and rated voltage, the field ampere-turns are larger than the armature ampere-turns by from 30 to 50 per cent.

177. Resultant Armature M.M.F in Polyphase Alternators Rotates with Respect to the Armature.—We have seen that in an alternator with a rotating armature, the resultant m.m.f. of the armature is fixed with respect to the field. It follows that this armature m.m.f. must rotate backwards with respect to the armature with the same speed that the armature rotates forward. In a revolving-field alternator, the armature being stationary, its resultant m.m.f., in order to remain fixed with respect to the field, must revolve in the same direction as the field is rotating, and with the same speed.

It should be evident then, that if a current is passed through the armature winding of an alternator whose field has been removed, the armature will set up an m.m.f. of the form shown in Figs. 270 and 271. At each peak of the m.m.f. wave a magnetic pole will be found, and since the m.m.f. revolves with respect to the armature, there is thus set up a revolving magnetic field. Such a revolving field is of prime importance in the operation of the induction motor and in this kind of motor is set up by a winding identical with that of a revolving-field alternator; in the induction motor, however, a rotor of different construction is used, as will be shown in the chapter dealing with motors of this type.

178. Rating of A.-C. Machinery.—The rating of alternating-current machines, generators, transformers, etc., is practically never given in kilowatts, but in kilovolt-amperes. A generator having a safe current capacity of 10 amperes and giving an e.m.f. of 100 volts would be rated as having a capacity of one kilovolt-ampere or abbreviated, 1 kv.a.

To get the rating of a single-phase generator or transformer the safe capacity in amperes is multiplied by the terminal voltage and this product is divided by one thousand. For example a 2300-volt single-phase alternator having a safe current capacity of 100 amperes would have a rating of $\frac{2300\times100}{1000}=230$ kv.a.

If the machine being considered were a two-phase alternator having a capacity of 100 amperes per phase and giving 2300 volts in each phase, the kv.a. rating would be twice that of one phase, or 460 kv.a. If it were a three-phase alternator, giving 2300 volts between any two lines and having a capacity of 100 amperes per line the kv.a. rating would be equal to $\frac{2300\times100\times\sqrt{3}}{1000}=398$ kv.a.

Why should the capacity of an alternating-current machine be given in kilovolt-amperes instead of in kilowatts? We have shown that the output of an electric machine is fixed by the safe heating, and that this heating in the conductors fixes, therefore, the safe current the machine may carry. Its voltage is fixed by the saturation of the magnetic field, number of conductors, speed, etc., and cannot safely exceed that for which the machine was designed. *The two limiting factors of the output are thus the current and voltage*, and it may be that the power factor of the load to which the generator is furnishing current is less than unity.

Example.—Let us consider the 2300-volt, 100-ampere, single-phase generator; its rating was 230 kv.a. Now if the load power factor were 0.5, when the machine was delivering 100 amperes at 2300 volts the watts output would be $2300 \times 100 \times 0.5 = 115,000$ watts $= 115$ kw. So that the generator would be working to its safe limit to supply 115 kw., whereas if the power factor of the load were unity the possible output would be 230 kw. Thus the possible power output of a generator depends entirely upon the power factor of the load; alternating-current machinery is therefore rated in terms of volts and amperes, and not watts. If a machine is rated in kilowatts a load of power factor equal to unity has been assumed.

179. Armature Reactance.—When current flows in the conductors of the armature of an alternator, magnetic lines will be set up around these conductors, so that the armature winding possesses self-induction. The self-induction when multiplied by 2π times the frequency, gives the *reactance* of the armature. When current flows through the winding there must then be a reactance drop, i.e., a component of the voltage generated in the armature must be used to supply the reactance drop of the winding.

180. Armature Resistance.—When a current flows through the alternator armature the magnetic lines set up around the conductors cause hysteresis and eddy-current losses in the surrounding iron. In order to take these losses into account with the I^2R losses occurring in the conductors themselves, the resistance of the armature is expressed as an effective resistance, such that, when multiplied by the square of the current, the total losses in the armature may be determined. Evidently the effective resistance of the armature is greater than its conductor resistance (i.e., the resistance measured by c.c.); in practice the effective resistance is generally about 20 per cent greater than the conductor resistance; it may, however, be as much as 200 per cent greater.

The vector sum of the resistance and reactance gives the impedance, Z, of the armature winding.

181. Characteristic Curves.—There are many kinds of so-called characteristic curves, but the two to be considered here are the external characteristic and the armature characteristic; the latter is often called the field-compounding curve. The efficiency curve is similar in shape to that of a continuous-current generator and, as the same factors determine the form of the curve for both machines, it will not be taken up here. It was discussed and analyzed in Volume I in detail.

The external characteristic of an alternator shows the relation between the terminal voltage and the load current, the field current being held constant at that value which gives rated voltage when rated full-load current is being carried. The shape of this curve resembles that of a continuous-current, separately excited generator, but the shape of the curve for an alternator depends somewhat on the power factor of its load. Fig. 272 serves to illustrate this point; the rated full-load voltage is *oD* and the external characteristic for a non-inductive load is shown by the curve *BGF*, for an inductive load by the curve *CGE*, and for load of leading power factor by the curve *QGP*. It will be noticed that the drop in terminal voltage, for a given increase in load current, is greater for an inductive than for a non-inductive load.

182. Factors Affecting the Form of the External Characteristic.—The factors which act to change the terminal voltage of an alternator as the load is varied are: the armature resistance, the armature reactance, and the armature reaction. In obtaining the external characteristic the field current is maintained constant, and hence, if the armature current did not affect the field strength, the generated e.m.f. would remain constant as the load varied. But, as previously shown, the armature reacts to affect the strength of the main field, a lagging current demagnetizing the field and a leading current magnetizing it. Thus, as a lagging load is increased, the *generated e.m.f. of the armature decreases* even though the field current and speed are held constant. With a load of leading current, the *generated e.m.f. increases* with an increase of load.

The terminal voltage is obtained by subtracting (vectorially) the armature *IR* and *IX* drops from the generated e.m.f. These combined effects give external characteristics similar in form to those given in Fig. 272.

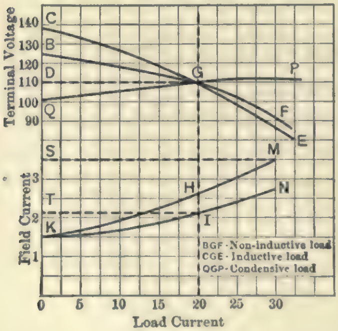


Fig. 272.—Showing the external characteristics and armature characteristics (field-compounding curves) for different kinds of loads.

183. Field-compounding Curve.—The armature characteristic, or field-compounding curve, shows the relation between the load current and the field current, the latter being so varied that the terminal voltage of the alternator remains constant as the load is changed. The field current must be increased to a considerable extent when the load is changed from zero to rated capacity and over, and this increase is greater for an inductive than for a non-inductive load.

Experimentally obtained results for a small alternator are given by the curves *KIN* and *KHM* of Fig. 272. The first curve is for a non-inductive load and the second for a load of $\cos \phi = 0.7$, lagging current.

184. Terminal Voltage of an Alternator.—In Fig. 273 are shown voltage diagrams for an alternator carrying current, in (*a*) with a lagging power factor, in (*b*) with unity power factor, and in (*c*) with a leading power factor. It will be seen that the relative effects of the resistance and reactance drops vary with power factor. With a power factor of unity, the *IR* drop has greater effect on the terminal voltage than with low power factors, while with a given generated voltage the *IX* drop produces greater reduction in terminal voltage with lagging power factors; with leading power factors the *IX* drop tends to increase the terminal voltage.

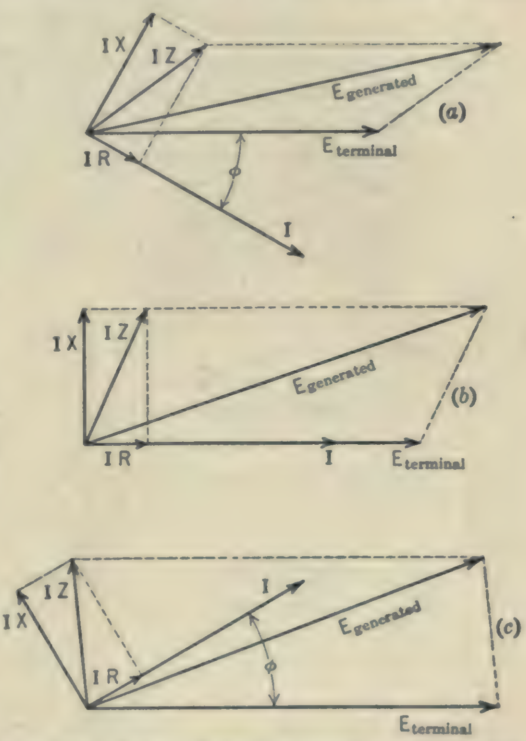


Fig. 273.—Simple voltage diagrams for one phase of an alternator for (*a*) inductive, (*b*) resistive, and (*c*) capacitive, loads.

It must be carefully noted that if the true reactance of the armature is used in constructing these diagrams, they take no account of armature reaction, which acts directly upon the flux and so affects the generated voltage. Thus, if the generated voltage in each of the diagrams of Fig. 273 is the same, we must assume more field current used in (*a*) than in (*b*), and more in (*b*) than in (*c*).

The effect of armature reaction is frequently combined with the true reactance of the armature, the result being then termed the total or synchronous reactance. More will be said of synchronous reactance later.

185. Regulation.—The regulation of an alternator at a given power factor is defined as the change in voltage from full load to no load, divided by the full-load voltage, both voltages being taken with the alternator running at rated speed; expressed in per cent, this may be represented by the equation

$$\text{Regulation} = \left(\frac{\text{no-load voltage} - \text{full-load voltage}}{\text{full-load voltage}}\right) \times 100.$$

If the external characteristics of an alternator have been experimentally determined as in Fig. 272, regulation may readily be calculated from the curves.

With a leading power factor the no-load voltage may be less than the full-load voltage; the regulation is then considered as negative.

186. Determination of Alternator Regulation.—The external characteristics of a small alternator may be determined by actually loading the machine, the amount of input power required, and the power output to be absorbed, being small. With large alternators, to actually load the machine before it is installed becomes a problem in itself, and is quite costly. Even after an alternator is installed it may be difficult to secure the desired balanced load.

The regulation of an alternator may be determined by methods requiring tests which involve little power, the results being sufficiently accurate for most purposes. In all such methods the armature reactance is increased by a sufficient amount to allow for the effect of armature reaction, because it is very difficult to separate these two effects.

Two methods of determining the regulation of an alternator will be considered, the e.m.f., or synchronous impedance method, and the A.I.E.E. method.

187. Electromotive Force, or Synchronous Impedance, Method.—In this method the generated voltage corresponding to a given terminal voltage, at rated load, is determined by the use of vector diagrams, as in Fig. 273. The IR drop is generally calculated from the conductor resistance increased by about 20 per cent, and the IX and IZ drops, now called the synchronous reactance and impedance drops, are obtained by special tests. The terminal voltage is the rated voltage of the machine.

To determine the synchronous impedance, the saturation curve of the machine is first determined in exactly the same way that the saturation curve of a c.c. machine was obtained; this curve expresses the relation between generated voltage and field current, being obtained with the alternator running at rated frequency with no load. After the saturation curve has been determined, the alternator field is reduced to zero, and the armature is short-circuited through ammeters, as in Fig. 274. With the machine still running at rated speed, the field is gradually strengthened, and a curve between armature current and field current determined, the armature current being increased to about 150 per cent of its rated value.

Such a "short-circuit current," or "synchronous impedance" curve is shown in Fig. 275.

With the armature short-circuited, to cause rated current, *AB*, to flow, requires a field current *OA*, but if the short-circuit is removed from the armature with the machine still running at rated frequency and the field current *OA* maintained, the generated voltage will be *CA*.

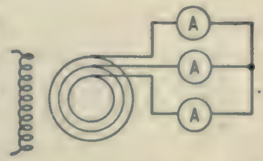


Fig. 274.—In getting the synchronous impedance of a three-phase machine the excitation of the generator is reduced to a suitably low value, and the armature short-circuited through three ammeters.

If it is assumed that the voltage *CA* was generated by the armature while short-circuited, then all of it must have been used up in overcoming the impedance of the armature. Thus we may calculate the synchronous impedance, per phase, Z_s (called "synchronous" because the impedance is determined while the machine is running at synchronous speed), by dividing the open-circuit voltage at a given field current by the short-circuit armature current at the same value of field current, that is

$$Z_S = \frac{CA}{BA}.$$

Knowing the effective resistance of the armature, per phase, we may determine the synchronous reactance, X_S since

$$Z_S = \sqrt{R_{\text{eff}}^2 + X_S^2}.$$

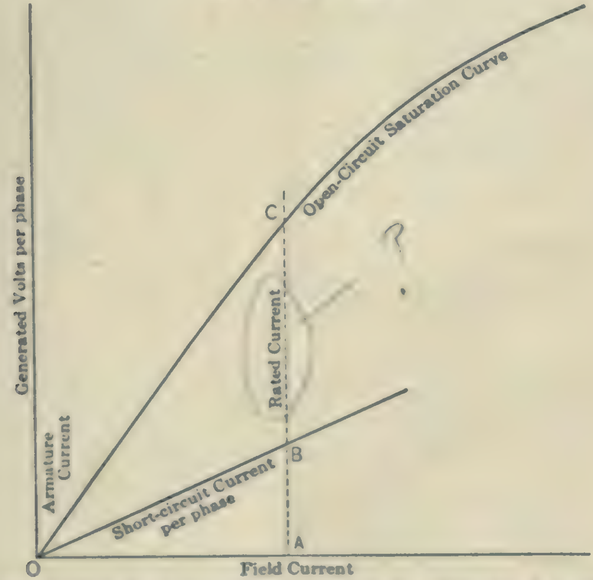


Fig. 275.—Two of the important characteristic curves of an alternator.

The value of synchronous reactance, as determined by the method just outlined, is larger than the actual synchronous reactance of the armature under operating conditions. The field current required to force rated current through the short-circuited armature is only about 40 per cent of that required to generate rated voltage at no load; the main field is therefore relatively weak. In normal design the reactance of an armature is generally many times its resistance, so that when the armature is short-circuited, the armature current will lag behind the generated voltage by an angle between 70° and 90°, and will therefore cause armature reaction which will strongly demagnetize the main field, which, as we have just seen, is weak to begin with. The value of the voltage actually generated when the armature is short-circuited is therefore much less than the value AC, and the synchronous impedance determined by dividing the voltage AC by the current BA is therefore larger than the actual impedance. The regulation calculated by using the synchronous impedance as determined is therefore pessimistic, i.e., the method gives higher regulation than that actually obtained by test. The method is, however, useful in comparing machines. If the regulation of one machine is actually known, another similar machine may be compared with the first by determining their regulations by the e.m.f. method.

To illustrate the method, let us consider a 100-kv.a., 1100-volt, three-phase alternator. When the armature is short-circuited it requires a field current of 25 amperes to force rated current $\left(\frac{100{,}000}{1100\times\sqrt{3}}=52.5\right.$ amperes per line) through the armature. With the short-circuit removed and the field current still 25 amperes, the alternator generates 420 volts between lines. The effective resistance of the armature is 0.3 ohm between lines.

Not knowing whether the machine is Y- or Δ-connected, let us assume the former; either assumption will give the same regulation. The open-circuit voltage per phase is then $\frac{420}{\sqrt{3}}=242$ volts, and the synchronous impedance per phase

$$Z_S=\frac{242}{52.5}=4.62 \text{ ohms per phase.}$$

The resistance per phase is $\frac{0.3}{2}=0.15$ ohm, and the synchronous reactance per phase, we find as

$$X_S=\sqrt{4.62^2-0.15^2}=4.61 \text{ ohms per phase.}$$

To determine the regulation with a power factor of $0.8(\phi=36.9°)$, lay off the vector E_t in Fig. 273*a*, equal to the rated voltage per phase, $\frac{1100}{\sqrt{3}}=635$ volts, and the vector I to represent the current per phase at full load, lagging 36.9° behind E_t. Lay off the full-load IR drop per phase equal to $0.3\times 52.5=16$ volts, and the full-load IX drop per phase equal to $4.61\times 52.5=242$ volts. The voltage generated at full load is then

$$E_g=\sqrt{(E_t+IR\cos\phi+IX\sin\phi)^2+(IX\cos\phi-IR\sin\phi)^2}$$

$$=\sqrt{(635+16\times 0.8+242\times 0.6)^2+(242\times 0.8-16\times 0.6)^2}$$

$$=815 \text{ volts per phase, or } 815\times\sqrt{3}=1410 \text{ volts between lines.}$$

$$\text{Regulation}=\frac{1410-1100}{1100}=\frac{310}{1100}=28.2 \text{ per cent.}$$

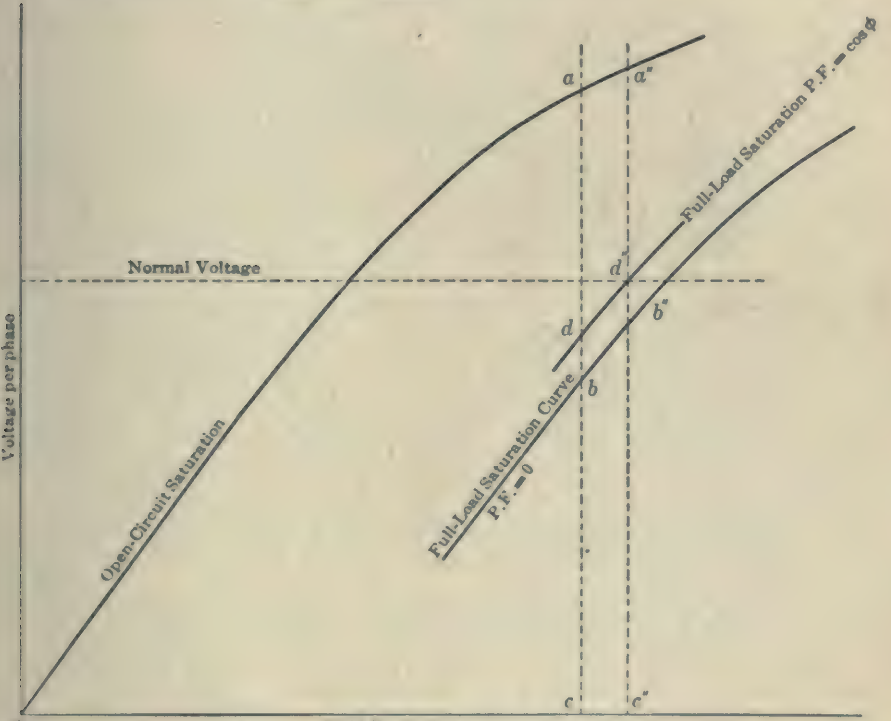


FIG. 276.—Necessary curves in using the A. I. E. E. method for predicting the external characteristic of an alternator.

188. A. I. E. E. Method.—This is the method recommended by the American Institute of Electrical Engineers; although an empirical method, it gives results for regulation that are close to the actual, especially for lagging power factors.

The saturation curve of the machine is first determined. Then one or more suitable, unloaded synchronous motors are operated from the alternator at voltages from about 50 to 150 per cent of the rated voltage of the alternator, the synchronous motors being adjusted at each reading to draw rated current from the alternator at lagging power factor as nearly zero as possible. (An unloaded synchronous motor with a weak field will draw a lagging current with a very low power factor; this will be explained in the chapter dealing with synchronous motors.) A curve plotted between field current and terminal voltage of the alternator, with the alternator carrying rated armature current at very low power factors, is called the full-load, zero-power factor saturation curve; this curve, together with the open-circuit saturation curve, is plotted in Fig. 276.

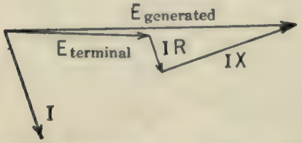


FIG. 277.—Vector diagram of an alternator supplying a load of low power factor.

For a field current *oc* on open circuit, the armature generates a voltage *ca*; at full load and a lagging power factor equal to zero, the terminal voltage falls to the value *cb*, the drop in voltage being due to the combined effects of armature resistance, armature reactance, and armature reaction. A vector diagram drawn for the alternator when supplying full-load current at a very low power factor is shown in Fig. 277; it will be seen that the effect of *IR* drop on the terminal voltage is very small and that the decrease in voltage is practically the synchronous reactance drop, that is, the reactance drop increased by the effect of armature reaction.

Having the saturation curve at no load, and at full load with zero power factor, we next construct a full-load saturation curve, at the power factor for which the regulation of the machine is desired; assume this to be $\cos \phi = 0.8$.

To calculate the full-load terminal voltage of the alternator at a power factor of 0.8 when supplied with a field current *oc*, we proceed as in Fig. 278. First lay off the dotted horizontal line, then an indefinite line, $b'f'$, making an angle ϕ ($\cos \phi = 0.8$) with the dotted line. From b' lay off the distance $b'e'$ equal to the full-load *IR* drop per phase and erect a perpendicular at e'. With b' as a center and with a radius equal to the voltage *ab* in Fig. 276, locate the point a', and with a' as a center and a radius equal to the open-circuit voltage *ca* in Fig. 276, locate the point c'. The distance $c'b'$ is the desired full-load terminal voltage per phase, with a power factor, $\cos \phi = 0.8$.

By drawing a line $c'g'$ parallel to the dotted line through the point c', the significance of Fig. 278 becomes apparent; the line $c'g'$ indicates the phase of the current, lagging $\phi°$ behind the terminal voltage $c'b'$; $b'a'$ is the full-load impedance drop, and $c'a'$ is the generated voltage.

The voltage $c'b'$ of Fig. 278 is now laid off as *cd*, along *ca* in Fig. 276, since it is the terminal voltage (per phase) at full load with a field current *oc*. By similar construction a sufficient number of points of the desired full-load saturation curve, at a power factor $\cos \phi = 0.8$, may be obtained. The curve indicates that at rated load and power factor 0.8, the required field current to give rated terminal voltage is oc''. The regulation of the machine is then the voltage $a''d''$ divided by the rated voltage $d''c''$.

In large alternators, where the resistance drop is relatively very small, it may be neglected; in this case Fig. 278 appears as Fig. 279. If it is impossible to experimentally determine the full-load, zero-power factor saturation curve, this curve may also be constructed. The student is referred to the standards of the A. I. E. E., which may be found in any good electrical handbook.

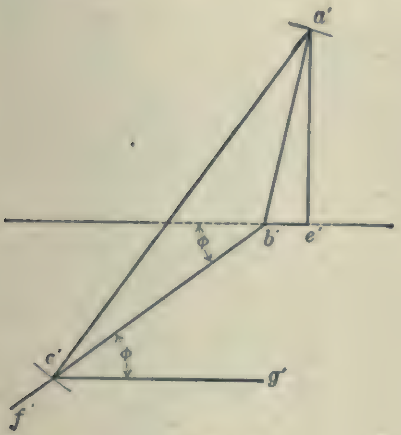


FIG. 278.

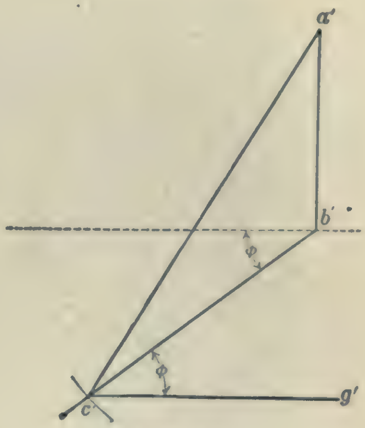


FIG. 279.

FIG. 278.—Showing how to get the terminal voltage of the alternator, at full load, for any power factor. The dimensions of the vectors used in constructing this figure have been taken much smaller than the values of Fig. 276, from which they were obtained.

FIG. 279.—In large alternators the resistance drop is generally very small compared to the synchronous reactance drop; by neglecting the IR drop the diagram of Fig. 278 is simplified, as shown here.

189. Short-circuit Current.—When an alternator generating voltage is suddenly short-circuited, it will deliver for an instant a current two or three times as great as will flow after conditions have become constant. When the short-circuit is first applied, only the resistance and inherent, or true, reactance of the armature winding limit the current, as it takes an appreciable period of time for the increased armature reaction to weaken the field. The highest value of the short-circuit current, for a given field excitation, is called the *instantaneous*, or *momentary short-circuit current*, and as this may reach such dangerously high values as to cause damage to the windings, circuit-breakers, etc., it is customary to put reactors in series with the alternator leads, between the machine and the switchboard.

Current-limiting reactors consist usually of turns of heavy copper cable or strap, set in concrete (Fig. 280), and may produce a reactance drop as high as 20 per cent of the rated alternator voltage when carrying rated alternator current.

As soon as armature reaction has weakened the field and conditions have become steady, the value of the short-circuit current is called the *steady* or *sustained* short-circuit current. The value of the sustained short-circuit current may be from three to ten times (without series reactors) the rated full-load current, depending upon the design of the machine; ordinarily it is about five times the rated full-load current.

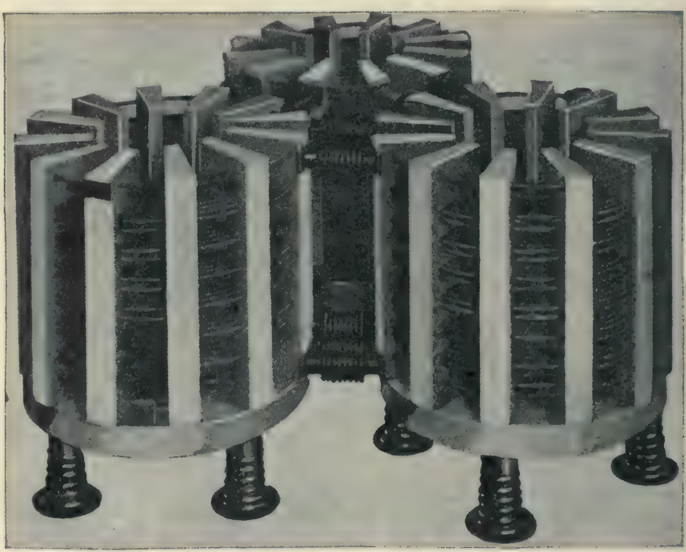


FIG. 280.—A group of reactors for limiting the short-circuit current of a three-phase alternator. They are made of coils of copper cable held firmly in place by concrete blocks; the three reactors are securely fastened to one another by suitable insulators.

If the synchronous impedance of the armature is known, the sustained short-circuit current can, of course, be approximately calculated. A certain machine generates 70.4 volts per phase if it is excited to give its rated terminal voltage, 110 volts between lines, when carrying full-load current of 50 amperes. The synchronous impedance per phase of the armature is measured and found to be 0.231 ohm. Now if the external circuit resistance is reduced to zero (i.e., the machine is short-circuited) all of the generated voltage must be used up in *overcoming the synchronous impedance drop in the armature.* So that we may put

$$E = IZ;$$

where E is the generated voltage per phase and Z is the synchronous impedance per phase. We than have

$$70.4 = I \times 0.231$$

from which

$$I = 305 \text{ amperes.}$$

The sustained short-circuit current of this machine is, therefore slightly more than six times the full-load, rated current.

190. Capacity of the Exciter.—The exciter for an alternator must be designed to carry safely the current required under the worst condition, i.e., overload, with a lagging current. Thus the exciter for the alternator whose curves are given in Fig. 272 should be designed to carry the current oS, although with a normal non-inductive full-load current, the required field current is only oT.

191. The Voltage Regulator.—Commercial loads always have a lagging current, and so if the excitation of an alternator were left unaltered as the load increased, the terminal voltage would fall to a considerable extent. It is desirable with most pieces of apparatus to which power is supplied (lamps, motors, transformers, etc.), to maintain a constant voltage at the *piece of apparatus*.

This necessitates an *increase* in the terminal voltage of the generator supplying the power, to overcome the increased line drop with increased load, but if left to itself the alternator would give a *decrease* in terminal voltage. Hence it is necessary either to have the operator vary the alternator field current as the load is increased and decreased, or else to have some automatic device to regulate the alternator field.

The Tirrill regulator (named after its inventor) serves the purpose by controlling a resistance in the field of the exciter furnishing the alternator field current. A cut showing this ingenious device is given in Fig. 281 and a diagram of its connections is given in Fig. 282, by reference to which its action will be described. A diagram of the exciter with its field rheostat and its connection to the alternator is given, to make the operation of the regulator easier to understand.

Essentially, the regulator consists of three magnets, A, A', and F and two pairs of contacts C, C' and E, E', which are opened and closed by these magnets.

The operation of the differential relay magnet F will first be described. Its winding consists of two coils, H and H', both of which are excited from the terminals of the exciter. One of them, H, is excited all the time the exciter is running, while the other H' is excited only when the two contact points C and C' make connection with each other. These two coils are differentially wound, so that when both of them are excited there is no pull on the armature S and the relay contacts E and E' remain closed by the action of the spring K, but when contacts C and C' are opened so that only coil H is excited the solenoid F is magnetized, the armature S is pulled down, and the contacts E and E' are separated.

The contacts E, E' are connected to the field rheostat of the exciter, *so that when they are together the rheostat is short-circuited but when they are apart the rheostat is in the field circuit* as it is ordinarily. Hence, when C and C' touch each other, the exciter voltage rises (due to cutting out the field rheostat) and when they separate the exciter voltage falls. When the regulator is in operation, the armature S is continually vibrating (due to actions described later) so that the exciter voltage is continually oscillating above and below its proper normal value.

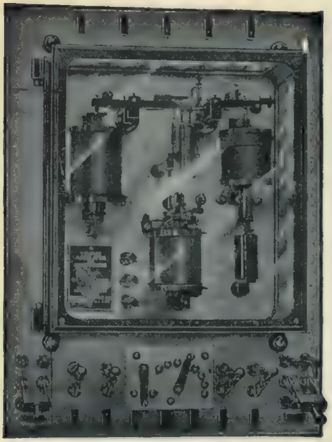


FIG. 281.—The voltage regulator originated by Tirrill, frequently called the Tirrill regulator.

Suppose now that contact C' remains in a fixed position; we shall examine the action of the solenoid A, which is connected across the exciter terminals. The plunger, P, is carried on a lever, L, and is held in its highest position, when A is not excited, by the spring B. The magnetic pull of A is about equal to that of the spring B. As the exciter voltage *rises and falls* therefore, the plunger, P, *falls and rises* accordingly and so the contacts C and C' are correspondingly opened and closed.

We have seen that the operation of the contacts E, E' is effected by the opening and closing of the contacts C, C'. Hence the combined action of the relay magnet and exciter magnet is to maintain the exciter voltage constant (and hence maintain the generator field current constant). For suppose (after the regulator has been properly adjusted) that the exciter voltage rises above normal. The plunger P is pulled down, opening the contacts C, C', thus (by the action of the relay magnet) opening the contacts E, E'. This inserts the rheostat in the exciter field, by opening the short-circuit path in parallel with the rheostat.

This action will decrease the field current of the exciter, and therefore will decrease its voltage. The magnetic pull on the plunger P is thus lessened, and it rises because of the action of the spring B. This closes the contacts C, C', closing the contacts E, E' and so short-circuiting the field rheostat once more. The exciter voltage again rises and the process is repeated. This action keeps the armature S in continual vibration at a rate of several vibrations per second.

Now consider the action of the generator solenoid A'. Its winding is connected across the terminals of the generator (through a potential transformer for high-voltage machines) and the action of its magnetic pull is to *raise plunger* P'. The plunger P' is suspended from a lever L' and counterbalanced to any desired degree by the adjustable weight D. When the regulator is in adjustment and the voltage of the generator is normal, P' (and hence C' which is carried on the same lever as P') occupies a certain mean position. Voltage on the alternator above normal will lift P' above its normal position and so depress C' below its normal position. A decrease in alternator voltage will have the opposite effects.

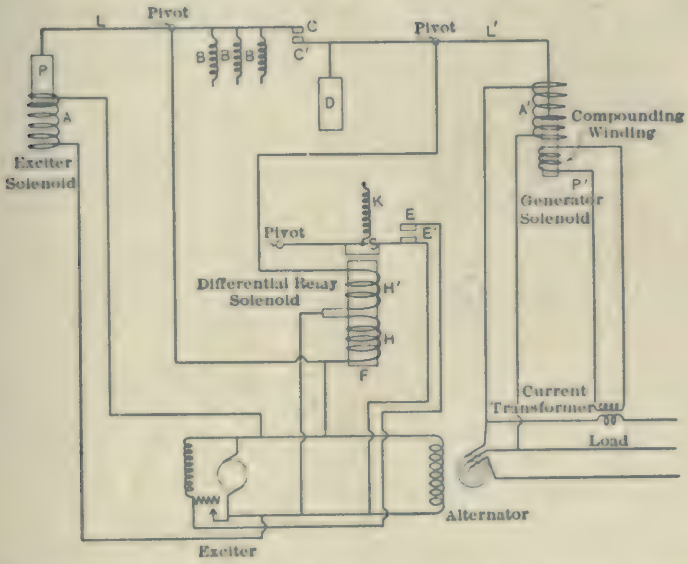


FIG. 282.—Simplified circuit diagram of the Tirrill regulator; suitable condensers and resistances are used across the breaks to eliminate sparking.

But if C' is lowered it will require less pull on P to make the contacts C, C' open, so they will open on less than normal exciter voltage, and when they open the exciter voltage falls as explained before. But if the exciter voltage falls, a corresponding decrease in the alternator voltage must occur. Hence the total action of the regulator is to bring the alternator voltage back to normal if it tends to rise; also if the alternator voltage falls, an increase in the exciter voltage (and hence in the alternator voltage) must take place.

The Tirrill regulator has been so perfected that it is possible by its use to maintain the voltage of an alternator *constant to within one-quarter of one per cent.*

It is often desirable to have the alternator voltage *increase with the load* so as to overcome the line drop and give constant voltage at some distant point on the line. This is easily done by winding on A' another coil and connecting it to a current transformer in the generator line, the connection to be so made that the action of this second winding is to neutralize the other. By properly proportioning this second differential winding it is possible to obtain any degree of compounding desired.

There are several auxiliary features to the regulator which we have not considered, such as condensers across the contacts, reversing switches for the contacts, the amount of resistance required in the field rheostat for proper operation, etc. These may be found in trade publications describing the regulator.

192. Alternators Running in Parallel.—Electrical power for a city or community can be generated most efficiently when all the generators supplying the power are located in one central power station. The cost of electrical power may be divided into several parts: coal, water, oil, station attendance, repairs, line maintenance, interest on investment, and various other costs grouped under the title of "overhead" charges. These are made up of office rent, salaries of clerks, executive officers, taxes, etc.

Practically all of these charges may be reduced if one power station, instead of several, supplies all the power. But one staff of executive officers need be employed, but one set of books need be kept, and but one office occupied; the cost for station attendance will not be as great as if several stations were used, etc. Also, larger generators will be used in the larger station than in the smaller ones, and we know that large generators have a higher efficiency than small ones.

The use of electrical power is becoming so general that for large cities perhaps 200,000 kw. of power may be required to supply all the demands. A generator to supply 100,000 kw. has never been built; the largest built so far has a capacity of about 50,000 kw. Hence to supply 200,000 kw. it would require several of these big machines and *they would all be operated in parallel on the same bus-bars.* In some large stations, as many as ten or more alternators have been installed to supply the load and they all operate in parallel.

193. Number and Capacity of Machines.—The considerations which would govern the selection of the number and size of machines for any given station are the same as those discussed in Volume I in the study of c.c. machines operating in parallel, but they may be reviewed here. In general *it is best to have the machines in a station of the same size and type,* because only a few spare parts need be kept in stock for making repairs. Coils are likely to burn out at any time and a few should always be kept in the station for making quick repairs in case of necessity. If all machines are of the same size and type, only one set of coils need be kept on hand; but if several sizes are installed in a station, a proportionally greater number would have to be kept in stock. Also, if the same size machines are installed there always exists the possibility of interchanging parts from different machines if necessary.

In any ordinary station there should be, however, at least one small-sized machine for carrying the load during the early hours of the morning when it is very light.

194. Parallel Connection of Alternators always Used.—Assuming, then, that there are several alternators in a station to be operated on the same bus-bars, the question arises as to how they shall be connected together, in series or parallel. Practically all power and lighting systems are designed for operation at *constant voltage* irrespective of the load. If the alternators were connected for series operation it is evident that the voltage on the buses would vary directly with the number of alternators operating. If for example three machines were operating in series, the bus-bar voltage would be three times as high as if one only machine were operating. *Hence it is evident that series connection of alternators would not satisfy the requirement of constant voltage.* But even if it were all right to use the series connection in so far as the load was concerned, it could not easily be done because *alternators operating in series are in unstable equilibrium*, unless the machines are mechanically coupled together. If the two alternators (not mechanically coupled) are connected in series to a load, the voltage on the load is likely to continually vary from zero to twice that of one alternator. It is not worth while to further analyze this point here as it has no commercial importance.

In considering alternators operating in parallel we have three points to investigate. What precautions are necessary in connecting an alternator to bus-bars already alive? What will happen if the *generated* voltage of this machine is made greater or less than that of the bus-bars to which it is connected? (We emphasize the word generated because, of course, the terminal voltage of the alternator cannot be made different from that of the bus-bars to which it is connected.) How may the load be divided between the different alternators, as desired by the operator? These three points will be taken up in the order named.

195. Synchronizing.—Suppose several alternators are already operating in parallel on the same bus-bars and another alternator is to be connected to these same bus-bars; what conditions must be fulfilled before the switch, connecting this *incoming machine* to the bus-bars, may be closed? They may be briefly stated thus:

1st. *A machine voltage equal to the voltage of the line (bus-bars).*

2d. *A frequency for the machine which is practically the same as that of the line.*

3d. *The phase of the machine voltage opposite to that of the line.*

4th. *A machine e.m.f. wave form similar to that of the line.*

The first three conditions can be satisfied by various manipulations which the operator can carry out, but the fourth cannot be accomplished by any variation of speed, field excitation, etc.

The e.m.f. wave form of an alternator is fixed by the design of the machine and cannot be affected by the operator; however, as practically all modern machines give wave forms very nearly sinusoidal, the fourth condition is nearly always satisfied to a degree sufficient for successful parallel running.

The operation of manipulating the incoming machine to bring about the first three conditions and of closing the switch which connects it to the bus-bars is called *synchronizing*. It is evidently easy for the operator to satisfy the first two requirements; the voltage of the incoming machine may be varied by its field rheostat, and its frequency (depending directly upon the speed) may be altered by changing the speed of its prime mover. To satisfy the third condition a *synchronizing device* may be used; *either incandescent lamps, properly connected between the machine and bus-bars, or a synchronoscope* (generally called a *synchroscope*) may be used.

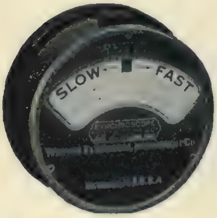


Fig. 283.—The oscillating synchroscope.

196. The Synchroscope.—The synchroscope is an indicating instrument having a dial over which a finger rotates or oscillates. In the better type (illustrated in Fig. 283) the finger merely oscillates back and forth, while the type shown in Fig. 284 has a circular dial and the finger rotates over the complete circle. In the oscillatory type the finger actually seems to rotate, instead of oscillating, because of the peculiar lighting used behind the dial.

As described in Chapter V, in either type there are two independent electrical circuits, one of which is connected to the bus-bars and the other to the incoming machine. The magnetic fields set up by these two circuits so react on one another as to make the needle either oscillate or rotate; a mark on the dial shows the proper "synchronizing" position for the needle, and the synchronizing switch must not be closed until the first two conditions have been satisfied and the synchroscope needle points to this mark.

197. Use of Lamps Instead of a Synchroscope.—Figure 285 shows how lamps may be used on low-voltage machines. If the machine were a

110-volt alternator, three 110-volt lamps would be used; with a 220-volt alternator, 220-volt lamps would be proper, or, of course, two 110-volt lamps in series might be used instead of each 220-volt lamp.

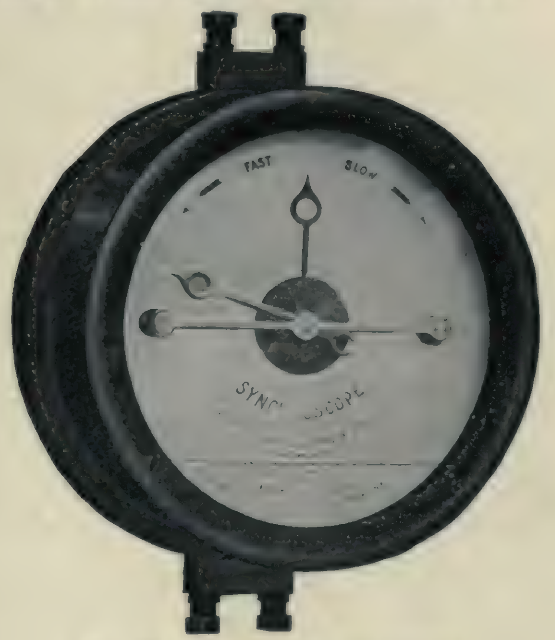


Fig. 284.—The rotating synchroscope.

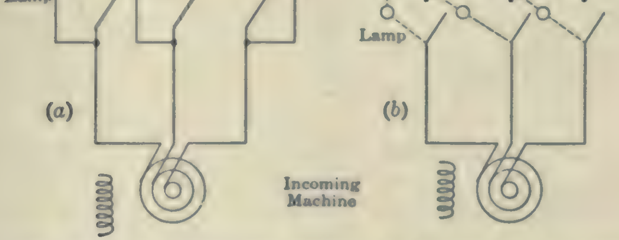


Fig. 285.—Two methods of connecting synchronizing lamps; in (*a*) the switch is closed when the lamps are dark, and in (*b*) when they are bright.

In synchronizing, either with lamps or a synchroscope, the incoming machine is brought up to approximately its proper speed and then its voltage is adjusted until equal to the bus voltage. In case a synchroscope is used it is then "plugged in" and it will rotate, in one direction if the incoming machine is slow and in the opposite direction if it is running too fast. The number of revolutions per second gives the *difference in frequency* between the incoming machine and the line. In case lamps are used they will flicker, being alternately bright and dark; *the number of flickers per second* gives the difference in frequency of the incoming machine and line.

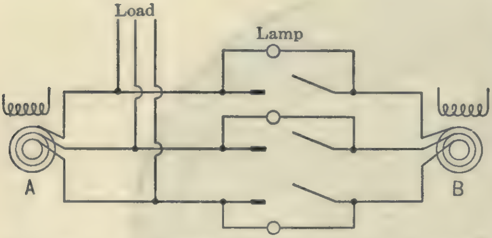


FIG. 286.—The synchronizing lamps form part of a closed circuit, involving the two alternator armatures in series.

198. The "Dark" and "Bright" Connections of Lamps.—If the lamps are connected straight across the switch blades (as shown by the full lines in Fig. 285*a*), the proper time for closing the synchronizing switch is in the middle of a dark period, while if they are connected zigzag across the switch (as in the dotted lines of Fig. 285*b*), the proper time for closing the synchronizing switch is at the center of a bright period.

The reason for the flickering of the lamps may be seen by reference to Fig. 286. Machine *A* is already supplying load and machine *B* is to be synchronized with it. The *lamps form part of a closed local circuit* made up of the two armatures and the lamps in series.

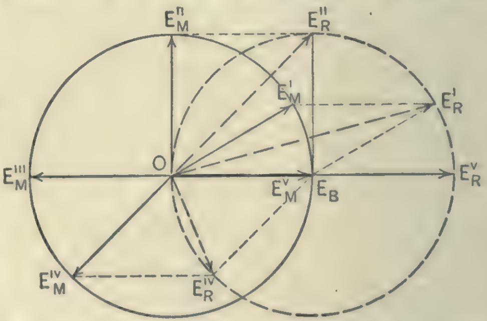


FIG. 287.—As one of the alternators of Fig. 286 pulls ahead of the other, the voltage effective in forcing current through the lamps, goes through the series of values indicated for E_R of the figure.

If the incoming alternator is running at a higher frequency than the bus, its voltage vector will rotate faster than that of the bus. Consider the vector representing the bus voltage, E_B, as stationary (Fig. 287), and the vector representing the incoming machine voltage, E_M, as rotating about it at a speed equal to the difference in frequency of the incoming machine and the bus-bars. When the machine voltage is in a position E'_M, the resultant voltage across the two synchronizing lamps ("dark" connection) is the vector sum of E'_M and E_B, or E'_R; as E_M continues to move ahead of E_B, the resultant E_R decreases. When the machine voltage E'''_M is equal and opposite to the bus voltage, the resultant voltage, E'''_R, is zero; at this instant there is no voltage across the lamps, so that they are dark, and the finger of the synchroscope is over the proper mark. If, at this instant, the difference in the machine and bus frequencies is sufficiently small, the machine switch may be closed. Figure 287 indicates that the locus of the resultant voltage across the lamps is a circle of radius equal to $E_B = E_M$, the center of which is at the head of the vector E_B.

199. Effect of Phase Rotation on Synchronizing Lamps.—In the laboratory it frequently happens that when two machines not permanently connected are to be synchronized, the lamps instead of becoming simultaneously bright and dark, so that all are bright or dark together, brighten and darken in rotation, at least one of the lamps always being bright. This is due to the fact that the phase rotation of the two machines is not the same; thus if phases A, B, and C of each machine are connected through lamps, and the phase rotation of one machine is A, B, C, and that of the other is A', C', B', the lamps will not brighten and darken together. Evidently the voltage of the phases as connected cannot have relatively the same values at the same instant. If the voltage of phase A were equal and opposite to that of phase A', that of phase B would be at its 120° position, and B' at its 240° position; although the instantaneous values are numerically equal, their phase positions are such as to produce a resultant across the lamps joining the phases.

In stations, after an alternator has been once properly installed, it will always come in with proper phase rotation, so that a synchroscope, or two lamps, placed across two of the three switches is sufficient, but the best practice is to use both a synchroscope and three lamps. When a newly installed alternator is first synchronized, however, proper precautions must be taken to insure its proper phase rotation and connections.

200. Lamps Used with High-voltage Machines.—When the machines are of high voltage, lamps cannot be used directly on the line, but small transformers must be used. If three synchronizing lamps are desired, each may be supplied from the secondary of a small transformer, the primary of which is connected across one of the switches.

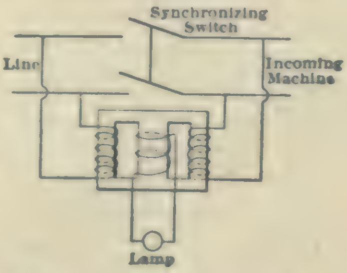


FIG. 288.—Instead of using two or more lamps for synchronizing purposes, one lamp only may be used, provided it is connected properly to a transformer, as shown here.

Figure 288 shows one scheme for using one transformer and one lamp, with a single-phase machine. The coils on the outside legs of the transformer core are so connected that when the two e.m.fs. are in phase opposition, practically no flux goes through the center leg, hence the lamp is dark; and when the two e.m.fs. are 180° out of their proper phase relation, the flux from both outside legs goes through the center leg and so the lamp glows.

201. Sequence of Operations in Synchronizing.—To synchronize the incoming machine, then, the proper sequence of operations is as follows: The alternator is brought up to approximately its proper speed and then its excitation is put on and the voltage is adjusted to approximately equal that of the line; the synchronizing device is then put in the circuit and the speed is again adjusted until the difference in frequency is very small, say, ten cycles per minute (this figure will vary with different operators and different machines). A final adjustment of the voltage is now made to bring it within one or two per cent of that of the line and, when the synchronizing device indicates the proper phase relation, the synchronizing switch is quickly closed. If the operator makes a bad "shot," i.e., closes the switch either too soon or too late, an excessive current will result in the armature and the protective devices will open and then another attempt must be made.

This discussion of synchronizing holds good for any synchronous machine, either alternator, synchronous motor, or synchronous converter.

202. Circulating Current.—Immediately after the alternator has been synchronized, *it takes no load at all;* in this respect it is similar to a c.c. machine which has just been paralleled with the station bus-bars. When c.c. machines are operating in parallel and it is desired to have one of the machines take more load, the voltage of this generator is raised and *it will increase its load in proportion to the increase in its generated voltage.*

If it is desired to make the incoming alternator take load, the natural thing to do is to increase its generated voltage by cutting out some of the field rheostat, as is done with a c.c. generator. *But such a procedure does not make the alternator take load at all;* the generator ammeter indicates a current increasing almost proportionally to the increase in excitation, but the generator output wattmeter gives scarcely any reading.

Figure 289 shows the alternator connected to the bus-bars through its ammeter and wattmeter and switch. Increasing the excitation increases the reading of A but does not make W read to any extent. Hence this current indicated by A must be a *reactive current,* 90° out of phase with the terminal voltage of the generator. The reactive current circulates in the local circuit between the various armatures and does not go into the load circuit at all; it is therefore called a *circulating current.*

The reason for this circulating current may be seen by reference to Figs. 290 and 291. Let us suppose that only two alternators in parallel are supplying some power to a load circuit; also that No. 2 has just been synchronized and so is furnishing practically none of the power to the load. Consider one of the local circuits as made up of two phases of each and the bus-bars, all in series with each other. If the voltage of No. 2 is just the same as that of the bus-bars (this condition is obtained before synchronizing), there is no resultant voltage in the local circuit and no current flows around this circuit. The vector diagram for this condition is shown in Fig. 291, where OE_1 gives the phase voltage of machine No. 1 and OE_2 gives that of machine No. 2. Evidently the resultant of these two e.m.fs. is zero.

203. Effect of Increasing or Decreasing the Voltage.—If the voltage of No. 2 is increased to OE_2', there is a resultant voltage per phase, OR', acting in the local circuit. As the armature circuits are highly inductive, the resultant phase voltage, OR', will produce a current nearly 90° behind it in phase, shown in Fig. 291 by OI', of magnitude

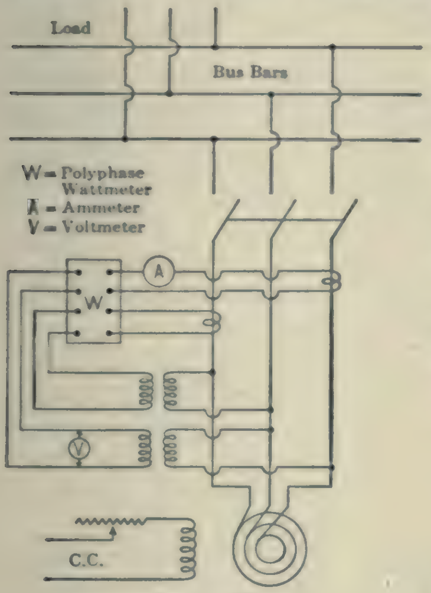


FIG. 289.

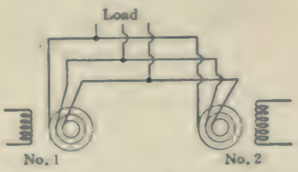


FIG. 290.

FIG. 289.—If the excitation of the alternator is increased, *A* will proportionally increase its reading, whereas *W* will show practically no change in power output.

FIG. 290.—To show the effect of increasing the field current in Fig. 289, consider the two armatures connected as shown here.

$$OI' = \frac{OR'}{\text{phase impedance of both armatures}}$$

and lagging behind OR', by the angle θ, where

$$\tan\theta = \frac{\text{reactance of both armatures}}{\text{resistance of both armatures}}.$$

The current OI' is practically 90° behind the phase of OE'_2; hence it does not represent a load on machine No. 2. Also it is practically 90° ahead of OE_1, and so does not represent a load on machine No. 1. It is merely a reactive current, circulating between the two armatures and representing no appreciable power in either of them.

If the voltage of machine No. 2 is decreased to OE''_2, the resultant voltage is OR'' and the current is again reactive, lagging nearly 90° behind OR''. It is now opposite in phase to what it was before, as shown at OI'' in Fig. 291; it now leads OE_2 and lags behind OE_1. This circulating current is really a magnetizing current for one machine and a demagnetizing current for the other. When No. 2 is over-excited (i.e., excited to give a voltage higher than that generated by No. 1) the current OI' tends to demagnetize No. 2 and magnetize No. 1, while current OI'' tends to magnetize No. 2 and demagnetize No. 1

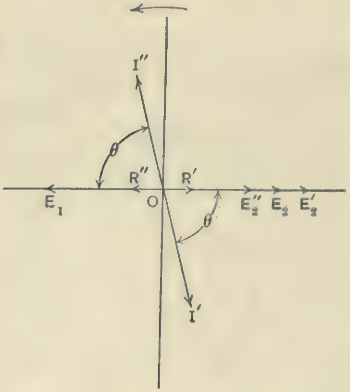


Fig. 291.—Vector diagram of voltages of the circuit given in Fig. 290.

This circulating current, of course, heats the armatures and so decreases the safe current which the machines can furnish to the load circuit because the total armature current is made up of the vector sum of the load current and the circulating current.

Summing up these ideas about circulating current, we may say that if the voltage of one generator (operating in parallel with others) is altered *it does not appreciably affect the load division between the different machines, but produces a circulating current which unnecessarily heats the armatures and tends at the same time to so react on the fields as to equalize the generated voltages of all machines.*

204. Division of Load.—The next question which arises is this: If the load division between two alternators (operating in parallel on the same bus-bars) is not affected by a variation of the field excitation, how can the load be shifted from one machine to the other? It may be shown both experimentally and theoretically that if two machines are sharing a load equally and it is desired to have machine No. 2 take more load, *this can be accomplished only by increasing the torque of the steam engine (or turbine) driving machine No. 2.*

To analyze the question let us suppose that two alternators are operating in parallel, supplying power to some outside load and that the load is equally divided between the two machines. Under such conditions the e.m.fs. of the two machines are exactly opposite in phase. We further assume that the two e.m.fs. are equal, so that there will be no reactive circulating current between the two machines. The two e.m.fs. are shown by OE_1 and OE_2 in Fig. 292.

If the prime mover of No. 2 is given more steam it will, of course, try to speed up generator No. 2. *But this can occur for only a small fraction of a second, until No. 2 has pulled slightly ahead of No. 1 in phase;* the two machines must turn the same number of revolutions per minute (same number of poles on both machines assumed) because they are operating *in synchronism* with each other, so that the increased speed of No. 2 can only last during that small time while No. 2 is pulling ahead of No. 1 in phase.

After No. 2 has pulled ahead in phase, its e.m.f. vector is represented in Fig. 292 by OE'_2, which is ahead of its original phase by the angle α. Now the resultant of OE_1 and OE'_2 is not zero, but OR. This voltage acting in the local circuit will force a current through the two armatures and this current is shown in Fig. 292 by OI, lagging behind OR by the angle θ, the angle θ being the same as it was in Fig. 291.

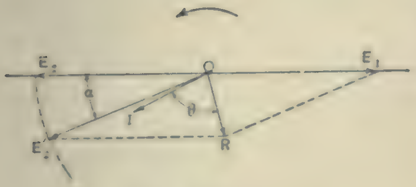


FIG. 292.

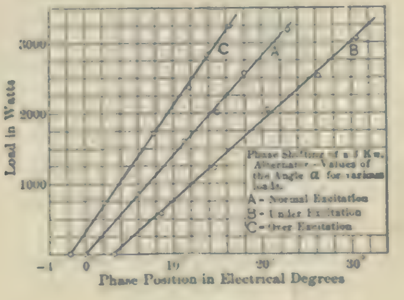


FIG. 293.

FIG. 292.—If the torque of the prime mover of machine No. 2 is increased (by giving it more steam), alternator No. 2 will pull ahead of its previous phase, giving a resultant current shown at OI. This represents increased load on the machine with increased torque, thus taking load away from the other machine.

FIG. 293.—Experimentally determined curves showing how the power which an alternator furnishes to the bus bars, increases with phase advance. For higher excitation smaller phase shift is required for a given load increase.

The current OI is nearly in phase with OE'_2 and nearly 180° out of phase with OE_1. *The result of this current is to add to the load No. 2 was already carrying, a load represented by the current OI, and to decrease the load No. 1 was carrying by nearly the same amount.*

The Load Varies Directly with the Phase Shift. The value of OR (hence of OI) depends upon the value of the angle α (Fig. 292) and this angle depends upon the torque of the prime mover of machine No. 2. To change the load on one of the alternators from zero to full load it is only necessary to change α from zero to about 20°, and for these small values of α it is approximately true that the load on the alternator is directly proportional to the angle α. Experimental results, illustrating this point, are given in Fig. 293. It is shown there that for a given load increase a somewhat smaller change in α is required when the generator is over-excited than when it is under-excited.

The electrical operator in a large power plant generally has control of the steam supply to the various prime movers so that he can, by variation of the supply, shift the load from one alternator to another as he desires.

205. Synchronizing Power.—It sometimes happens that two alternators will not operate satisfactorily in parallel. The division of load is not constant; the load shifts quickly from one machine to the other and back again by reason of periodic fluctuations of the angle α in Fig. 292, so-called "*hunting.*" The excessive current between the two machines, as the load shifts from one to the other, may be sufficient to open the circuit-breakers and so throw the two machines out of step. Such machines are said to have a low *synchronizing power*, whereas machines which operate smoothly, maintaining a fairly uniform division of load, are said to have a high synchronizing power.

PROBLEMS

1. A two-pole, single-phase alternator has 24 full-pitch coils equally spaced on its armature. Each coil generates a maximum voltage of 65 volts. What will be the effective voltage rating of the generator, if the coils are properly connected for a single-circuit winding? A two-circuit winding? If each coil can safely carry 10 amperes, what is the current rating and kv.a. rating for each connection?

2. If the winding of Prob. 1 is connected for a two-phase winding (independent circuits) with two groups of coils per phase, what will be the voltage, current, and kv.a., rating? If connected for a two-phase mesh winding?

3. If the winding of Prob. 1 is properly arranged for a Y-connected, three-phase scheme (one group of coils per phase), what will be the voltage, current, and kv.a., rating? What if connected Y, with two groups of coils per phase? Answer the same questions if the winding is arranged for the delta connection.

4. Determine the frequency in cycles per second given by a ten-pole alternator turning 720 r.p.m. If a generator is to be direct connected to a reciprocating engine turning 90 r.p.m., how many poles must it have to deliver 60-cycle current? How many poles must an alternator have to generate 25-cycle current when driven by a turbine running 750 r.p.m.?

5. A single-phase alternator rated at 150 kv.a. delivers 75 kw. of power to an induction motor load of 0.8 power factor. How many 60-watt lamps can be supplied in addition before the machine is loaded up to its rating?

6. Lay out (on cross-section paper) the winding of Prob. 1, and construct the armature m.m.f. curve of the winding when carrying 10 amperes load.

7. Do the same thing for the winding connected two-phase, carrying the same current.

8. Construct the same diagram for the winding connected three-phase.

Check (from the diagrams) the relative maximum values of armature m.m.fs. for the three windings, with the relative values given in the text.

9. A three-phase generator is loaded with 150 kw. per phase of the load, at cos $\phi = 0.70$. How many kv.a. is it delivering? If the line voltage is 2200, what is the current per line?

10. A 125-volt, 250-kv.a., delta-connected alternator is reconnected in Y. What will be the new rating in volts, amperes, and kv.a.?

11. On short circuit a certain 60-cycle, 2200-volt, 450-kv.a. alternator requires 35 amperes field current to give a line current of 200 amperes. The magnetization curve shows a generated voltage between lines of 1140 volts with 25 amperes field current. What is the synchronous impedance per phase, if Y-connected and if Δ-connected? If the resistance per phase is 0.3 ohm, what is the reactance per phase, for the two cases?

12. A single-phase, 60-cycle generator has an armature resistance of 0.3 ohm and self-inductance of 0.007 henry. The terminal voltage at full load of 30 amperes, is 220 volts. What will be the no-load voltage if the load is non-inductive? Leading with cos $\phi = 0.8$? Lagging with cos $\phi = 0.7$? If the generated voltage is adjusted for 220 volts and the machine is short-circuited, what will be the current?

13. Three generators, one single-phase, one two-phase, and one three-phase, are each rated 100 amperes and 250 volts. What is the proper kv.a. rating of each?

14. A Y-connected alternator, the open-circuit voltage of which is 8600 volts between wires, is short-circuited and in each line there flows a current of 250 amperes. The resistance per phase is 0.3 of the reactance per phase. What is the impedance, resistance, and reactance of one phase?

15. A Δ-connected alternator supplies a Δ-connected, resistive load, each phase of the load absorbing 50 kw. at 1100 volts. What is the line current? If the alternator is reconnected in Y, what power will it supply to the load, the speed and excitation of the alternator being the same as before?

16. The magnetization curve of a 800-kv.a., 4400-volt, Y-connected alternator is given as follows:

Field amp.-turns. . .	2,000	4,000	6,000	8,000	10,000	12,000	14,000	16,000	18,000
Generated volts per phase.	960	1,900	2,600	2,950	3,160	3,340	3,640	3,500	3,620

On short-circuit, rated current is obtained when the field ampere-turns are 3400. The resistance per phase is 0.29 ohm.

By the A.I.E.E. method, and by the e.m.f. method, determine the regulation for loads of unity power factor and of 0.7 power factor (lag).

17. Two 60-cycle, single-phase alternators are in parallel on the same bus-bars; the first has an armature of 0.1 ohm resistance and 0.001 henry inductance, the other, 0.15 ohm resistance and 0.0008 henry inductance. The generated voltages being exactly opposite to one another, one is adjusted for 145 volts (generated) and the other for 110 volts (generated). What is the current flowing between the two machines, and what is the bus voltage?

18. The two machines of Prob. 17 are each adjusted to generate 125 volts and machine No. 2 is pulled ahead of No. 1 by 25° (electrical) by giving its prime mover more steam. What change in load (in amperes and kw.) does this bring about for each machine?

CHAPTER VII

THE TRANSFORMER

206. Principles Involved.—A transformer is a piece of stationary apparatus by means of which a.c. power may be changed from one voltage to another. It consists essentially of a closed magnetic circuit (made of laminated iron and called the *core*) on which are placed two coils of insulated wire, the two coils being entirely separate and insulated from one another. (This later qualification will be modified when a special type called the *autotransformer* is discussed.) The two coils generally have a widely different number of turns; a certain transformer, for instance, has 93 turns in one coil and 1800 in the other.

One of the coils (called the *primary*) is connected to an a.c. line of suitable voltage; the current which flows in it (the other coil being open-circuited) serves to magnetize the iron core and is called the *exciting current*. The magnetic flux through the core will evidently be an alternating one, being in one direction with positive current and in the reverse direction when the current has reversed; hence any turn of wire surrounding the core will have an e.m.f. induced in it because of this varying flux.

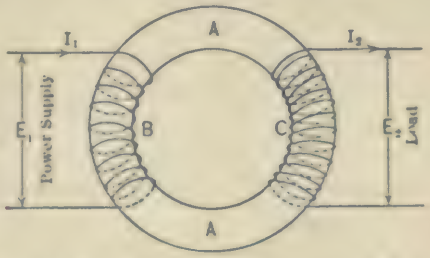


FIG. 294.—An elementary representation of a transformer; the current in the coil *B* produces an alternating flux in the laminated core *A–A* and this produces an e.m.f. in coil *C*. Lamps, etc., connected to coil *C* will be supplied with power which comes from the line to which coil *B* is connected.

The second coil (the one to which power is *not* supplied, called the *secondary*) is also wound on the core and therefore will have an e.m.f. induced in it. Lamps, motors, etc., may be connected to this secondary coil and will be supplied with power just as well as though they were connected to the line which is feeding power to the primary coil. The terms "high-tension coil" and "low-tension coil" are often used instead of the terms "primary coil" and "secondary coil"; evidently the terms "high-tension" and "low-tension" coil may designate either the primary or secondary coil, respectively.

The power input to the primary depends directly upon the amount of power being supplied by the secondary. It is not at once evident how the power gets from the primary to the secondary and we shall first consider that point.

207. Equation of the Transformer.—Figure 294 represents an elementary transformer, having a laminated, closed magnetic circuit, core, A, a primary coil, B, and a secondary coil, C.

Let I_1 = the primary current;
I_2 = the secondary current;
N_1 = the number of turns in the primary coil;
N_2 = the number of turns in the secondary coil;
R_1 = the resistance of the primary coil;
R_2 = the resistance of the secondary coil;
e_1 and E_1 = the primary impressed voltage (instantaneous and effective);
ϕ = the magnetic flux in the core at any time.

As the impressed e.m.f. of the primary must be balanced by the reacting forces in the primary coil, we may write (for an open-circuited secondary)

$$e_1 = R_1 i_1 + N_1 \times (\text{the rate of change of flux}). \quad . \quad . \quad . \quad (117)$$

where

$R_1 i_1$ = the resistance reaction;

$N_1 \times (\text{the rate of change of flux}) = e'_1 =$ the back e.m.f. of self-induction.

In ordinary transformers the term $R_1 i_1$ is very small compared with $N_1 \times$ (the rate of change of flux), so that we may neglect it without introducing an appreciable error. We may therefore write

$$e_1 = N_1 \times (\text{rate of change of flux}) = e'_1, \quad . \quad . \quad . \quad . \quad (118)$$

or, in effective values,

$$E = 2\pi f N_1 \phi_{eff} = E'_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

where E is the voltage impressed on the primary;
and E'_1 is the back e.m.f. of self-induction in the primary.

From this equation we see that there must always be a certain flux (effective) through the core in order to balance the e.m.f. impressed on the primary; *this flux must exist in spite of any demagnetizing action the secondary current may produce.*

208. Increase in Primary Current to Balance the Demagnetizing Effect of a Secondary Current.—For a secondary current equal to zero there exists a small magnetizing current in the primary; if some load is put on the secondary so that its current is some value I_2, the secondary coil will produce a demagnetizing action on the core equal to $0.4\pi N_2 I_2$. But the primary current must always be sufficient to produce in the core the flux ϕ_{eff}, hence when the secondary current increases, the primary coil automatically draws from the line an increased current. *This increase in primary current will be just sufficient to overcome the demagnetizing action the secondary current has produced.*

209. Ratio of Currents and Voltages.—We may, therefore, put

$$0.4\pi N_1 I'_1 = 0.4\pi N_2 I_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

where I'_1 is the increase in I_1 to overcome the demagnetizing effect of I_2. But as the magnetizing current of a transformer is very small compared to its full-load current (generally about 5 per cent), we may put, without much error,

$$0.4\pi N_1 I_1 = 0.4\pi N_2 I_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (121)$$

or

$$N_1 I_1 = N_2 I_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

which gives the approximate fundamental relation between the currents in the primary and secondary. This may be stated in words thus: *The primary ampere-turns are always just equal to the secondary ampere-turns (the magnetizing current being neglected).*

If all the flux which the primary produces threads the secondary coil (and this is approximately true), the volts generated per turn will be the same in each winding. If we call this voltage e, we must have

$$E_2 = N_2 e. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (123)$$

and

$$E_1 = N_1 e. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (124)$$

Eq. (124) is true because the right-hand member is the counter e.m.f. of self-induction and this must be equal to the impressed e.m.f. (approximately).

Substituting Eqs. (123) and (124) in Eq. (122), we have

$$E_1 I_1 = E_2 I_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (125)$$

This is the approximate equation for apparent power in the two coils. Also from Eqs. (123) and (124) we get the relation

$$E_1/E_2 = N_1/N_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (126)$$

which is the fundamental voltage relation of the transformer. Expressed in words this relation is: The primary voltage bears the same relation to the secondary voltage as the number of turns in the primary coil bears to the number of turns in the secondary coil. Of course, from Eq. (125) it is seen that any change in voltage brought about by the transformer must be accompanied by a change in current in the opposite sense.

Eq. (126) may also be written

$$E_1 = E_2 \times \frac{N_1}{N_2} \quad \text{and} \quad E_2 = E_1 \times \frac{N_2}{N_1}. \quad . \quad . \quad . \quad . \quad (127)$$

Eqs. (122), (125), and (126) give only the approximate relations of the transformer quantities; the derivation of the exact relations requires us to consider the loss due to hysteresis and eddy currents in the core, the I^2R losses in the windings, and the magnetic leakage.

210. No-load Vector Diagram of a Transformer.—When voltage is applied to the primary of a transformer, the impressed voltage is balanced almost completely by the back e.m.f. of self-induction, which is equal to the product of primary turns and the rate of change of the flux. The flux, being common to both primary and secondary, is chosen as the reference vector; it is shown in Fig. 295 by *OA*.

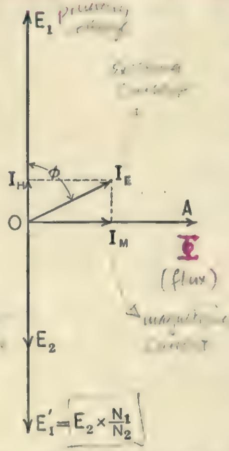


FIG. 295.—At no load the primary coil draws a small current which is nearly 90° behind the line voltage; it may be divided into an active and a reactive component, the latter being called the magnetizing current.

To set up the flux the primary must draw a *magnetizing current*, I_M, which is in phase with the flux; the circuit being almost purely inductive (disregarding the iron losses, to be discussed later), this current must lag 90° behind the impressed voltage, E_1.

The back e.m.f. of self-induction, E'_1, as well as the induced voltage of the secondary, E_2, both being induced by the rate of change of the flux, must both lie 90° behind the flux; E'_1 and E_2 must thus be in phase, and it follows that $\frac{E'_1}{N_1}$ must be equal to $\frac{E_2}{N_2}$; E_1 must therefore be equal and opposite to E'_1.

211. Iron Losses.—The alternating flux set up by the primary causes both hysteresis and eddy-current losses in the iron core, and as these losses represent energy, there must be a power component of current in the primary to supply them. This energy, or *iron-loss*, component of primary current is represented by the vector OI_H (so designated because the hysteresis losses are always the predominating ones), in phase with the primary impressed voltage.

The vector sum of the magnetizing and iron-loss components is then the exciting current, I_E, the current taken from the supply when the secondary is open. In well-designed transformers the iron loss is made small for reasons to be seen later, although the exciting current may be 5 per cent of the rated primary current; the no-load power factor of the transformer, the cosine of the angle ϕ, is therefore small, generally about 0.3.

212. Elementary Vector Diagram of a Loaded Transformer.—When the secondary of a transformer is connected to a load, current will flow, being urged by the induced voltage of the secondary. The angle by which the secondary current will lead or lag the secondary voltage will depend practically upon the resistance and reactance of the load. In Fig. 296 the secondary current, I_2, is represented as lagging by an angle ϕ_2 behind the secondary voltage, E_2.

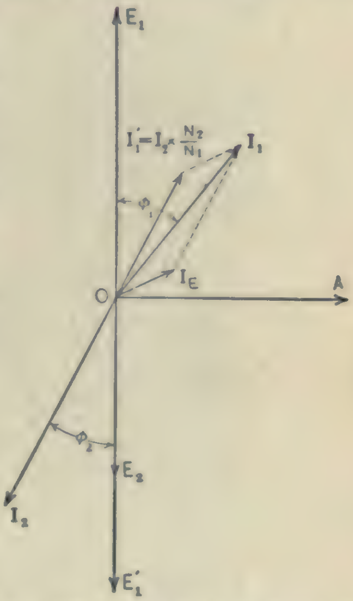


FIG. 296.—When the secondary coil is carrying current, the primary coil current is increased by a current just opposite in phase to the secondary current, and fixed in magnitude by the turn ratio of the transformer.

Since the back e.m.f. of the primary is opposite to the primary impressed voltage, and since the primary back e.m.f. and the secondary induced voltage are in phase, it follows that the secondary voltage is opposite in direction at every instant to the primary impressed e.m.f. The flux set up by the primary *must* remain constant, as its first function is to balance, by its rate of change, the primary impressed voltage. The secondary current, I_2, flowing through the secondary turns, will give a certain number of ampere-turns which would affect the magnetic flux in the core if no counter action of the primary coil were brought into play.

The magnetic flux in the core, as stated above, must remain constant and equal to what it was before secondary load was added, so that whatever ampere-turns are set up by the secondary must be neutralized by an equal number of ampere-turns in the primary. This requires a current I'_1 in the primary, in addition to the exciting current, I_E, which was flowing before the secondary current was supplied. This primary current, I'_1, to completely neutralize the ampere-turns of the secondary, must be 180° from the secondary current, and must be of such value that $N_1 I'_1 = N_2 I_2$. The vector sum of the *load component* of primary current, I'_1, and the exciting current, I_E, will be the primary current, I_1.

The elementary vector diagram considered above does not take into account all the factors involved when a transformer is loaded. Before taking up the actual vector diagram, we shall first take up the construction of the transformer.

213. Commercial Importance of the Transformer.—From the previous simple analysis it is seen that, by means of a transformer, a.c. power may be changed from one voltage to another; if the secondary coil has more turns than the primary, the voltage will be raised (i.e., "stepped up"); and if the secondary coil has fewer turns, the voltage will be lowered ("stepped down"). Some of the greatest power developments to-day are hydro-electric (using water turbines for prime movers); in such cases the power station must be situated close to the falls from which the power is taken. As the place where the electric power is consumed may be a hundred miles or more distant, a *long-distance transmission line* is necessary to carry the power to the place of consumption.

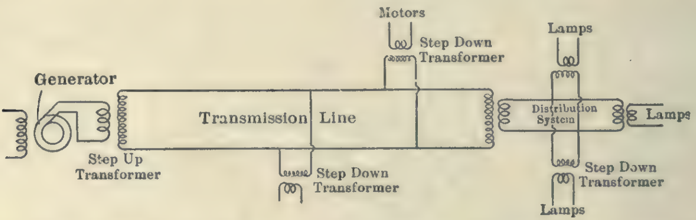


FIG. 297.—Showing how the transformer fits into an electrical transmission and distribution system.

214. Long-distance Transmission Requires a High Voltage.—It is commercially impossible to carry electric power any great distance unless high voltage is used, because of the high I^2R losses in the line, or else the heavy investment for the copper wires. An approximate rule for the voltage of ordinary transmission lines is from 500 to 1000 volts per mile length of the line. A line 150 miles long might be run at 80,000 volts or 100,000 volts, although special conditions may cause departure from this rule.

215. Location of Transformer in a Power-transmission System.—These high voltages cannot be generated directly in an alternator, neither can the power be supplied to the consumer at such a high voltage, hence the problem the electrical engineer has to face is this: The power may be generated at medium voltage (2300–13,000 volts), must be transmitted at high voltage (perhaps 50,000–200,000 volts), and must be supplied to the customer at low voltage (110–440 volts). The transformer raises or lowers the voltages where required; without the transformer, the successful development of a.c. transmission systems would have been impossible.

Figure 297 shows how the transformer forms links in a transmission and distribution system. For railway substations the voltage is changed, in one step, from the line voltage to that required in the sub-station; for lighting work the high voltage of the transmission line is generally stepped down by a few large transformers to 2300 volts, and is then distributed throughout the city at 2300 volts to small transformers located on the poles, which step it down again to 110 volts or 220 volts.

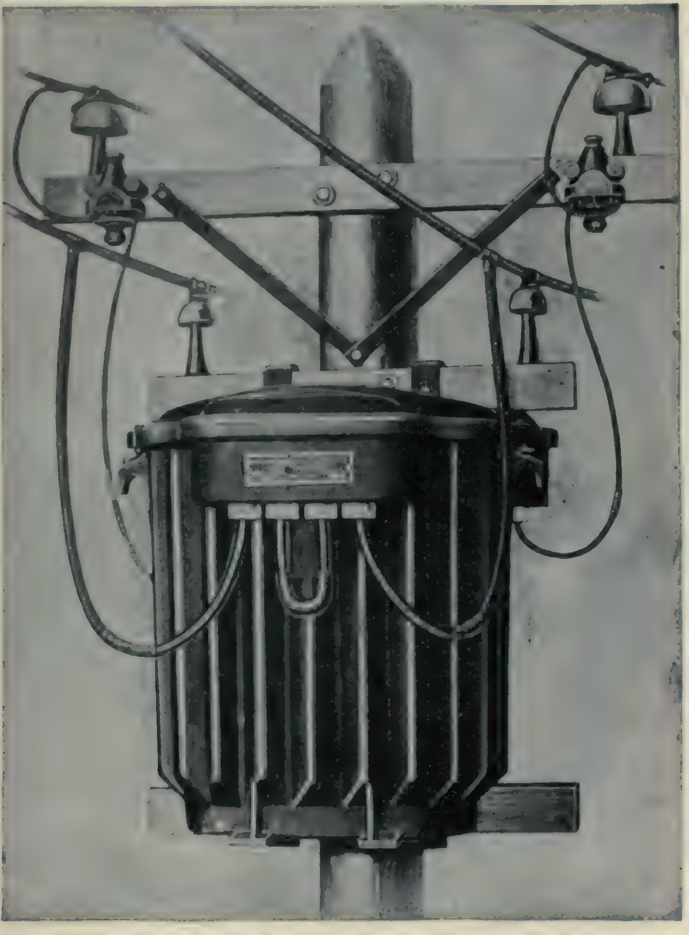


FIG. 298.—Lighting transformer of about 5 kv.a. capacity, mounted on a pole. The 2300-volt wires on the upper cross-arm enter the back side of the transformer (through suitable fuses) and the 110–220-volt wires come out of the front of the transformer case.

Each of these small transformers (generally of less than 15 kv.a. capacity) supplies perhaps twenty-five to fifty customers. Figure 298 shows one of these small transformers located on a pole, just under the cross arms; in the cut only the outside containing case of sheet iron can be seen, because the coils and core are always put inside cases to protect them from moisture, dirt, mechanical injury, etc. The small wires going into the case at the back (next the cross arms) are the 2300-volt wires and the larger wires coming from the front are the 220-volt wires which connect directly to the lighting wires in the customer's house.

216. Construction.—In actual construction the ordinary transformer does not closely resemble that given in Fig. 294. The cores can be most economically built up of rectangular, instead of circular, sheets, and the two coils are always placed as closely together as possible.

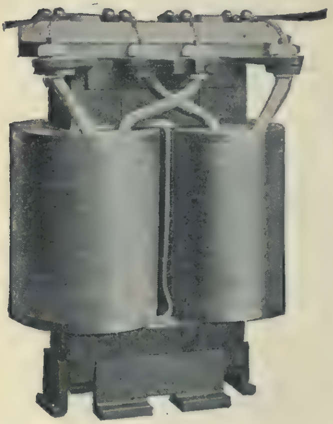


FIG. 299.

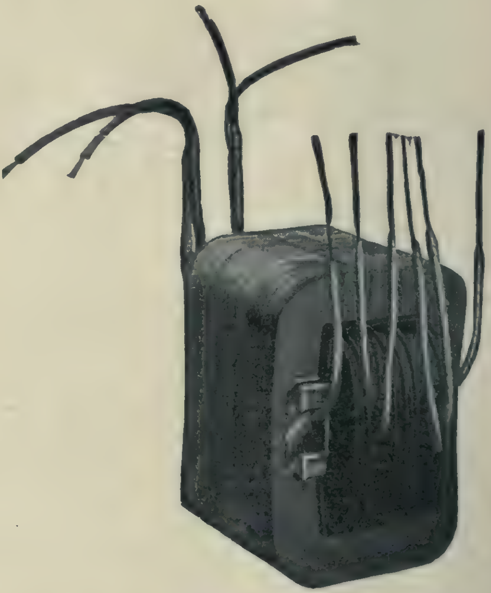


FIG. 300.

FIG. 299.—The core-type transformer somewhat resembles in form the elementary type shown in Fig. 294. Primary and secondary coils are placed on both legs of the core.

FIG. 300.—The shell type of transformer, showing how the coils are completely surrounded by the laminated iron core.

Two types of transformers have been developed in which both of these ideas are incorporated; they are the *core type* and the *shell type*. In both these types the cores are built up of thin sheets of iron, generally rectangular in shape, and the primary and secondary coils are placed in very intimate relation with one another.

The core type somewhat resembles the elementary transformer type shown in Fig. 294; however the shape of the core is rectangular (its cross-section is nearly square) and the two coils are placed one over the other

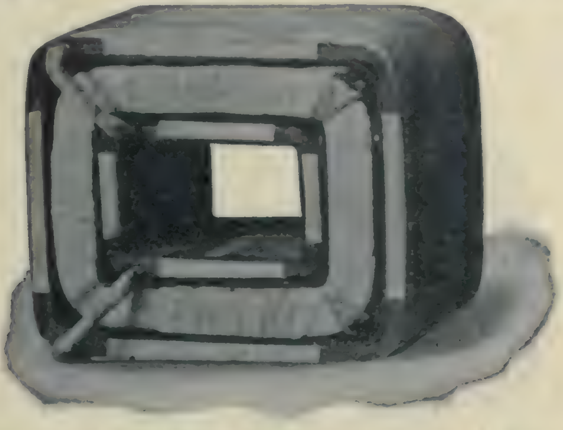


FIG. 301.—An assembly of coils for a shell-type transformer; the low voltage winding is in two parts in this transformer.

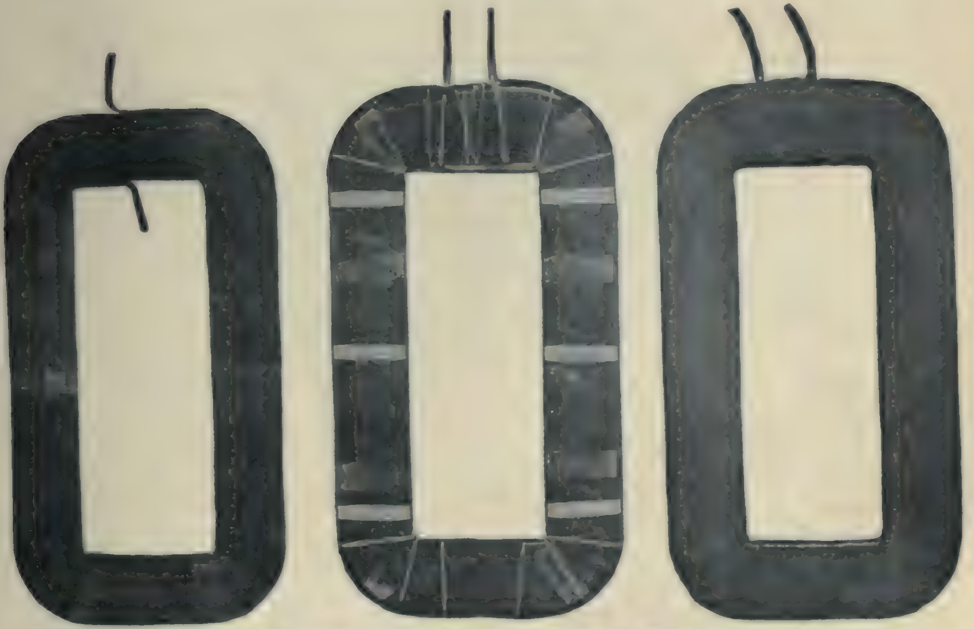


FIG. 302.—Large transformers generally have their coils built up of pancake-like sections, the sections of the primary and secondary coils being well interspersed.

as close together as possible. Each coil is divided into two sections, the two sections being placed on opposite legs of the core. Figure 299 shows a view of an assembled core-type transformer ready to be placed in its containing case. Space is left between the two coils so that oil for cooling purposes may circulate through the windings.

The shell-type transformer is shown in Fig. 300. The coils of the core type are generally cylindrical in form, but in the shell type they form a rectangular structure (as shown in Fig. 301) *similar in form to the core of the core type*, and the laminations (sheets of which the core is made) are built up around two legs of the assembled coils.

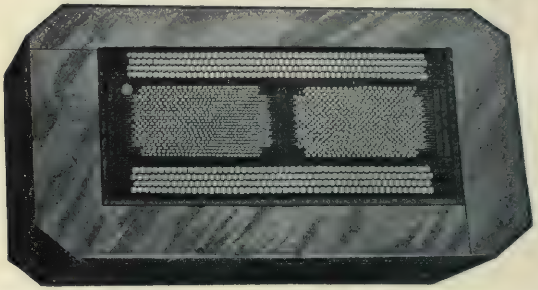


FIG. 303.—A cross-section through the windings of a shell-type transformer; the assembled coil structure is thoroughly impregnated with insulating compound before the transformer is assembled.

217. Lamination of Coils.—In order that the primary and secondary coils may be placed close together, they may be built up in thin sections (called " pancake " coils) and the sections of the two coils are sandwiched together, first a thin section of the primary, then one of the secondary, etc. One of these thin sections is shown in Fig. 302. In Fig. 304 is shown a cross-section of the assembled coils showing how the sections are interspersed; there may be a dozen or more of these sections in a large transformer. Figure 303 shows another method of winding shell-type transformers; here pancake coils are not used but the two coils are placed intimately together by having the low-tension coil built in two parts, the high-tension coil being placed between the two sections of the low-tension coil.

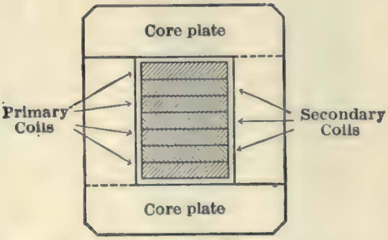


FIG. 304.—Each of the core laminations consists of two parts, for ease in assembly; every other lamination is reversed, thus " breaking joints," making the core assembly mechanically rigid and keeping the magnetic reluctance low.

218. Construction of the Core.—Although the iron core of a modern transformer forms a continuous iron path around the coil, the laminations, in the form of a hollow rectangle, are not in one piece, because if such were the case, the coil would have to be wound through them instead of being built up independent of the core.

Each lamination is made of two pieces as shown by the full lines of Fig. 304. In stacking them up around the coils, every other one is reversed so that the joints come as shown by the dotted lines. Because of this alternation of the joints, the core, when built up and clamped together, is very solid and has but little more reluctance than it would have if each lamination were one solid piece. In Fig. 305 is shown a partly assembled transformer; as the laminations are stacked up around the coil structure; frequent ducts are left for the circulation of oil, in which the transformer is to be immersed.

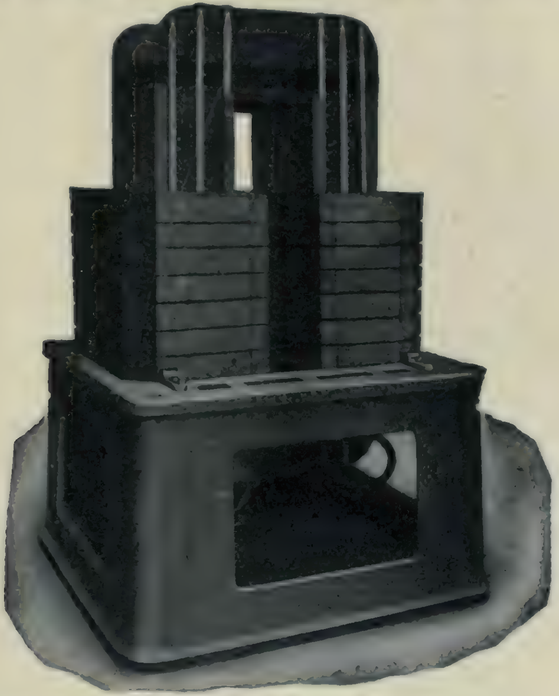


FIG. 305.—Showing how the laminations are assembled around the thoroughly insulated coil assembly; every inch or two, spaces are left in the core assembly to allow the cooling oil to circulate freely.

The most recent type of construction is really a modification of the core type; a view of one of these transformers removed from its containing case is given in Fig. 306, from which it may be seen to resemble a core-type transformer having winding on but one leg. The other leg, used as a return path for the magnetic flux, is divided into four parts, symmetrically disposed around the center leg, on which the coils are placed. This type uses concentric, cylindrical coils (which are the easiest to manufacture), is compact in its arrangement, and is more easily cooled than either of the other types. A sketch of the transformer is given in Fig. 307; it shows how freely the cooling oil may circulate between the coils and around all parts.

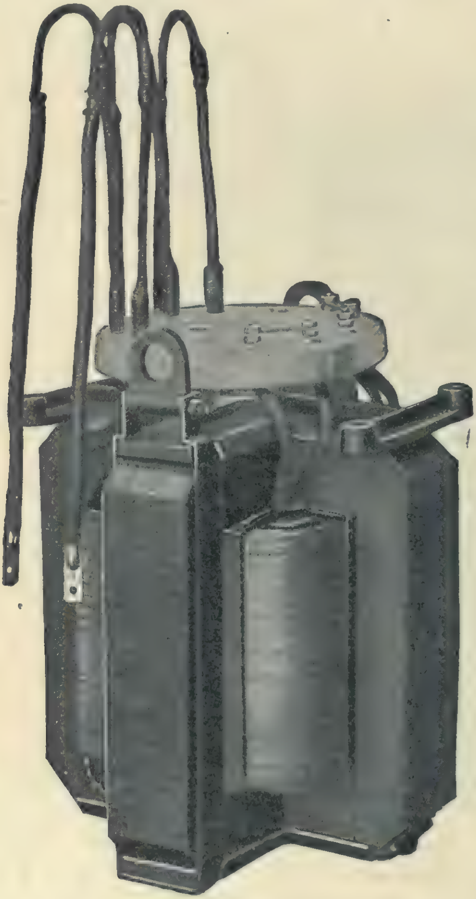


FIG. 306.—In this type of transformer the central leg of the core is surrounded by the coils and there are four parts to complete the magnetic circuit.

219. Size of Wire in Transformer Coils.—The size of wire used on the two coils is not the same; it is much smaller for the high-voltage coil than for the low-voltage coil. The most efficient design is one in which the I^2R in the primary coil is equal to that in the secondary. We have shown in Eq. (125) that $I_2/I_1=E_1/E_2$, or, in other words, the currents in the two coils are in the inverse ratio of their voltages. Let us put

$E_1/E_2=a$, the ratio of the transformer.

Then

$$I_2=aI_1$$

and if we are to have

$$I_2{}^2R^2=I_1{}^2R_1,$$

then evidently

$$a^2R_2=R_1. \quad . \quad (128)$$

If, for example, a transformer has a ratio of 20 : 1, the high-voltage coil should have 400 $(=20^2)$ times as much resistance as the low-voltage coil. Now there are twenty times as many turns on the high-voltage as on the low-voltage coil, so that *the cross-section of the wire in the high-voltage coil should be* $\frac{1}{a}$ *times that of the low-voltage coil.* This ratio of cross-sections gives approximately *the same weight of copper for both coils.*

Transformers designed with this ratio, however, will get hotter in the high-voltage coil than in the low-voltage coil, because the greater thickness of insulation around the high-voltage coil makes it more difficult for the heat to escape from the copper. In practice, therefore, a somewhat greater weight of copper is used in the high-voltage winding than in the low-voltage.

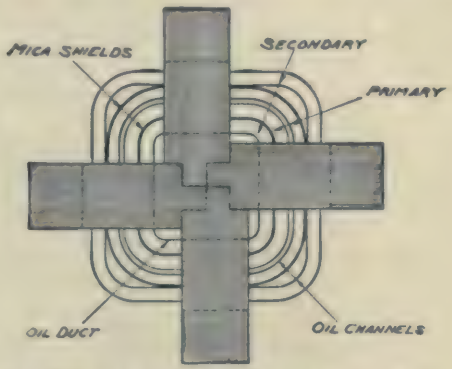


FIG. 307.—A plan of the transformer shown in Fig. 306; oil can circulate freely around the coils and core.

220. Insulation of Windings.—The insulation of the windings of a transformer must be very carefully looked after because of the high voltages to which a transformer is subjected. The high-voltage coil must be insulated not only from the low-voltage coil, but also from the core; a breakdown in the insulation in either of these places would connect to the high-voltage line a part of the transformer on which the operator supposes there is either a low voltage or none at all, and he would probably receive a fatal shock in working around the transformer. Or a customer touching what he supposes to be a low-voltage line might receive a fatal shock by connecting a high-voltage wire to ground through his body.

221. Danger of a "Broken-down" Transformer.—This possibility is illustrated in Fig. 308; it is supposed that the high-voltage insulation has broken down so that the two coils are connected together through the point of breakdown at *E*. Then the low-voltage line (either *C* or *D*) becomes charged with a high voltage of nearly the same potential with respect to the ground as the line *A*. If a partial ground exists on line *B* (as it generally will) and a person standing on the ground touches either of the low-voltage wires, *C* and *D*, he will receive a shock, just as if he had grounded the high-voltage line, *A*. And even if *B* has no partial ground, a fatal shock might still be received owing to the *charging current* (the earth serving as a condenser plate) his body will have to carry.

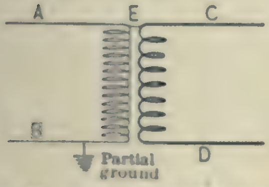


FIG. 308.—Illustrating the danger of a connection (due to faulty insulation) between high- and low-voltage coils. A person touching either of the low-voltage wires, *C* or *D*, might receive a fatal shock.

222. Construction of Coils.—To obtain good insulation, the coils of a transformer are wound with insulated wire and covered with a thick layer of tape. After they are wound they are impregnated with some insulating compound, and oiled cambric and special insulating papers are freely used between the coils and between the coils and the core. After being completed, the transformer insulation is tested for breakdown by using another high-voltage transformer and at least twice the voltage for which the high-voltage coil was designed. This figure varies for different transformers, being greater for low-voltage than for high-voltage transformers; for further information the Standards of the A. I. E. E. should be consulted.

In order to protect transformers from the high voltages due to *surges* (electrical oscillations in a transmission line) it is customary to insulate the first few turns of the high-voltage winding much better than the rest of the turns. It is found in practice that a transformer not so built nearly always breaks down in these first turns.

223. Methods of Cooling Transformers.—Owing to the losses which occur in the core and windings of a transformer, it heats up like any other piece of electrical apparatus. But the heating in a transformer would be greatly exaggerated unless means were taken to prevent it, as there are no moving parts to create air currents and so dissipate the heat. Special means are always used to keep down the temperature of a transformer.

Air-cooling, oil-cooling, or water-cooling may be employed, oil-cooling being most generally used for small transformers and water-cooling for large transformers in power plants.

224. Air-cooled Transformers.—In the air-cooling method a fan is used to force cool air from the bottom of the transformer, past the windings and core, to carry off the heat from the top of the transformer. Sometimes the fan connects to a *pressure chamber* in the basement of the station and the row of transformers is placed directly over this chamber. The bottom of each transformer case is open and connects to the chamber below; the air pressure used in the chamber depends upon the load the transformers are carrying, being sufficient at heavy loads to send a very swift air current through the transformers. The air should be well cleaned, otherwise dirt and foreign particles will collect in the transformer case and tend to cause short-circuits, etc. One disadvantage with air-blast transformers is that, if an arc occurs in the windings, it is rapidly fanned into flame by the air currents.

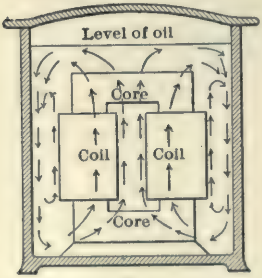


FIG. 309.—Showing the circulation of the oil in an oil-cooled transformer.

225. Oil-cooled Transformers.—The method of cooling most generally used is to place the transformer in a water-tight case, *filled with oil*. The oil serves both as an insulator and as a cooling medium. Currents circulate in the oil, owing to convection. These currents are "up" next to the transformer core and coils and "down" on the outside, next to the case. The heat is thus carried by the oil currents from the transformer to the case, through which it is conducted to the atmosphere. The direction of these currents is shown in Fig. 309.

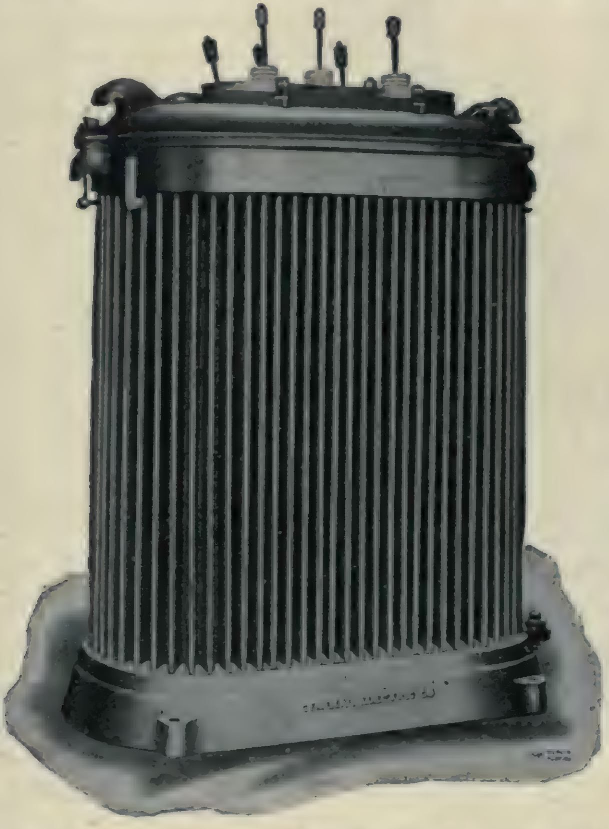


FIG. 310.—In the larger sizes of oil-cooled transformers the case is made corrugated, to expose more cooling surface and thus keep the temperature of the oil down to a safe value.

The use of oil in a transformer increases its capacity in the same way that forced ventilation increases the capacity of a motor or generator. This method of cooling is the most reliable there is; there is nothing to get out of order as is the case with air- or water-cooling.

In the larger sizes difficulty is encountered because of the great amount of heat generated in the transformer; the smooth iron case does not expose enough surface to the air for proper cooling. Because of this the case is made of *corrugated metal*, as shown in Fig. 310, or a nest of suitably disposed pipes is used so that the radiating surface of the case is very much increased. But in very large station transformers the heat cannot be radiated fast enough, even with a corrugated case, so that resort is had to water-cooling.

In a modern oil-cooled transformer the transformer tank is kept completely filled with oil by having a small auxiliary oil tank connected to the main tank and placed above. This lessens the chance of explosion of oil-vapor laden gases in the top of the transformer tank itself.

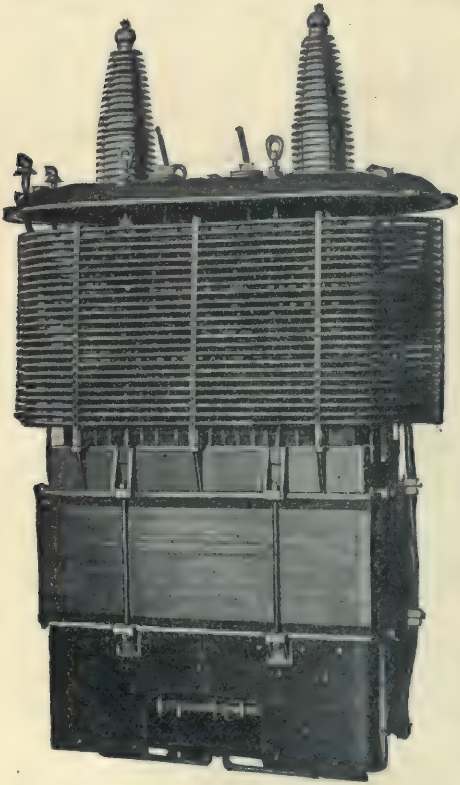


Fig. 311.—A transformer arranged for water-cooling, completely assembled and ready to be put into its containing case. The coil of pipe, in which the cooling water circulates, will be in the hottest part of the oil, when the transformer is in its tank.

226. Water-cooled Transformers.—A water-cooled transformer is one in which the transformer is immersed in oil (as in the oil-cooled transformer) but the oil itself is cooled by circulating between coils of pipes, which are immersed in the oil and through which cold water is circulating. This method of cooling is required in very large transformers, such as are frequently located in stations. Evidently the method could not be used for transformers located on poles or in out-of-doors installations because of trouble due to the freezing of the water and other causes. A large water-cooled transformer removed from its case is shown in Fig. 311. The water piping is made very carefully to guard against leaks, because the least trace of water in the oil would spoil its insulating power.

227. Losses in a Transformer.—The two sources of energy loss in a transformer are the *core loss* and the *copper loss*. These two losses are generally about equal at full load, each of them being from 1 to 3 per cent of the full-load capacity of the transformer. The larger percentage applies to the smaller sizes.

228. Core Loss.—The core loss in a transformer is due to hysteresis, and also to the eddy currents which are set up in the laminations of which the core is built.

As explained in Chapter III, Volume I, it requires energy (work) to carry iron through cycles of magnetization, a measure of this energy being the *area of the hysteresis loop*. As it is desired to keep this loss in the transformer as low as possible, the iron for transformer cores is carefully selected; only that grade of iron which has a narrow hysteresis loop is used. For a given transformer the hysteresis loss *varies directly with the frequency and with the 1.6 power of the maximum flux density*.

The eddy currents in the core circulate in the laminations as shown in Fig. 312, in which the laminations are represented as much thicker than they really are. These currents heat the iron and so represent loss in the transformer; this loss is kept low by using thin laminations. Fig. 312 gives the cross-section of one leg of a core-type transformer.

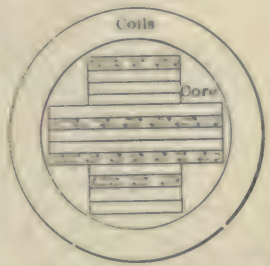


FIG. 312.—To show how the eddy currents circulate in the laminations of a transformer core; of course the laminations are actually much thinner than indicated here.

For a given transformer, the eddy-current loss *varies with the square of the frequency and with the square of the maximum flux density.*

The flux density in a transformer is *nearly independent of the load on the secondary* and, as the frequency of the supply current does not vary with the load, we may conclude that both hysteresis and eddy-current losses are practically independent of the load; hence the core loss (which is the sum of these two) *is independent of load.*

229. Copper Loss.—The I^2R loss in the windings evidently increases with the square of the load current. This is true for both transformer coils because we have previously shown that the primary current increases proportionately with any increase in the secondary current. The total copper loss may be calculated from the equation:

$$\text{Copper Loss} = I_1^2R_1 + I_2^2R_2 = I_1^2(R_1 + a^2R_2), \quad . \quad . \quad (129)$$

where the term $(R_1 + a^2R_2)$ is called the *effective resistance of the transformer* (of both coils) *in terms of the primary current.*

The term $I_1^2(a^2R_2)$ expresses the secondary copper loss in terms of the primary current; we may consider that the resistance of the secondary is zero, but that a sufficient resistance, R', has been added to the primary to make

$$I_1^2R' = I_2^2R_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (130)$$

At full load we have approximately,

$$I_1N_1 = I_2N_2;$$

hence

$$I_2 = I_1\left(\frac{N_1}{N_2}\right) = I_1\left(\frac{E_1}{E_2}\right) = I_1a. \quad . \quad . \quad . \quad . \quad . \quad . \quad (131)$$

where a is then the ratio of the transformer.

Substituting for I_2 in Eq. (130), its value from Eq. (131), we have

$$I_1^2R' = I_1^2a^2R_2,$$

from which

$$R' = a^2R_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

Thus the equivalent resistance of the secondary in terms of primary current is equal to the secondary resistance times the square of the ratio of the transformer.

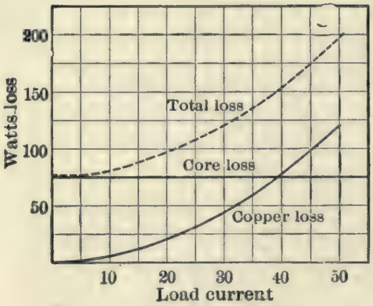


FIG. 313.

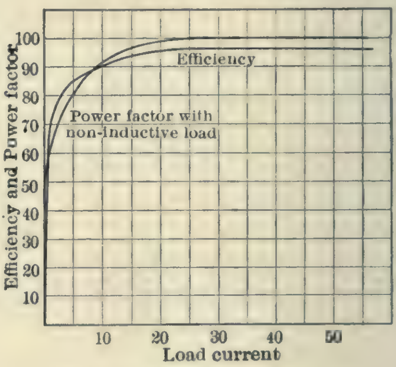


FIG. 314.

FIG. 313.—Loss curves for a small transformer.

FIG. 314.—Typical power factor and efficiency curves for a small transformer.

230. Loss Curves Efficiency.—Figure 313 shows the two loss curves of a 4-kv.a. transformer plotted against the secondary current; the total-loss curve is obtained by adding the two.

From the total-loss curve the efficiency can easily be calculated. We know that

$$\text{efficiency} = \text{output}/\text{input} = \text{output}/(\text{output} + \text{losses.})$$

Now assume any output, as for example, 10 amperes. As this transformer is rated 2200 volts–110 volts, the output $= 110 \times 10 = 1100$ watts. By reference to the " total loss " curve we see that the losses in the transformer for 10 amperes output amount to 80 watts.

Hence the efficiency of this output $= \frac{1100}{1100+80} = 93.3$ per cent.

At 20 amperes output the efficiency $= \frac{2200}{2200+92} = 96.0$ per cent.

At 30 amperes output the efficiency $= \frac{3300}{3300+120} = 96.5$ per cent.

At 40 amperes output the efficiency $= \frac{4400}{4400+153} = 96.7$ per cent.

At 50 amperes output the efficiency $= \frac{5500}{5500+195} = 96.6$ per cent.

The efficiency curve is thus determined and has the shape given in Fig. 314. This method of calculating efficiency is not rigidly correct, because the secondary voltage does not remain quite constant as we have assumed. The resulting error is, however, practically negligible.

231. Power-factor Curve.—The power-factor curve of the primary circuit (supposing a non-inductive secondary load) has the shape given in Fig. 314. The power factor is about 0.30–0.40 at no load and quickly rises to approximately 1.00, maintaining this high value until the transformer is heavily overloaded. The method for determining the power factor curve is explained in the experiment on the transformer in Volume III.

232. Decrease in Secondary Terminal Volts as the Load Increases.—The secondary voltage falls slightly with an increase of load (a constant impressed primary voltage being assumed), the total change from no load to full load being 2–5 per cent, expressed as a percentage of the full-load voltage. This percentage is called *the regulation* of the transformer. Its value depends upon the *resistance of the windings and the magnetic leakage,* an increase in either of these factors producing an increase in the regulation. The regulation of a transformer will be further considered in section 240.

233. All-day Efficiency.—The "all-day" efficiency of a transformer is a very important factor for the central station manager to consider; in fact, it is much more important than the efficiency as ordinarily defined.

By all-day efficiency is meant the ratio of watt-hours output to watt-hours input during a whole day's operation. An example will make this clear. Suppose a 5-kv.a. transformer is supplying a lighting load; it would, in a normal day's run, operate at full load $1\frac{1}{2}$ hours and perhaps at half load $1\frac{1}{2}$ hours; during the rest of the day there would be no load on the transformer. Suppose the iron loss is 200 watts and the full-load copper loss (I^2R) is 200 watts; the copper loss at half load would be equal to $(\frac{1}{2})^2 \times 200$ watts $= 50$ watts.

As the transformer is connected to its supply line all day, the core loss for all day $= 24 \times 200 = 4800$ watt-hours.

The copper loss for the $1\frac{1}{2}$ hours full load $= 1\frac{1}{2} \times 200 = 300$ watt-hours and during the $1\frac{1}{2}$ hours half load $= 1\frac{1}{2} \times 50 = 75$ watts, so that the total copper loss for the day's run is 375 watt-hours. The total loss in the transformer for 24 hours $= 4800 + 375 = 5175$ watt-hours.

The energy output in one day $= 1\frac{1}{2} \times 5000 + 1\frac{1}{2} \times 2500 = 11{,}250$ watt-hours.

The energy input in one day $= 11{,}250 + 5175 = 16{,}425$ watt-hours.

The all-day efficiency $= \frac{11250}{16425} = 68.5$ per cent.

Now the efficiency of this transformer in the ordinary sense

$$= \frac{5000}{5400} = 92.7 \text{ per cent.}$$

234. Proper Design to Get High All-day Efficiency.—If the transformer were redesigned (so as to have the same full-load efficiency) with 100 watts iron loss and 300 watts copper loss at full load, the all-day efficiency would be better than before.

Core loss, 24 hours $= 24 \times 100 = 2400$ watt-hours.
Copper loss, 24 hours $= 1\frac{1}{2} \times 300 + 1\frac{1}{2} \times 75 = 562$ watt-hours.
Output in 24 hours $= 11{,}250$ watt-hours (the same as before).

The all-day efficiency $= \frac{11250}{11250 + 2400 + 562} = 79.3$ per cent.

It is therefore evident that if a transformer is to be used for a service on which the full-load demand exists for a short time only, the transformer should be designed with as low a core loss as possible, even if the I^2R loss is much increased thereby. This idea must not be carried too far, however, because a transformer with high resistance regulates poorly and it would be unsatisfactory for supplying the power for lamps.

235. Determination of Losses.—The iron loss in a transformer is easily determined by reading the watts input (at normal voltage and frequency), when the secondary is open-circuited; connections as in Fig. 315. Although this wattmeter reading does include a small I^2R loss due to the exciting current, it is ordinarily not considered because it is so small. The total wattmeter reading is reckoned as core loss.

To determine the full-load copper loss, connections are made as in Fig. 316, *a very low voltage* being impressed on the primary. This voltage is increased until the ammeter in the short-circuited secondary indicates the full-load current. A reading of the primary wattmeter then gives the *full-load copper loss in both coils.* (There is a very small core loss included in this reading but it is ordinarily neglected.) The copper loss for other loads may be determined by proportionality, if desired, without being actually measured. For instance, the half-load copper loss will be one-quarter of the full-load loss; the quarter-load loss will be one-sixteenth of the full-load loss, etc.

The reading from these two tests may also be used to predetermine the power factor and the regulation of the transformer; the student is referred to Volume III for the method.

236. Effective Resistance of the Transformer.—If the wattmeter reading in Fig. 316 is divided by the secondary ammeter reading squared, the effective resistance of the transformer, in terms of secondary current, is obtained. The wattmeter reading must equal the sum of the copper losses occurring in both windings, i.e.,

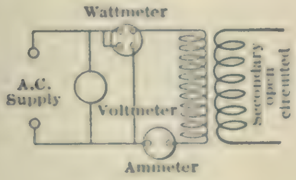


FIG. 315.

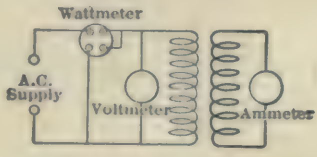


FIG. 316.

FIG. 315.—Proper connections for measuring the core loss of a transformer.

FIG. 316.—Proper connections for measuring the copper loss of a transformer.

$$W = I_1^2 R_1 + I_2^2 R_2. \qquad \qquad (133)$$

From Eq. (131)

$$I_1 = \frac{I_2}{a}.$$

Substituting this value of I_1 in Eq. (133)

$$W = \frac{I_2^2}{a^2} R_1 + I_2^2 R_2 = I_2^2 \left(\frac{R_1}{a^2} + R_2 \right),$$

so that

$$\frac{W}{I_2^2} = \frac{R_1}{a^2} + R_2.$$

The expression $\frac{R_1}{a^2}$ is the equivalent resistance of the primary in terms of secondary current, and may be derived in the same way as Eq. (132) was obtained.

If the wattmeter reading is divided by the primary current squared, the value of which may be determined either by an ammeter inserted into the primary circuit, or from the relation given in Eq. (131), i.e.,

$$I_2=I_1a \quad \text{or} \quad I_1=\frac{I_2}{a},$$

the effective resistance of the transformer in terms of primary current will be obtained, since

$$W=I_1^2R_1+I_2^2R_2=I_1^2R_1+I_1^2a^2R_2$$

so that

$$\frac{W}{I_1^2}=R_1+a^2R_2.$$

Knowing the effective resistance of the transformer in terms of either primary or secondary current, the effective resistance in terms of the other may easily be determined, since

$$\frac{\text{Effective resistance in terms of primary}}{\text{Effective resistance in terms of secondary}}=\frac{R_1+a^2R_2}{\dfrac{R_1}{a^2}+R_2}=a^2. \quad (134)$$

In practice, only the effective resistance of a transformer in terms of either primary or secondary is usually desired, and it is obtained by the method indicated in Fig. 316. It is, of course, also possible to calculate the copper losses from measurements of the conductor resistances of both primary and secondary windings, and assumed or measured values of currents.

It is to be pointed out that if the copper losses are determined by the method of short-circuiting the secondary, the total loss can be actually and properly divided between the primary and secondary windings, if desired. We know that both windings have resistance and carry currents, so that in each winding there occurs an IR drop and an I^2R loss.

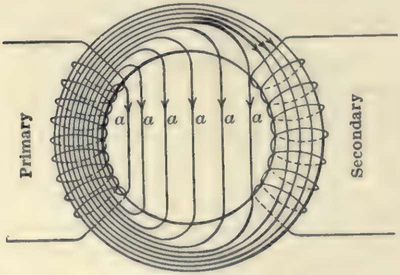


FIG. 317.—A conventional diagram to illustrate magnetic leakage; part of the flux which is set up in the primary does not link with the secondary and this is called the leakage flux.

237. Magnetic Leakage.—The magnetic field, upon which the operation of the transformer depends, is generated by the primary coil; all of this magnetic flux does not thread the secondary winding, as some magnetic lines leak across from one part of the core to another without interlinking with the secondary coils. In the elementary form of transformer, shown again in Fig. 317, the leakage flux takes the path indicated at a,a, etc., while the normal flux threads both the secondary and primary. The amount of leakage depends directly upon the load, because the secondary current produces a back m.m.f. on the magnetic circuit, forcing out of the secondary more and more flux as the load increases. It is generally disadvantageous to have much leakage in a transformer because of the poor regulation which results; the two coils, primary and secondary, are, therefore, placed as close together as possible.

In the vector diagrams of Figs. 295 and 296, the flux vector represented the *mutual* flux, the flux which interlinked both the primary and the secondary, and we said that the rate of change of this mutual flux generated a voltage in the secondary, E_2, and induced in the primary a back e.m.f. of self-induction, E'_1; leakage flux was therefore not considered in Fig. 296.

A circuit capable of setting up flux which links with itself is said to have reactance, and so the primary winding is said to have *leakage reactance*. The primary leakage flux induces an e.m.f. in the primary, which, while it may be considered a back e.m.f. of self-induction, is always considered as an I_1X_1 drop, to distinguish it from the back e.m.f. of self-induction due to the mutual flux.

238. Determination of Leakage Reactance.—The leakage reactance of a transformer is obtained from the short-circuit test of Fig. 316. By dividing the voltmeter reading by the ammeter reading, the *impedance*, Z, of the transformer in terms of the primary is obtained; and since

$$Z = \sqrt{R^2 + X^2}$$

and the effective resistance of the transformer in terms of the primary is known, $\left(\frac{W}{I_2^2}\right)a^2$, the reactance in terms of the primary may be calculated.

It is customary to divide the reactance of a transformer between the primary and secondary in the same way as the effective resistance is divided between the windings, and treat it as though part of it were in the primary and part of it in the secondary. It is, however, impossible to divide the leakage reactance experimentally, in the manner in which it is possible to divide the effective resistance between the primary and secondary. It seems simpler, then, to consider that all leakage reactance is concentrated in the primary, and that there occurs only an I_2R_2 drop in the secondary.

239. Vector Diagram of the Transformer.—The voltage impressed upon the primary must overcome three separate reactions, the reaction due to mutual flux, the primary I_1R_1 drop, and the primary I_1X_1 drop due to leakage flux. In constructing the vector diagram (Fig. 318), we again start with the vector of mutual flux, OA. The component of impressed voltage which overcomes the back e.m.f. due to this flux, E'_1, leads the flux by 90°, and is shown as the vector $-E'_1$; the secondary induced voltage, E'_2, lags 90° behind the flux. The secondary current is indicated by the vector I_2, and the load component of the primary current as $I'_1\left(=I_2\times\frac{N_2}{N_1}\right)$; the primary current is then the vector sum of I'_1 and the exciting current, I_E.

The primary I_1R_1 drop, drawn parallel to I_1, and the primary I_1X_1 drop, leading the current by 90°, are added to E'_1, giving the primary impressed voltage. The secondary terminal voltage is obtained by subtracting the secondary I_2R_2 drop (drawn parallel to I_2) from the induced voltage in the secondary.

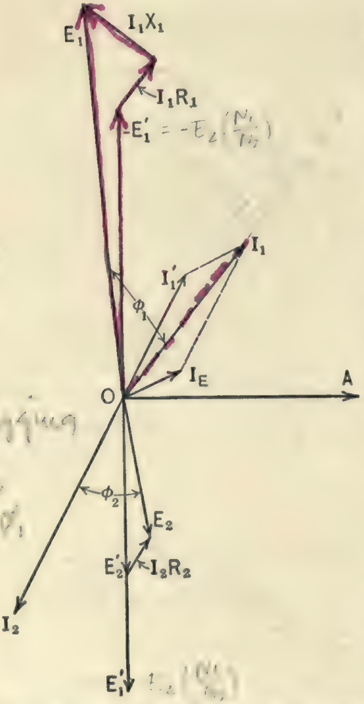


FIG. 318.—Showing how the required primary voltage may be calculated from the secondary terminal voltage and transformer constants.

*
p. 292

φ₁ = pf lagging
current lags voltage by φ₁

240. Regulation of the Transformer.—It will be seen from the vector diagram of Fig. 318 that as load is put upon the transformer the primary I_1R_1 and I_1X_1 drops will increase. The back e.m.f. of self-induction need therefore not be as large, and as a result the mutual flux will decrease somewhat; this in turn decreases the secondary induced e.m.f. The secondary terminal voltage is however equal to the vector difference between the secondary induced voltage and the secondary I_2R_2 drop, and it is evident, then, that as load is put upon a transformer (the primary voltage being held constant), the secondary terminal voltage will decrease. In a well-designed transformer this drop in voltage, from no load to full load, will amount to from 1 to 3 per cent. The vector diagrams of Figs. 295, 296 and 318 are very much exaggerated, the IR and IX drops having been drawn with values much larger than actually occur. For the same reason, from about one-quarter load up, the power factor of the primary is practically equal to that of the secondary, the angles ϕ_1 and ϕ_2 being nearly equal.

With a non-inductive load, the resistance drops have the greater effect in causing the secondary voltage to fall with load; the IX drop, being practically at right angles to the primary impressed voltage, has practically no effect on the regulation. As the power factor of the load decreases, the effect of the *IX* drop becomes more and more marked, and that of the *IR* drops less marked. The *IX* drop being usually larger than the *IR* drop, the regulation of a transformer is usually poorer at a power factor of 0.8 than at unity power factor. The student may easily construct vector diagrams to test the accuracy of the above statements.

241. Polyphase Transformers.—In transforming the voltage of a two- or three-phase power system it is possible to use two or three single-phase transformers, or one polyphase transformer. A two-phase transformer has no commercial importance, but three-phase transformers are used to a considerable extent. If three single-phase, core-type transformers were wound each on one leg only, and the three empty legs were combined to make the return path for the flux of each transformer, we should have essentially a three-phase transformer. The flux carried by the common return path, however, would be zero (because three equal harmonic fluxes 120° apart in time have a zero resultant). Hence, as the common return leg carries no flux, it is useless and so may be dispensed with.

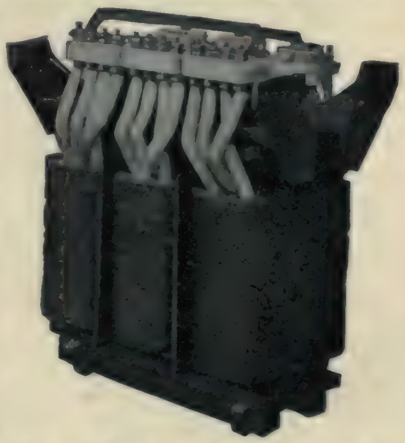


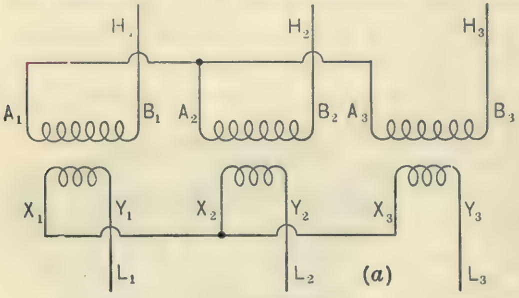
FIG. 319.—A core-type, three-phase transformer. Due to the phase relations, each leg of the core successively uses the other two legs as the return path for its magnetic flux.

By this modification, the three-leg, three-phase transformer is reached, having the primary and secondary windings of one phase on each leg. The design mentioned above, altered slightly to make the core easier to construct, yields a transformer of the form shown in Fig. 319. It may be seen that no common leg is used in such a design.

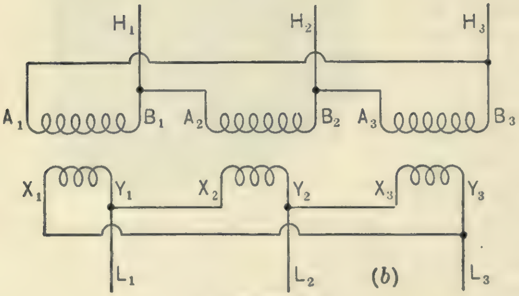
The three-phase transformer is slightly more efficient than three single-phase transformers, as there is less iron per kw. output, but the cost of upkeep and of spare units is greater. In general, three single-phase transformers are used, rather then one three-phase transformer, because of the greater reliability and flexibility of the installation

242. Polyphase Power Transformation.—It is not possible to transform a single-phase system into a polyphase system by means of ordinary transformers; single-phase power, as we have seen, is pulsating, whereas the power in any balanced polyphase system is constant. It is, however, possible to transform one polyphase system into another, also to raise or lower the voltage of a given system, with or without changing the number of phases.

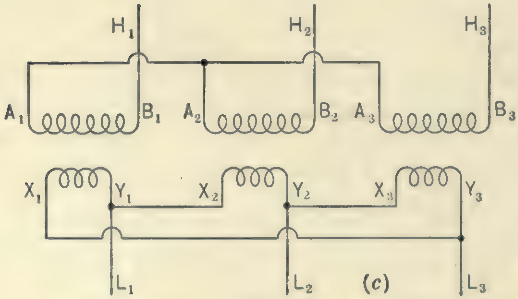
243. Three-phase Transformation.—Three-phase transformation may be accomplished by one three-phase transformer or by three single-phase transformers. In special cases two single-phase transformers may be used for three-phase transformation.



Y-Y Connection



Δ-Δ Connection



Y-Δ Connection

FIG. 320.—Various connections of three-phase banks of transformers; all of these are used in practice.

Three-phase transformation is the one most generally used, the voltage being stepped up or down as desired. Four possible combinations of three single-phase transformers (or the three phases of a three-phase transformer) are possible; these are known as the Y–Y, Δ–Δ, Y–Δ, and the Δ–Y. In Fig. 320 the connections for the first three schemes are indicated.

The ratio in which the voltage is changed depends not only upon the ratio of the transformers themselves, but also upon the way in which they are interconnected. With the Y–Y and the Δ–Δ connections, the voltage of the system is changed only in the ratio of the transformers; in (*a*), if there are 1000 volts between lines on the high-tension side, each transformer has impressed on its high-tension winding $\frac{1000}{\sqrt{3}} = 577$ volts. If the ratio of transformation is 10 : 1, there are induced 57.7 volts in each low-tension winding, giving $57.7 \times \sqrt{3} =$ 100 volts between lines. In the Δ–Δ connection, 1000 volts are impressed upon each high-tension winding, and 100 volts are induced in each low-tension winding.

3. When the Y–Δ connection is used, with 1000 volts between lines, each transformer has impressed upon its high-tension winding 577 volts, and there are induced 57.7 volts in each low-tension winding; with the low-tension windings connected in delta, the line voltages are 57.7. Thus the ratio of transformation with the Y–Δ connection is 1.732 times the ratio of the tranformers themselves instead of equal to this ratio as is the case with the Y–Y and Δ–Δ connections.

4. If the high-tension windings were connected in delta, 100 volts would be induced in each low-tension winding; these being connected in Y, the line voltages would be $100 \times \sqrt{3} = 173$ volts, giving a ratio of transformation of 0.577 times the ratio of the transformers themselves.

To transform a given amount of three-phase power, the transformers will each have the same kv.a. capacity, but their voltage and current ratings obviously depend upon the connection to be used.

244. Two-phase to Three-phase Transformation.—The transformation of a two-phase system into a three-phase system (or *vice versa*) is accomplished by two transformers grouped according to the *Scott connection.* Figure 321 shows how the connection is made. Transformer *A* has a ratio of 1 to 1, and transformer *B* has a ratio of 1 to 0.866, this condition being brought about by bringing out a tap, so that only 86.6 per cent of the turns are used, making the voltage *a–b* only 86.6 per cent of the voltage *d–e*.

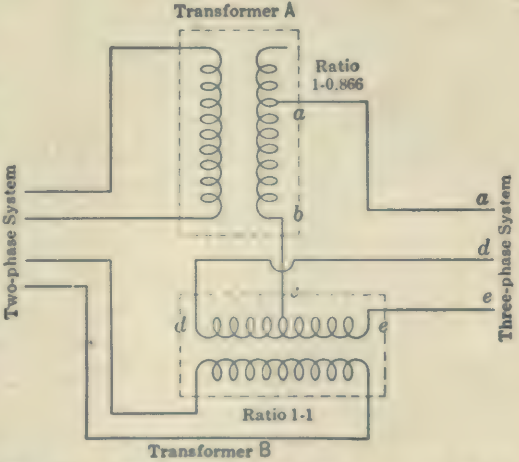


Fig. 321.—Two transformers, having proper ratios, and being properly connected, permit the transformation of a two-phase system to a three-phase system, or vice versa.

If it is assumed that the positive directions of the voltages induced in the secondaries of the transformers (three-phase side), are as in Fig. 322*a*, then the voltage E_{de} between lines *d* and *e* on the three-phase side is the sum of E_{dc} and E_{ce} (Fig. 322*b*); the voltage E_{ea} is the vector sum of the voltages E_{ec} and E_{ca} (Fig. 322*c*), and voltage E_{ad} is the vector sum of E_{ac} and E_{cd} (Fig. 322*d*). If E_{de} is 100 volts, E_{dc} and E_{ce} will each be 50 volts, and E_{ac} will be 86.6 volts. The voltages E_{ea} and E_{ad} will then each equal

$\sqrt{50^2+86.6^2}=100$. From Fig. 322*c*, $\cos\theta=\frac{50}{100}=0.5$, and $\theta=60°$. The three line voltages on the three-phase side are thus equal and 120° apart, the criterion for a three-phase system.

If it is desired to change from a high-voltage two-phase system to a low-voltage three-phase system (or *vice versa*), it may be done, provided only that the secondary voltage of A is 86.6 per cent that of B.

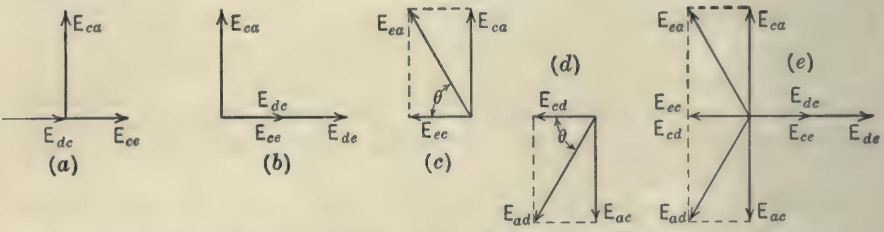


FIG. 322.—Showing how the balanced three-phase system is obtained by the Scott transformer connection.

This method of transformation is used in cases where a customer, previously supplied with two-phase power and having installed two-phase apparatus, is now being supplied with power from a modern three-phase system. Occasionally this system of transformation is used to tie together two-phase and three-phase power systems.

245. Special Cases.—In addition to the four possible connections for three transformers for three-phase transformation given above, it is possible to use only two transformers. It was shown in section 42 that if one phase of the three-phase Δ-connected alternator was opened, the alternator would still deliver three-phase voltages; the same applies to transformers.

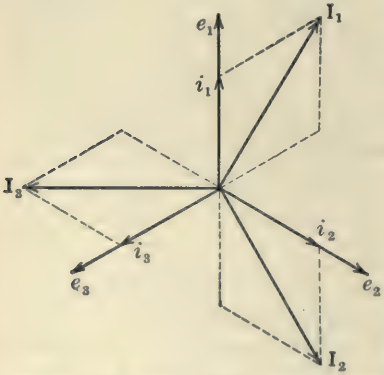


FIG. 323.—Current and voltage relations in the secondaries of transformers connected as in Fig. 320*b*.

This open Δ-, or " V ", connection gives balanced three-phase voltages, but two transformers so connected have a capacity of only 58 per cent of the capacity of three transformers connected in Δ, instead of $66\frac{2}{3}$ per cent as might be supposed.

In Fig. 323 is shown the vector diagram of the voltages and currents in the secondaries of the transformers of Fig. 320*b*; the phase currents are i_1, i_2, and i_3, and the line currents I_1, I_2, and I_3. If transformer No. 3 is removed, then the line current I_3 will have to pass through No. 1 and line current I_2 through transformer No. 2. It will be noticed that even though the load is non-inductive, the currents in the two transformers are 30° out of phase with their respective voltages. If each secondary of the transformers has a current capacity of 1000 amperes and induces 1000 volts, the power output of the transformers with 1000 amperes in the lines will be $1000 \times 1000 \times \cos 30° = 866$ kw., or a total capacity of 1732 kw. Yet if all three transformers are connected in delta, their combined capacity is 3000 kw.

The open Δ-connection is, however, sometimes used, as for example, when a power company is extending its distribution into territory which will not have many customers at first. When the number of customers has grown sufficiently, the third transformer can be added.

In addition to the few simple connections shown, there are many more complicated groupings possible. With one scheme called the "double Δ," it is possible to operate a six-phase, synchronous converter from a three-phase supply line. Thus the efficiency of the three-phase transmission is combined with the increased capacity of a six-phase converter.

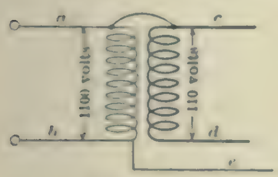


FIG. 324.

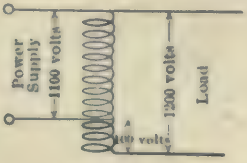


FIG. 325.

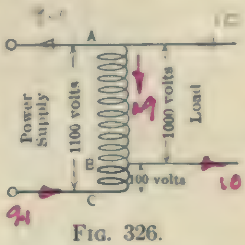


FIG. 326.

FIG. 324.—Connection of an ordinary two-coil transformer as an autotransformer.

FIG. 325.—Auto-transformer used to increase the line voltage by 10 per cent; that part of the winding common to both circuits carries a comparatively small current.

FIG. 326.—An auto-transformer used to lower the line voltage by 10 per cent.

246. The Auto-transformer.—This name is applied to an ordinary transformer the coils of which are connected together electrically. Generally, the two coils of a transformer *are connected together only by the magnetic field*, but in the auto-transformer *they are connected together not only magnetically but also electrically*.

Consider an ordinary 1100-volt 110-volt transformer, the coils of which are electrically connected together, as shown in Fig. 324. Now, instead of taking the secondary line from *c–d*, as would ordinarily be done, let the secondary load be taken from the points *d–e*. The voltage of this secondary circuit will be that of the primary, increased or decreased by the voltage generated in the coil *c–d*. If the coil *c–d* acts in the circuit so as to make the voltage of *d–e* greater than that of *a–b*, it shows that it is assisting the voltage of *a–b*; when the voltage of *d–e* is less than that of

a–b the secondary e.m.f. is opposing the primary e.m.f. The secondary e.m.f. in the two cases would be 1210 and 990 volts respectively.

The auto-transformer is represented as having but one coil with proper taps; Fig. 325 shows the connection for the 1100–1200 volt connection, and Fig. 326 shows the connection for the 1100–1000 volt connection. The auto-transformer may thus increase or decrease the voltage.

In that part of the coil common to both circuits, the primary and secondary currents tend to neutralize one another. Suppose the load in Fig. 326 is 10 amperes in the secondary circuit of 1000 volts; evidently 10 kv.a. must be supplied by the primary coil to supply this load and 10 kv.a. at 1100 volt swould require $\frac{10000}{1100} = 9.1$ amperes. Therefore, the part of the coil AB (Fig. 326) carries $10 - 9.1 = 0.9$ ampere, neglecting the exciting current. The part B–C, however, carries the full 9.1 amperes.

Because of this neutralization of currents, the auto-transformer is very economical in the use of copper, *especially when the secondary and primary voltages are nearly the same.*

247. The Constant-current Transformer.—For street lighting, the high-voltage series system has proved to be commercially successful and many street lighting systems are of this type. The incandescent lamps used in such work are of the *constant-current type* and require a transformer which supplies a constant secondary current.

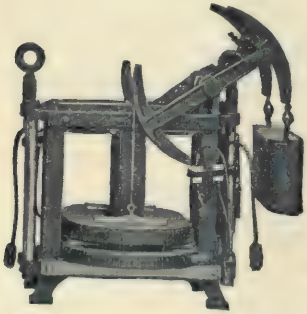


FIG. 327.—The common form of constant-current transformer; if the secondary current is greater than it should be, the movable coil is pushed away from the stationary coil, thus increasing the leakage flux and reducing the secondary voltage and current.

In the operation of a series system of tungsten street-lamps, it often happens that some lamps go out, for some reason or other, and, of course, this changes the resistance of the load circuit. If the voltage impressed on the circuit were constant, the current would rise and fall as the number of lamps burning was decreased or increased and the lamps would operate poorly. Hence there is required in the station a transformer the *secondary voltage of which varies with the number of lamps in circuit.*

248. Operation of the Constant-current Transformer.—The constant-current transformer accomplishes this variation by a variation of the leakage flux between the primary and secondary. The secondary coil is made movable and the primary fixed, and the leakage flux will evidently depend upon the separation of the two. A view of this type of transformer is given in Fig. 327. (This type is sometimes called the "tub" transformer.) The secondary coil is suspended and nearly counterbalanced so that but little force is required to move it up, or to separate it from the primary.

The operation may be understood by reference to Fig. 328, which shows a simplified cross-section of a constant-current transformer. The leakage flux is shown in dotted lines, and the leakage path is made noticeably large in such a transformer.

The weight used for counterbalancing the secondary is not quite sufficient to make the coil float, but, of course, when the secondary is carrying current, the leakage lines repel the coil and help the counterweight to make the coil float. This force of repulsion is proportional to the current and *the coil is under-counterbalanced to such an extent that when the rated secondary current is flowing the coil floats.* If more than rated secondary current flows, the secondary is repelled farther from the primary, so that more leakage flux is produced. But the more leakage there is, the lower is the secondary voltage, and so the secondary current is lowered and reduced to its proper value.

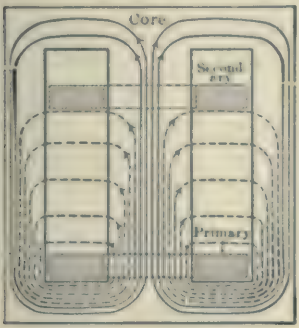


FIG. 328.

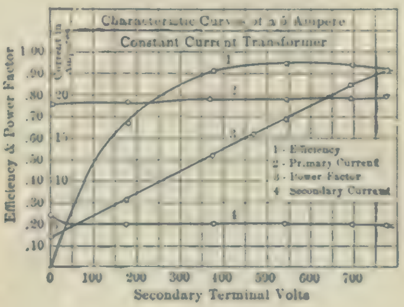


FIG. 329.

FIG. 328.—A diagram of the flux distribution of a constant-current transformer like that shown in Fig. 327.

FIG. 329.—Curves showing the behavior of the ordinary constant-current transformer, as the resistance of the secondary circuit is varied.

Suppose the transformer is operating on a certain load of series arc lamps and one lamp goes out, thus reducing the resistance of the external circuit. The secondary current will immediately increase, thus increasing the repelling force between the secondary and primary, thus raising the secondary coil. But this decreases the generated e.m.f. in the secondary coil because of the increase in leakage flux, and so the secondary current decreases to its normal value. Hence we see that this type of transformer will give a practically constant secondary current so long as the secondary coil is floating freely on the leakage flux.

249. Characteristics of the Constant-current Transformer.—Because of the excessive leakage flux the primary of such a transformer has a rather low power factor; the value of the power factor depends upon the separation of the coils, being smaller the greater the separation. The efficiency of this type of transformer is somewhat lower than that of a constant-potential transformer. This is due to the construction; there is more iron in the core and more resistance in the coils than there would be for a constant-potential transformer of the same capacity.

The characteristic curves of a constant-current transformer are given in Fig. 329, in which are reproduced the results of a laboratory test on a small constant-current transformer. Because of the low power factor and efficiency at light loads, it is customary to have several taps on the secondary and to use that tap which makes the transformer operate (on a given circuit) with its secondary close to the primary, thereby producing a high power factor. To prevent violent oscillation of the secondary coil as the load varies, a dash-pot is employed to prevent rapid motion of the secondary. From Fig. 329 it may be seen that the *primary current as well as the secondary current* of such a transformer is practically independent of the load. The explanation of this would require more space than can be afforded here.

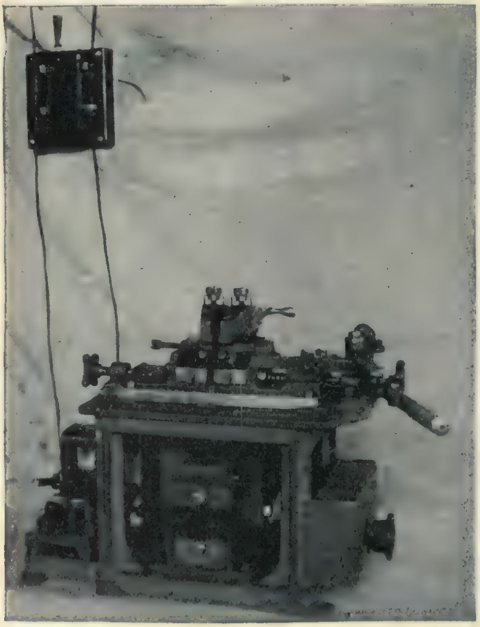


FIG. 330.—A small welding transformer.

250. Welding Transformer.—The welding transformer is designed for the purpose of making welded joints by heating the junction of the metal with an electric current. The two pieces to be joined are clamped tightly together and the secondary of the transformer is short-circuited through this contact. Of course the resistance of the contact is high as compared with that of the rest of the secondary circuit, and it becomes intensely heated. After a few seconds the two pieces fuse together at the joint. When they fuse the contact resistance immediately falls and the joint cools off and is finished. The primary coil of such a transformer is generally wound for 110-220 volts while the secondary has only one or two turns and so generates only a few volts; the current capacity of the secondary is, however, very high, as perhaps thousands of amperes are required to make a weld quickly. A small transformer designed for making joints in pieces not more than $\frac{5}{16}$ inch diameter is shown in Fig 330. The jaws, in which the pieces to be welded are clamped, are seen at the top.

251. Instrument Transformers.—Instrument transformers are of two types, current and voltage transformers. It is not easy to build an ammeter to carry thousands of amperes, or voltmeters to measure the very high voltages used on transmission lines, and so in places where high currents and voltages are to be measured, meters of low ranges are used in connection with suitable instrument transformers. If, for example, the voltage of a 2200-volt line is to be measured, a step-down transformer with a ratio of 20 : 1 might be used in connection with a voltmeter having a range of 125 volts. Such a transformer is called a *potential transformer;* it generally has a full-load capacity of only a few watts, i.e., just sufficient to operate the voltmeter.

In using potential transformers it must be remembered that they will not be accurate if more than the rated load is taken from them; also, they cannot be used as step-up transformers if accurate results are required. A transformer having a ratio of 60 : 1 stepping down, might have a ratio of 1 : 57 when used as a step-up transformer.

The *current transformer* is used when very heavy alternating currents are to be measured. An ammeter to carry 5000 amperes would be very cumbersome and expensive; besides, a 5-ampere meter in connection with a current transformer answers just as well. A current transformer is used also when it is desired to measure the current in a high-voltage transmission line. As it is not advisable to connect an ammeter directly in series with such a line, a current transformer is used, and the ammeter, connected in the secondary, is thus insulated from the high-tension line. Such a transformer is shown in Fig 193.

Fig. 331 shows a transformer designed to be placed over the station bus-bar to indicate the station current. It has a ratio of 1500 : 5 and a 5-ampere ammeter is connected in its secondary circuit. Of course, when

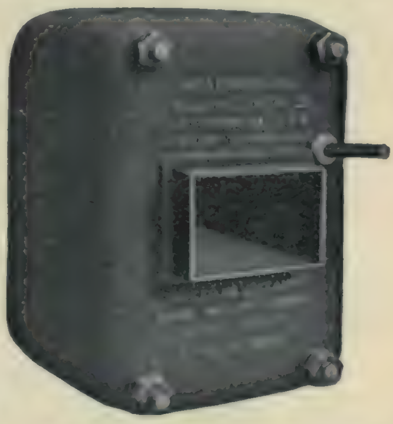


FIG. 331.—A current transformer intended for installation on the station bus-bar. There is a laminated iron core surrounding the hole through which the bus-bar passes and around this core is wound the secondary coil which connects to the ammeter.

the meter indicates the full-scale deflection (i.e., 5 amperes on the usual calibration) it signifies that the bus-bars are carrying 1500 amperes and so the full scale of the meter is marked 1500 amperes, etc.

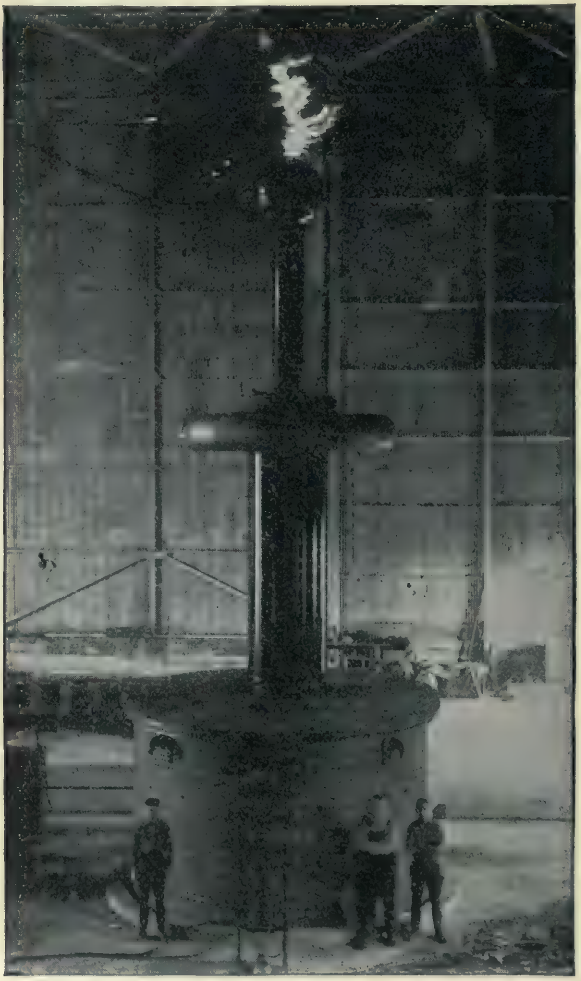


Fig. 332.—A one-million-volt testing transformer; one high-tension terminal of the transformer is grounded to the case, and the other comes out through the specially formed bushing on the top of the transformer case.

When a current transformer is carrying current, its secondary circuit must not be opened. Two bad effects may result: either the transformer may become overheated or else a dangerously high voltage may be generated in the open secondary.

252. The Testing Transformer.—All apparatus must be tested for insulation before it is put in use. The voltage used in such a test is specified at perhaps twice the normal voltage of the piece of apparatus being tested. Whenever a high-voltage power transformer, for example, is to be tested, there must be available, for making the test, a transformer which can generate about twice that of the transformer being tested. Also there is required a high voltage for making corona-loss tests, tests on the dielectric strength of air, etc. So it is necessary to build transformers (for test purposes only) that can generate between 500,000 and 1,000,000 volts. These test transformers do not have as much capacity as their dimensions would indicate; they are very bulky because of the space needed for the separation of the coils to give proper insulation.

Fig. 332 shows a testing transformer built for one million volts. One side of the high-tension winding is grounded to the case and it will be seen that the one high-tension terminal required is actually larger than the transformer itself.

PROBLEMS

1. A 110-volt, 5-kv.a., 60-cycle transformer has a core cross-section of 60 sq. cm. and average length of magnetic path in the core of 50 cm. If the primary winding has 150 turns, what is the effective value of the magnetizing current, and what is the maximum value of the flux density? (Neglect effect of current distortion.) Use the B–H curves given for silicon steel, in Fig. 49 of Vol. I. If the secondary voltage is to be 1200, how many turns must be used in this coil?

2. If the full-load copper loss of the above transformer is to be 120 watts (equally divided between the two windings) what will be the resistance of each coil?

3. The resistances of a transformer marked 110 volts on the low side, are 0.3 and 0.012 ohm. If the current density and average length of turn are the same for both windings, for what voltage is the high-tension side designed?

4. A 3-kv.a., 10-1 transformer gives 110 volts on the low-tension side. The copper loss is equally divided between the two windings and the full-load regulation is 2 per cent. On the assumption that there is no reactance drop in the windings (no magnetic leakage), what is the resistance of each coil, and what is the voltage drop in each at full load?

5. A 5-kv.a., 60-cycle, 10–1 transformer has 120 sq. cm. cross-section of core and 600 turns in the high-tension side. When the voltage impressed on the coil is 1000, the maximum flux density in the core is 6500 gausses. What would be the density if the impressed voltage is increased to 1200? If with this voltage the frequency was decreased to 50 cycles, what would the flux density become? What if the turns were decreased to 500, the core cross-section increased to 130 sq. cm. and the frequency changed to 55 cycles?

6. Two transformers having equal weights of iron, of the same quality, have flux densities of 4500 and 6500 gausses, respectively. What is the ratio of the two components of the core loss in the two?

(The eddy-current loss varies as the square of the flux density and as the square of the frequency; the hysteresis loss varies as the 1.6 power of the flux density and as the first power of the frequency.)

7. A 50 kv.a., 60-cycle transformer has 220 watts hysteresis loss and 45 watts eddy-current loss with a density in the core of 8500 lines per square centimeter. What would be the effect on the iron loss of increasing the flux density to 11,000, by raising the voltage. How will it be changed if the frequency is changed to 25 cycles and the voltage impressed is dropped to one-half its first value?

8. If a 25-cycle transformer is operated on a 60-cycle line (same voltage as that for which the transformer is rated), how will the core loss compare with its normal value?

9. If the density is not to exceed 4800 gausses, what must be the number of turns used in a 2200-volt, 60-cycle transformer having a core cross-section of 165 sq. cm.?

10. A 25-kv.a., 2200-110 volt transformer has a no-load loss of 175 watts and primary resistance of 0.07 ohm. What must be the resistance of the secondary? Calculate the efficiency of the transformer for every quarter load to 50 per cent overload.

11. Assuming the iron loss constant, show by calculus that maximum efficiency occurs when iron loss equals copper loss.

12. A 1100-110-volt, 10-kv.a. transformer is loaded as follows: 2½ hours full load, 5 hours half load, 5 hours quarter load. Maximum efficiency occurs at full load and is 96.2 per cent. What is the all-day efficiency of the transformer?

13. A 2200-110 volt, 25-kv.a. transformer has primary and secondary copper losses equal. The resistance of the 110-volt coil is 0.007 ohm. Total full-load leakage reactance (in terms of low voltage winding) is 0.0012 ohm. At no load the input reads 2200 volts, 1.3 amperes, 600 watts, and secondary voltage reads 112. What is the secondary terminal voltage when delivering 250 amperes to a resistive load? Find efficiency for loads of 250, 150, 50, and 10 amperes.

14. Having three 10-1 transformers with 950 turns each in the high-voltage windings, how many turns must be used in the secondary coils, if, when the primaries are connected in delta to a 1100-volt system the secondaries connected in Y, are to give 119 volts between lines?

15. Three 60-cycle transformers to transform from 229 to 110 volts, three-phase, are to have primaries in Y and secondaries in delta. If the flux density is 6500, and the core cross-section is 260 sq. cm., how many turns must there be in each coil?

16. A 10-kv.a. 2200-220 volt transformer has coil resistances of 5.45 and 0.0545 ohm respectively; the normal iron loss is 126 watts and the magnetizing current is 5 per cent of full-load current. Find the primary current and efficiency for secondary loads (resistive) of 40, 30, 20, 10, and 5 amperes.

17. Two transformers of identical ratings have equal copper losses. Transformer A has an iron loss of 125 watts and transformer B has an iron loss of 250 watts. How much more will it cost to operate transformer B per year, if power costs $0.04 per kilowatt-hour delivered at the transformer? If each dollar invested is required to earn 13 per cent, how much more could we afford to pay for transformer A?

CHAPTER VIII

THE INDUCTION MOTOR

253. General Construction.—An induction motor, like any other motor, consists of a stationary part and a revolving part. The stationary part is called the *stator* and the revolving part the *rotor*. The induction motor derives its name from the fact that the rotor is not connected *electrically* to the source of power supply; whatever currents circulate in the rotor conductors are not produced directly by the e.m.f. of the power supply, but *result from e.m.fs. induced in the rotor by the action of the magnetic field set up by the stator.*

254. Principle of Operation.—The principle of operation of the induction motor is just the same as that of any other type of motor; the rotating member (rotor) is fitted with a set of conductors in which currents flow and, as these conductors lie in the magnetic field produced by the stationary part (stator), a force is exerted on the conductors and the rotor tends to revolve.

The motor depends, for its operation, upon the generation of a *rotating magnetic field* and the first characteristic of the motor to be examined is this rotating magnetic field.

255. Rotating Magnetic Field.—A picture of the stator of a small induction motor, with its windings, is given in Fig. 333. From this illustration it may be seen that the stator coils are distributed over the inside surface of a laminated iron core. The coils are of few turns and resemble very much the coils for the armature of a c.c. generator. In fact, the stator winding resembles exactly the distributed or lap winding of an alternator.

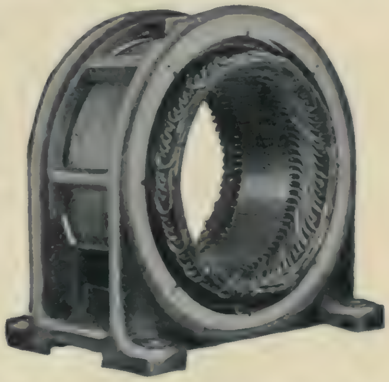


FIG. 333.—The stator (wound) of an induction motor; the winding resembles that of the armature of a small revolving-field alternator.

We saw, when studying armature reaction in a polyphase, rotating-field alternator, that when the armature was carrying current, and the field was rotating at synchronous speed, the armature windings produced an m.m.f. which was constant in magnitude and fixed in position with respect to the main field. With the field rotating at synchronous speed, for the m.m.f. produced by the armature to be fixed with respect to the field, the m.m.f. must be rotating with respect to the armature at synchronous speed, in the direction in which the field is rotating. In the alternator air gap, flux is produced by the main field, excited by continuous current; in the induction motor the m.m.f. set up by the stator, which is wound exactly like the armature of the rotating-field alternator, produces the air-gap flux.

256. Elementary Representation of a Stator Winding.—It is seen that the stator core, like the armature of a revolving-field alternator, has no projecting pole pieces, but to explain the production of the rotating field by the stator in another and an easier way, we shall suppose that there are salient pole pieces, each wound with an ordinary field coil.

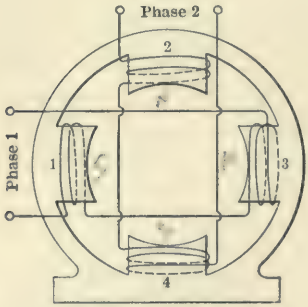


FIG. 334.

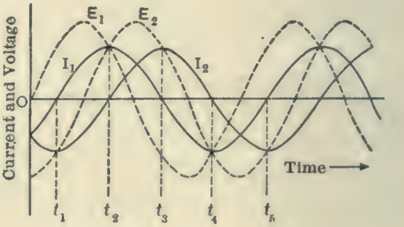


FIG. 335.

FIG. 334.—A conventional diagram of the stator of a two-pole, two-phase induction motor. The actual motor has no salient poles on its stator; they are shown here to make the conception of the rotating magnetic field easier to grasp.

FIG. 335.—Showing the voltages and currents supplied to the winding of Fig. 334.

The diagram of such a stator, for a two-phase, two poles per phase, motor is shown in Fig. 334. Poles 1 and 3 are wound in series and connected to the terminals marked *phase No. 1;* poles 2 and 4 are similarly wound and connected to the terminals marked *phase No. 2.* Moreover, these poles are so wound that a current through coils 1 and 3 produces a *S* pole at 1 and a *N* pole at 3; coils 2 and 4 are similarly wound in such a way that when a *N* pole is produced at 2, a *S* pole is produced at 4, etc.

Suppose that the terminals of the motor are connected to a two-phase power supply, the e.m.f. waves of which are shown in dotted lines in Fig. 335. In the two windings of the stator there will flow equal currents, each lagging behind the impressed e.m.fs. by the same amount, because of the similarity of the windings.

257. Analysis of the Field at Successive Instants.—We are to consider the magnetic field produced by the stator windings at different times, and therefore Fig. 335 is marked, t_1, t_2, t_3, etc. to represent the successive times. At a time t_1, the current I_1 is zero and hence poles 1 and 3 (Fig. 334) are not excited and have zero magnetic flux in them. At the same time the windings on poles 2 and 4 are carrying a maximum current because I_2 is a maximum at the time t_1; suppose the polarity is as shown, there being a N pole at 2 and a S pole at 4. Then, at the time t_1, the magnetic field is in the condition shown in the circle marked t_1 in Fig. 336, and in the sketch marked t_1 in Fig. 337. At a time t_2 (Fig. 335) the current I_1 is a maximum and I_2 has zero value; poles 1 and 3 are therefore magnetized while poles 2 and 4 now have zero magnetism. This gives a field as shown in the circle marked t_2 in Fig. 336.

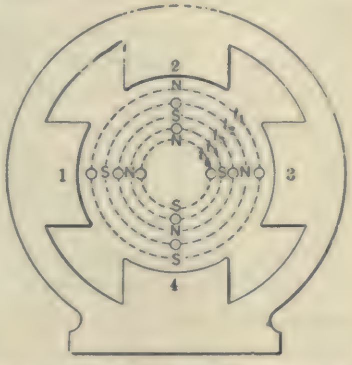


FIG. 336.—If we imagine the winding of Fig. 334 to be supplied with the currents of Fig. 335, a magnetic field will be set up, having polarities (at times t_1, t_2, etc., of Fig. 335) as indicated in the circles marked t_1, t_2, etc.

At a time t_3, windings 1 and 3 carry no current and windings 2 and 4 carry a maximum current, but this current is in the opposite direction to what it was at the time t_1. Hence the magnetic field is as shown on the third circle of Fig. 336, marked t_3. At a time t_4 poles 1 and 3 are magnetized but in the opposite direction to what they were at the time t_2, and poles 2 and 4 have no magnetism. This condition is shown on the circle t_4. At a time t_5 the magnetic field is in the same direction as it was at the time t_1, and after t_5 the previous cycle is repeated.

Direction of magnetic field for each 90° of a cycle.

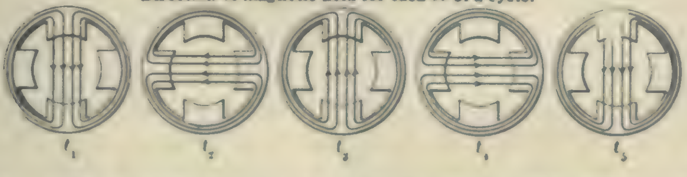


FIG. 337.—Another way of representing the sequence of magnetic fields given in Fig. 336.

At an instant between t_2 and t_3, when each phase is carrying positive current of instantaneous value equal to 0.707 of its maximum, there will be a N pole at both 3 and 4 and a S pole at both 1 and 2. The resultant flux will then be inclined upwards to the left in Fig. 337.

This succession of conditions in the magnetic circuit is evidently equivalent to a *rotating magnetic field;* this may be seen from Fig 337 which shows the direction of the fields given in Fig. 336.

258. Action of a Three-phase Winding.—A three-phase winding supplied with three-phase power gives a rotating field in exactly the same manner as has just been described for a two-phase winding; this may be seen from Figs. 270, and 271. These figures were used in the study of armature reaction, or of the effect of the m.m.f. set up by the armature of an alternator upon the m.m.f. of the main field. The stator winding of an induction motor being identical with the lap windings used for alternators, these figures may be used, by disregarding the field poles and considering that the armature is stationary, to show that when three-phase currents pass through the windings a rotating field is set up by them.

Figs. 256 to 259 showed that when one phase of a three-phase winding carried current, an m.m.f. was set up which varied proportionately with the current in magnitude, but which was fixed in position with respect to the phase winding. Fig. 270 considered the resultant m.m.f. of three phases, the current in phase No. 1 being at its maximum value; this “ stepped ” resultant was found to have a certain magnitude and was approximately replaceable by a sine wave. Fig. 271 represented an instant 30° later, the current in phases No. 1 and No. 2 being equal to 0.866 of their maximum values, and that in phase No. 3 being zero. The “ stepped ” resultant was found to have moved 30° with respect to the armature, and to be replaceable by a sine curve of magnitude equal to the sine curve in Fig. 270.

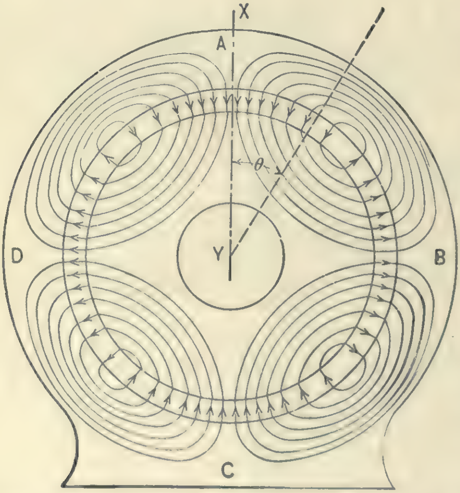


Fig. 338.—Magnetic field distribution of a four-pole induction motor.

The conclusion to be drawn is that the m.m.f. set up by the three-phase armature is constant from instant to instant and moves at synchronous speed with respect to the armature. The fact that the m.m.f. set up by the armature is replaceable by the same sine wave at any instant means that the field set up is sinusoidal in its space distribution, this follows because of the uniform reluctance of the magnetic circuit all around the machine; this being true because there are no salient poles on the stator or rotor. This sinusoidally distributed magnetic field remains of the same strength as it rotates, that is, the maximum strength of magnetic field, which will, of course, be at different parts of the stator at different moments, will always be of the same magnitude.

In Fig. 338 is shown the magnetic field distribution of a four-pole motor; it has maximum field density at four points around the air gap, A, B, C, and D, and these positions of maximum density are the poles of the motor, in so far as it has any. One pole is centered in the line $X-Y$, where the flux has its maximum density, $B_{\max}$. The sinusoidal distribution of flux is determined by the fact that at any other position, distant θ electrical degrees from $X-Y$, the flux has a density $B_{\max} \cos \theta$.

As this magnetic field rotates at a uniform velocity, it follows that at any fixed point in space, the flux density goes through sinusoidal variations with respect to time, that is, $B = B_{\max} \sin 2\pi ft$, where f is equal to one-half the number of poles (points of density $B_{\max}$) passing the point in question each second.

259. Speed of Rotation of the Field.—By referring to Figs. 336 and 337, it is seen that, in one cycle of current, the magnetic field travels from pole 1 all the way around the stator and back to pole 1. If we had represented a stator having *two pairs of poles per phase*, an analysis of the rotating field would have shown that it moves *half way around the stator* in the time required for one cycle of the current. It is seen that the magnetic field travels over *one pair of poles per phase for one cycle of current*, and this irrespective of the number of poles or phases. A two-phase motor having four poles per phase requires two cycles of current for a complete revolution of the magnetic field; this holds true whether the motor is two-phase, three-phase, or of any number of phases.

When it is stated that an induction motor has four poles, four poles per phase is always meant. A four-pole, three-phase motor, for example, would actually have twelve coils (or sets of coils). How fast will the magnetic field revolve in such a motor if it is supplied with 60-cycle power? As the motor has *two pairs* of poles per phase, it requires two cycles of current to make the field travel all the way around the stator. As there are 60 cycles per second the field will make $(60 \div 2) \times 60$, or 1800 r.p.m. Consider an eight-pole motor supplied with 25-cycle power; it would require 4 cycles of current to give the magnetic field one complete revolution; hence the field would turn $(25 \div 4) \times 60$ or 375 r.p.m.

260. Rotor Speed. Slip.—The speed at which the field of an induction motor turns is called the *synchronous speed of the motor*. We shall find that the rotor never turns as fast as the magnetic field, but that, when the motor is carrying load it turns perhaps 5 to 10 per cent slower.

The difference between rotor speed and field speed is termed the *slip*.

Slip may be expressed in r.p.m., but is generally expressed as per cent of the synchronous speed.

$$\text{Thus slip (per cent)} = \frac{\text{synchronous speed} - \text{rotor speed}}{\text{synchronous speed}} \times 100. \qquad . \quad (135)$$

Suppose a 60-cycle, sixteen-pole, three-phase motor has a slip at full load of 6 per cent; at what speed does the rotor turn at full load?

Synchronous speed $= 450$ r.p.m.;
Slip $= 450 \times 6$ per cent $= 27$ r.p.m.;
Rotor speed $= 450 - 27 = 423$ r.p.m.

261. Rotor Construction.—There are two types of rotor, one called the *squirrel-cage rotor* and the other the *wound rotor*. The cores of both must, of course, be of laminated iron; the distinction comes from the manner in which the conductors of the rotor winding are connected together.

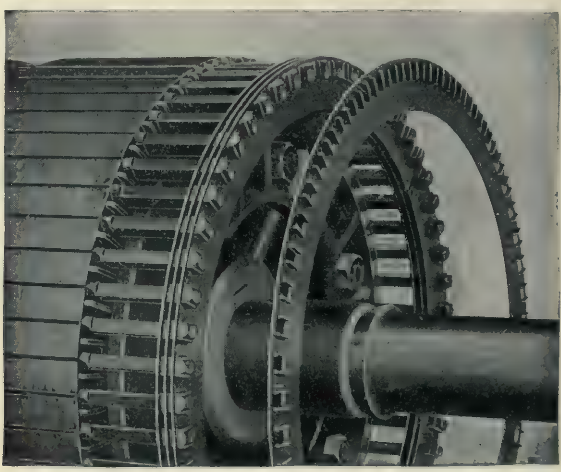


FIG. 339.—Showing how the heavy copper bars of a squirrel-cage rotor are all connected together at their ends by soldering them to heavy copper rings, these having holes properly punched to slip over the ends of the bar winding.

In the squirrel-cage rotor the winding consists of heavy copper bars embedded in slots in the periphery of the rotor, and *these copper bars are all short-circuited* at both ends of the rotor by being soldered to heavy copper rings. The rotor slots are always semiclosed (see p. 158, Volume I), so that the bars must be pushed in from the ends of the rotor core. As the voltage generated in such a winding is low, very little insulation is required on the bars; in fact, a heavy coat of insulating enamel generally proves sufficient.

An illustration of a squirrel-cage rotor, showing the bars and short-circuiting ring is given in Fig. 339. The bars are not always placed parallel to the shaft, but are sometimes "skewed" to some extent. This skewing tends to give a more uniform torque in starting and also to reduce the humming noise made when the machine is running.

The wound rotor is equipped with a winding very similar to that of a revolving-armature alternator. The different conductors are insulated from one another and arranged in coils. These coils are generally grouped to form a Y-connected three-phase winding and the three ends are connected to slip rings mounted on the rotor shaft as shown by the left-hand rotor of Fig. 340. This type of winding is therefore open-circuited until the brushes, bearing on the slip rings, are connected together.

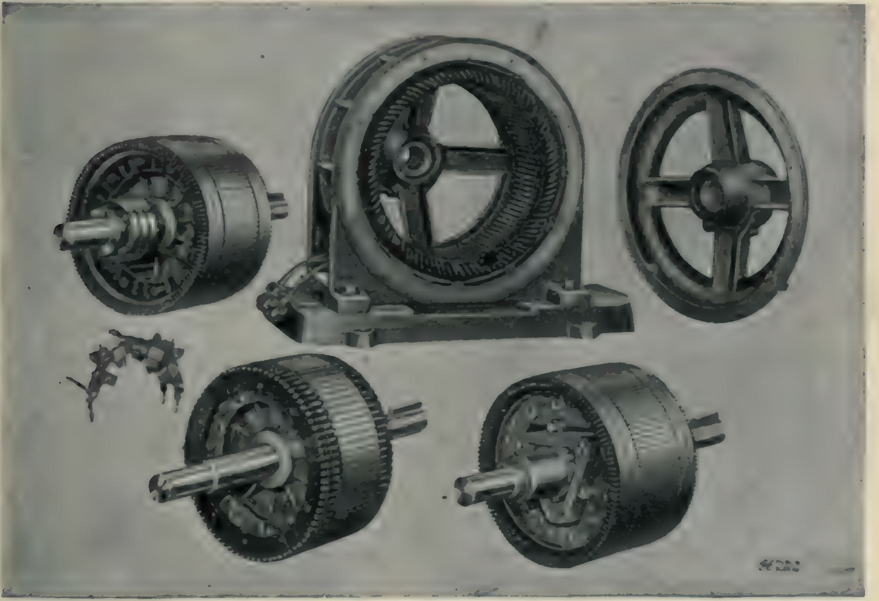


FIG. 340.—Three types of rotor designed to operate with the same stator. The lower right rotor has an enclosed resistance, the low left has a squirrel-cage winding, and the upper left is a wound rotor, having its winding connected to slip rings for insertion of external resistance.

The external circuit through which these brushes are connected may be of low or high resistance, and thus the resistance of the rotor circuit can be controlled by varying the resistance of this external circuit. This is done through a controller switch similar in appearance and construction to the railway controller.

The resistance may sometimes be located on the spider inside of the rotor itself. In such a rotor no slip rings are necessary; the ends of the rotor windings are connected to sliding fingers which make contact with the resistance. The position of these fingers controls the amount of resistance in the rotor circuit, and the position of the fingers is regulated by a lever which operates a sliding sleeve mounted on the rotor shaft.

The three types of rotor are shown in Fig. 340; the brush rigging for the slip-ring rotor is also shown.

For induction motors of less than 5 h.p., the squirrel-cage rotor is nearly always used; it may be used in sizes as large as 150 or 200 h.p. For larger motors the wound rotor with external resistance is generally used, for reasons given in the succeeding paragraphs.

262. Development of Torque.—Consider a squirrel-cage rotor in a revolving magnetic field. A cross-section of the magnetic field and rotor conductors is shown in Fig. 341. A two-pole motor is represented and it is supposed that the field turns counter-clockwise. If the rotor is stationary (or turning at any speed less than that of the field) the conductors on the top of the rotor will have a positive e.m.f. induced in them (into the paper in the figure) by application of Fleming's right-hand rule,[1] and those on the bottom will have a negative e.m.f. (out of the paper in the figure). And as the rotor conductors are all connected together by the end rings, currents will flow in the rotor in the same direction as these e.m.fs., so that the top conductors carry current away from the reader and the bottom conductors carry current toward the reader.

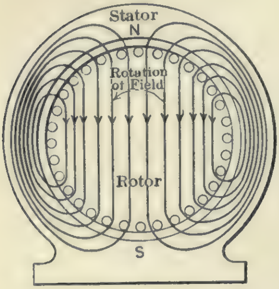


FIG. 341.—Conventional representation of rotor winding and stator field, to show how torque is developed.

But we know that a conductor carrying current in a magnetic field is acted on by a force which tends to move the conductor in a direction at right angles to itself. It is seen that the top conductors (Fig. 341) have a force urging them toward the left and the bottom ones have a force urging them toward the right. Hence the rotor develops a turning effort which tends to make it revolve *in the same direction as that in which the magnetic field is turning;* in other words, if the rotor can revolve it follows the magnetic field.

It has been shown that the rotor is urged in the same direction as the magnetic field. Another question now arises: How fast will the rotor turn? Suppose, first, that there is no load on the motor and that the friction is negligible, so that the rotor is perfectly free to revolve. As long as the rotor is turning at a speed less than that of the magnetic field, e.m.fs. are induced in its conductors, currents flow in the conductors, and a torque is developed which tends to make the rotor " catch up " with the field.

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[1] Fleming's right-hand rule considers the conductor as moving with respect to the flux; here the conductors are to be considered as moving in a clockwise direction if the field is considered as stationary.

263. Actual Rotor Speed Always Less than that of the Field.—When the rotor has caught up with the field so that field and rotor are turning at the same speed, *there is no cutting of the magnetic field by the rotor conductors*, and hence no currents flow in the rotor circuits. But when there is no current in the rotor there can be no torque developed, and hence the rotor can speed up no more. It follows, therefore, that the limiting speed for the rotor is the same as the speed of rotation of the magnetic field. Actually, there is always some friction for the rotor to overcome, and hence the rotor can never run at exactly synchronous speed (i.e., the speed of the field), but it approximates it very closely. In a certain motor having a synchronous speed of 900 r.p.m. the rotor turned 898 r.p.m. at no load, or, we might say, the rotor had a slip of 2 r.p.m. or about 0.2 per cent.

264. Frequency of Rotor Currents.—The magnetic field utilizes the rotor core for part of its path; any conductor on the periphery of the rotor will therefore have a sine wave of voltage induced in it, the frequency of this e.m.f. being the same as the frequency of the power supply if the rotor is stationary. In case the rotor is turning in the same direction as the field, a conductor is cut by the revolving magnetic poles less frequently, and therefore, the frequency of the rotor e.m.f. diminishes as the rotor speeds up, being zero if the rotor should happen to be turning at the same speed as the field, a condition which never occurs in motor operation. As the speed of the rotor increases (in the direction of the rotating field) not only does the frequency of the rotor e.m.f. diminish, but the magnitude of the e.m.f. diminishes also, this, like the frequency, being directly proportional to the difference in the speeds of the rotor and the field.

265. Distribution of Rotor E.M.Fs.—The ordinary small induction motor has a squirrel-cage winding, a set of bars imbedded in the rotor slots with short-circuiting rings connecting all the bars together at their ends. Evidently, at a certain instant, the various bars of the rotor will have different e.m.fs. induced in them, because they are lying in fields of different density. A crude idea of this voltage distribution is given in Fig. 342, where the length of the various arrows indicates the relative magnitude of the e.m.f. induced in the different conductors. The conductors at the positions marked A in Fig. 342 would correspond to those in the densest fields in Fig. 338. As the magnetic field rotates faster than the rotor, the inductors of the rotor which have the maximum e.m.f. in them continually move forward over the surface of the rotor, so that in any one rotor conductor the induced e.m.f. goes through alternating values, the number of cycles per second being fixed by the relative speeds of the rotor and rotating field.

266. Reactance of Rotor Circuits.—As the rotor circuits have reactance as well as resistance, the current which flows in the rotor bars will not be in phase with the rotor e.m.f., but behind it by an angle depending upon the ratio of the reactance to resistance. As the e.m.f. in each conductor is a sine wave, the current in any conductor will be a sine wave, and the current will be distributed among the rotor bars in just the same manner as is the voltage. An attempt to picture this is shown in Fig. 343. The full lines give the voltage distribution at a certain instant, and the dashed lines show the currents in the same conductors for the same instant. It is seen that the position of maximum current does not coincide with that of maximum voltage; in fact, the conductor having maximum current is behind that having maximum voltage by an electrical angle θ, fixed by the relation $\tan\theta = \frac{\text{reactance of rotor circuits}}{\text{resistance of rotor circuits}}$, which, in the case of the four-pole motor shown, is represented by a mechanical angle just half as large.

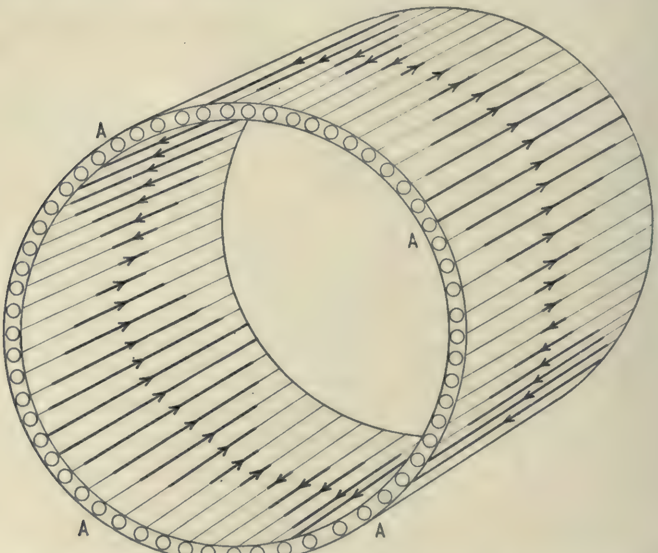


FIG. 342.—The distribution of induced voltages in the rotor conductors of a four-pole motor. The relative lengths of the heavy lines with arrow-heads, give the relative magnitudes of the voltages in their respective conductors.

267. Torque Exerted.—That torque will be exerted by the motor may be predicted from the fundamental principle that a conductor carrying current, and situated in a magnetic field, experiences a force which is proportional to the product of the current and field and is in a direction perpendicular to both. Now the magnetic field has its maximum density where the rotor conductor e.m.fs. are the greatest; hence the conductors having the greatest rotor current are not in the fields of maximum density, as they should be to get maximum possible torque.

As the current in the rotor bars always lags somewhat behind the e.m.f. in the bar, and as the e.m.f. takes its sign from the polarity of the field in which the bar is lying, it will be seen from such a diagram as Fig. 338 (or by analysis of Fig. 343), that some of the rotor conductors are carrying positive current but are situated in negative magnetic fields; owing to this effect the net torque is less than it should be. By increasing the resistance of the rotor circuit by some scheme or other, the angle of lag of the current behind the voltage can be lessened. This angle would also be lessened if the reactance of the rotor circuit could be lessened; but after the rotor is once built, this cannot be done economically. Of course, increasing the resistance of the rotor circuit will diminish the current in the circuit, but this diminution in current will not at first decrease the net torque as much as the decrease in angle of lag increases the torque. This is suggested in Fig. 344, in which the effect of increasing the rotor-circuit resistance is shown. Small increases in rotor-circuit resistance do

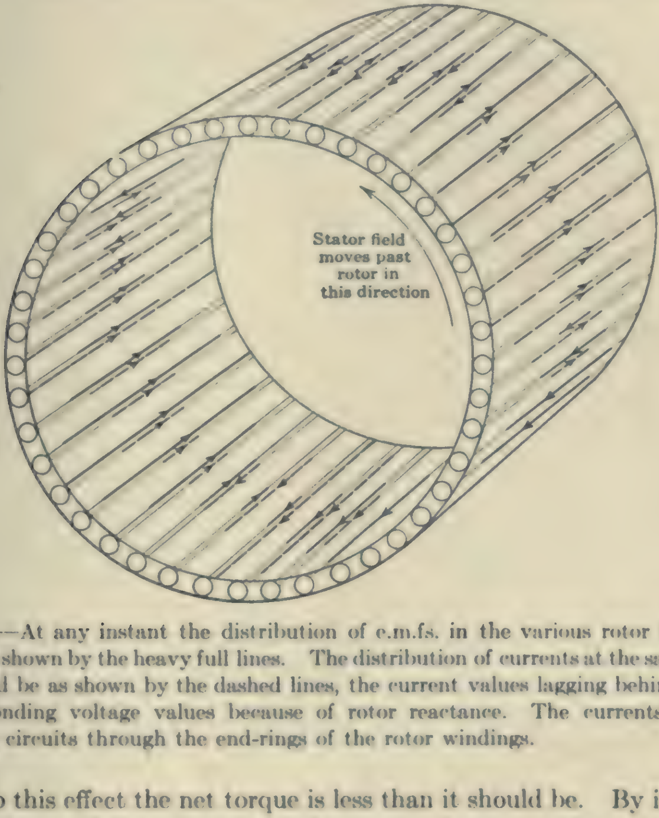


FIG. 343.—At any instant the distribution of e.m.fs. in the various rotor bars might be as shown by the heavy full lines. The distribution of currents at the same instant would be as shown by the dashed lines, the current values lagging behind the corresponding voltage values because of rotor reactance. The currents complete their circuits through the end-rings of the rotor windings.

not diminish the current very much, but they do materially increase the cosine of the angle of lag, and the greater the cosine (for a given current) the greater the torque.

It may be shown mathematically that maximum torque is exerted when the rotor-circuit resistance has been increased to equal the rotor-circuit reactance, a condition which exists with 0.95 ohm in Fig. 344. If the rotor-circuit resistance is further increased the resultant decrease in current decreases the torque more than the accompanying increase in the cosine of the angle of lag increases it. When the reactance and resistance of the rotor circuits are equal, the angle of lag of the rotor currents behind the rotor voltages is 45°.

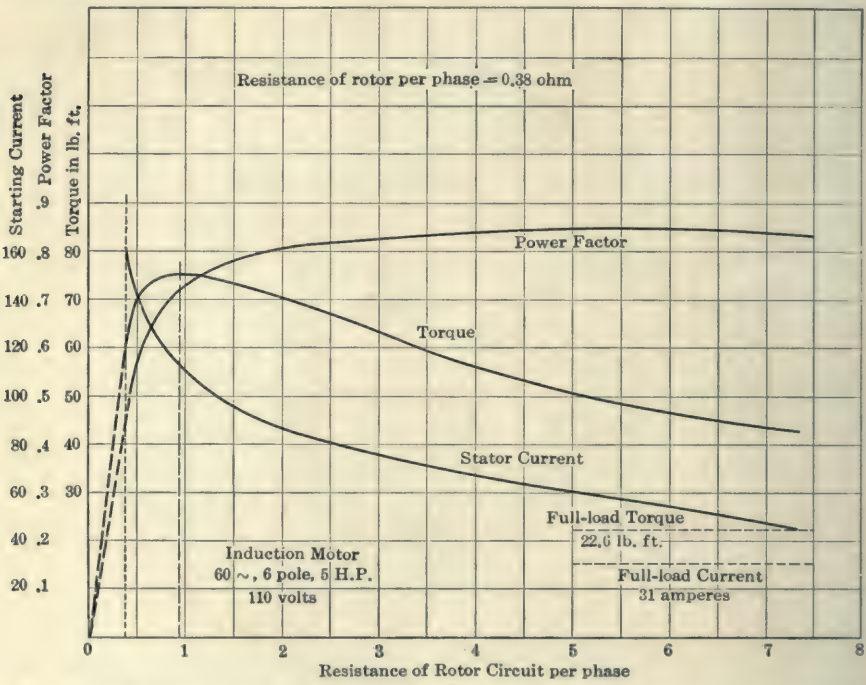


FIG. 344.—Showing the effect of rotor-circuit resistance in the starting characteristics of a three-phase induction motor. The starting current with no added resistance in the rotor-circuit is more than five times the rated full-load current.

268. Starting Characteristics of a Squirrel-cage Motor.—The induction motor, especially at standstill, is seen to be nothing but a transformer with short-circuited secondary. The current taken from the line, if the stator is directly connected to it, must thus be very large, much larger than full-load current. The case is not quite as bad as it would be with the ordinary constant-potential transformer, because the magnetic leakage between the stator and rotor circuits is much greater than it is between the primary and secondary of an ordinary transformer. This magnetic leakage (large because of the air gap between rotor and stator) helps to limit the current taken by the motor at starting. But even so, the starting current of a squirrel-cage induction motor may be from five to six times full-load current.

The second bad feature in connection with the starting of a squirrel-cage motor is that the starting torque, per ampere in the stator, is low; thus even with a starting current of five to six times full-load current, the starting torque will only be about twice full-load torque. This is due to the fact that the rotor reactance at standstill is high compared with the rotor-circuit resistance, so that the cosine of the angle of lag of rotor current behind rotor e.m.f. is low.

The third bad feature is that the power factor of the starting current is very low, and this, combined with the fact that the current is large, produces bad fluctuations in the line voltage when such a motor is switched to the supply line. A certain 5-h.p. motor caused the line voltage to drop from 110 volts to 102 volts when starting, although when taking its normal full-load current and running at its normal speed, the line voltage held up to 109 volts. Such a fluctuation in the line voltage is prohibitive if lamps are connected to the line; and even if there is no lamp load to suffer, synchronous motors or synchronous converters are likely to be disturbed in their operation and may even pull out of step if the fluctuation is excessive.

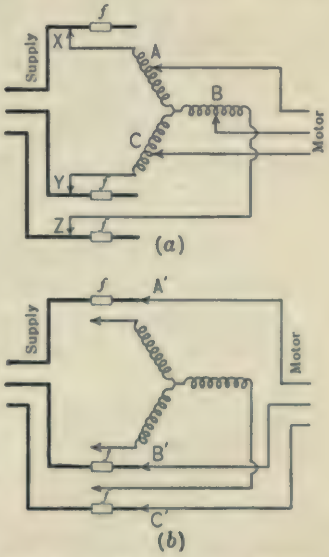


Fig. 345.—Connections of a compensator to the starting switch; its use limits the starting current taken by the motor to a reasonable value.

269. Starting of Squirrel-cage Motors.—Squirrel-cage induction motors are used where only occasional starting is necessary, and where the starting torque required is not above the full rated value of the motor. To limit the stator current to reasonable values, the voltage across the stator is properly reduced, this being usually done by means of an auto-transformer or *compensator*. Such a scheme is indicated in Fig. 345, the auto-transformer being three-phase, Y-connected.

The first position of the starting switch (Fig. 345*a*) performs two operations. It connects the terminals of the transformer to the line, on the line side of the fuses, thus exciting the transformer, and also connects the motor terminals to suitable taps, *A*, *B*, *C*, on the transformer, thus impressing a desired voltage (lower than line value) upon the stator. After the motor has come up to speed, a second movement of the starting switch (Fig. 345*b*) quickly transfers the motor terminals to the lines A', B', C', on the load side of the fuses, and disconnects the auto-transformer from the line. The fuses, being designed to protect against overload while the motor is running, must not be connected in the lines while the heavy starting current is being drawn.

The auto-transformer, besides reducing the voltage impressed upon the line, also reduces the current taken from the line. Thus, if a given motor required 600 amperes of current when rated voltage was impressed on the stator, and developed twice rated torque, when connected across half-voltage taps on the transformer the motor current would be 300 amperes, but the current taken from the line by the transformer would be only 150 amperes.

The torque of an induction motor varies as the square of the voltage impressed upon the stator, other things remaining constant. This follows when we consider that doubling the stator voltage doubles the flux, which induces twice as much rotor voltage, and results in the doubling of the rotor current. The torque, being proportional to the product of flux and rotor current, will therefore be doubled twice, or will be four times as great.

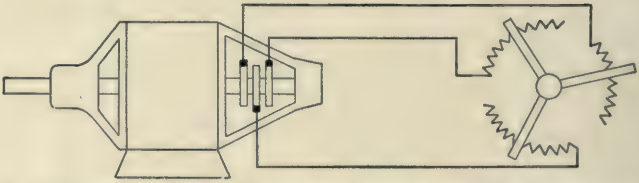


FIG. 346.—In starting, the slip rings of the motor are generally connected to a variable three-phase resistor, arranged in the Y-connection.

With the motor considered above, to develop rated torque, 71 per cent rated voltage must be impressed upon the stator, which results in a current of $0.71 \times 600 = 426$ amperes to the motor and $0.71 \times 426 = 300$ amperes from the line.

270. Rotor Circuit with Variable Resistance.—A high-resistance squirrel-cage rotor would have, when starting, none of the bad features mentioned above, but the efficiency of a squirrel-cage motor with a high-resistance rotor is necessarily low and the speed regulation is poor. It is therefore advantageous to have a rotor circuit the resistance of which may be made equal to or greater than the reactance at standstill, and then reduced as the motor speeds up and the rotor reactance decreases. As it is not convenient to vary the resistance of a squirrel-cage rotor, a wound rotor must be used. A wound rotor is practically always fitted with a three-phase, Y-connected winding, the open ends being brought to slip rings; a Y-connected resistance is then connected to the rings, as shown in Fig. 346.

The effect of rotor resistance upon the starting characteristics of a motor was shown in Fig. 344. These curves were obtained by a laboratory test of a small motor having a wound rotor. Various resistances were inserted in the external circuit of the rotor and the different quantities measured. As the rotor itself had a resistance of 0.38 ohm per phase, the quantities could not be measured for a rotor-circuit resistance less than this, but the curves were extrapolated, as shown by the dashed lines.

271. Effect of Rotor Resistance upon Starting Characteristics.—It is seen from these results that, as external resistance is added to the rotor circuit, the torque and the power factor increase, while the current decreases. A certain total resistance (in Fig. 344 this is 0.95 ohm) gives a maximum starting torque; if more than this is inserted the torque again falls off. The proper starting resistance for this motor would be about 7.3 ohms; for this resistance, twice rated torque is exerted, the power factor is comparatively high, and only about 150 per cent of rated current is drawn from the line. If greater torque is required to start the motor, the resistance may be decreased.

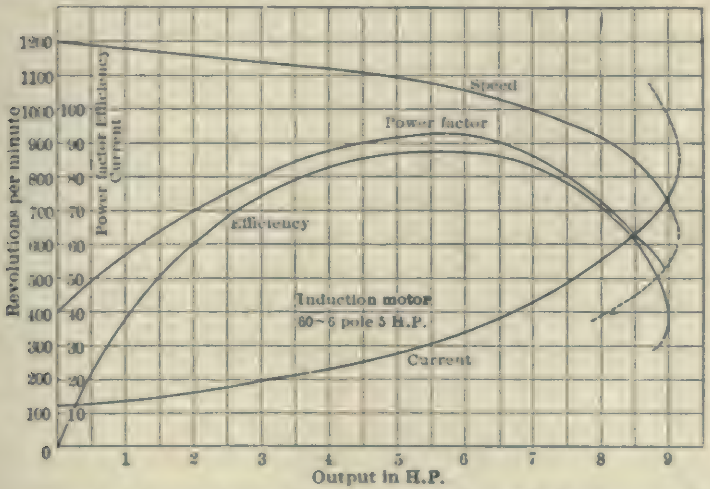


FIG. 347.—Running characteristics of the ordinary and small induction motor, having 6 poles and supplied with 60-cycle current.

272. Running Characteristics.—The running characteristics of the ordinary small induction motor, whether of the squirrel-cage or wound-rotor type, with no added resistance in the rotor circuit; are about as shown in Fig. 347. The no-load current (called the "running-light" current) is generally between 20 and 40 per cent of the full-load current.

The current increases with increase of load, and increases more rapidly for the overloads.

As an induction motor is loaded, it is called upon to increase its torque; in order to exert more torque, the product of stator flux and rotor current must increase. The stator flux remains practically constant, so that to increase the torque of the motor the rotor current must increase; this means that greater rotor e.m.f. must be induced, and this demands greater slip. Thus the speed falls off slightly as the load increases, the decrease from no-load to full-load being generally less than 10 per cent of synchronous speed. The difference between the field speed and the rotor speed (i.e., the slip) increases rapidly with overloads; if the motor is loaded too much, it stops altogether and is said to be "stalled." Under such conditions the circuit-breakers (or fuses) in series with the motor open the circuit, and the load must be removed, the circuit closed, and the motor again started. This maximum load point is generally 75 or 100 per cent greater than the rated load of the motor; it varies with different motors, but should always be considerably greater than the rated load, otherwise the motor is likely to be "stalled" frequently.

273. Effect of Rotor Resistance upon Operating Characteristics.—The motor whose characteristics are shown in Fig. 347 was equipped with a slip-ring rotor. For the purposes of the test the rotor-circuit resistance was increased by inserting extra resistance between the brushes on the slip rings and the motor characteristics were again obtained. The results are given in Fig. 348. They show that an increase in rotor resistance produces, for a given load, an increased slip, a decreased efficiency, and an increase in current. Also, the maximum load obtainable from the motor is now only 6 h.p., whereas with no extra resistance it was nearly 10 h.p.

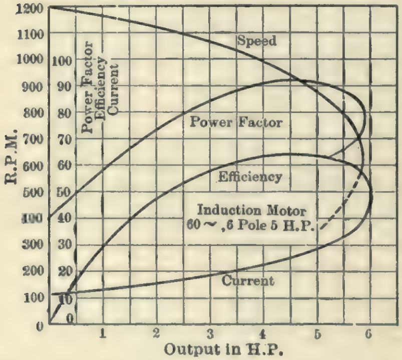


Fig. 348.—By adding resistance to the rotor circuit of the motor used in getting the curves of Fig. 347, its characteristics were changed to those shown here.

If still more resistance were added to the rotor circuit, the speed and efficiency for a given output would be still further decreased. If too much resistance is put in the rotor circuit, very little output can be obtained from it; *practically all of the power which goes into the stator of the motor is used up as heat in the rotor-circuit resistance grids.*

By using a motor with a wound rotor and inserting external resistance, it is possible to obtain good starting characteristics, i.e., low current, high torque and power factor. As the rotor speeds up the external resistance is gradually cut out, and the motor is normally operated with no external resistance in the rotor circuit. Thus the running characteristics are similar to those of the squirrel-cage motor, but the starting characteristics are much better than those of the squirrel-cage rotor.

274. Speed Control.—Several methods have been designed for varying the speed of an induction motor for a given output, but none of them is very successful; the induction motor is essentially a constant-speed motor.

Variation of Rotor-circuit Resistance.—The first method consists of inserting resistance in the rotor circuit. We explained in the last paragraph that this will cut down the rotor speed, for a given load, but that it results also in a low efficiency, poor speed regulation, and reduced h.p. output. Hence the rotor-resistance control scheme can be used economically only during the time the motor is being started; in this way it is similar to the resistance used for the control of series c.c. railway motors.

Power Supply of Several Frequencies.—It was at one time thought feasible to have lines of two or three frequencies and to switch the motor from one line to the other as a change in speed might be desired. We have shown that the rotor always turns with a speed somewhat less than that of the revolving field and that the speed of rotation of the field is determined by the frequency of the power supply. A motor designed for 25 cycles will run at more than twice its rated speed if connected to a 60-cycle line. In fact, the speed of the motor is practically proportional to the frequency of the power supply.

A motor designed for a low frequency can safely be run on higher frequency lines, provided the mechanical strains in the rotor are not dangerous at the higher speed. But a motor designed for 60 cycles should never be operated on a 25-cycle line because of the excessive densities resulting in the magnetic circuit and the correspondingly high hysteresis loss. The multiple-frequency scheme for speed control never came into prominence, owing probably to the complexity of the wiring and station apparatus.

Varying the Number of Poles.—In another scheme for speed control the windings of the stator are connected through a series of switches; by proper operation of these switches it is possible to change the number of poles on the stator. Thus a motor might have its windings so connected through these switches that when they are thrown one way the motor has eight poles and when thrown in the opposite way the motor has six poles. This scheme, while it gives an efficient speed control, requires rather complicated connections on the coils and so is not used as yet to any great extent.

Cascade Connection.—In some installations (notably railway equipment) two motors are connected in concatenation, or cascade. The stator of the first motor is connected to the line, the slip rings of the first rotor are connected to the stator windings of the second, and the second rotor circuit is closed through a variable resistance. When all the resistance is cut out of rotor No. 2, both motors operate at half the speed they would have if directly connected to the line (with rotor short-circuited). This scheme is only applicable where two equal motors are connected to the same load, as in electric cars.

There are other methods for controlling the speed of an induction motor, but they are all either inefficient or complicated and high in first cost. We must conclude as before that the induction motor is *essentially a constant-speed motor*.

275. Induction Motor Used as a Generator.—If an induction motor, connected to a polyphase line and running, is driven by some outside source of power at a speed greater than synchronous speed (in the same direction it had as a motor) the machine will act as a generator and so will *give off* electrical power instead of *absorbing* it. An induction machine so used is called an *induction generator* or sometimes an *asynchronous generator*. Such an induction machine cannot operate as a generator unless it is connected to a supply line of *fixed voltage and frequency* and is then driven above synchronous speed for this frequency.

The amount of load such a machine furnishes to the line is determined entirely by its speed; if it is driven 4 per cent above synchronous speed it will furnish about twice as much power as though its speed were only 2 per cent greater than synchronous speed. In the case of the induction generator, the rotor is turning *faster than the magnetic field* and the slip (i.e., difference between the field speed and the rotor speed) is said to be *negative*.

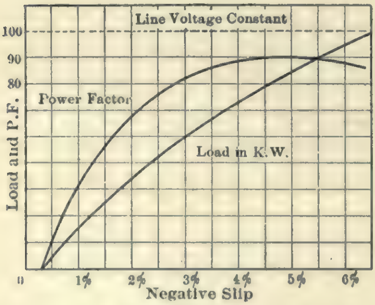


FIG. 349.—Showing how the load and power factor of an induction generator change as it is driven above synchronous speed.

276. Advantages of the Induction Generator.—The operating curves for a large induction generator are given in Fig. 349. The chief advantage of this type of generator, and the reason for its occasional use in large power stations, is its *short-circuit characteristic*. If an ordinary alternator were short-circuited it would burn up in a few seconds, but *if an induction generator is short-circuited it stops generating* and so cannot harm itself.

277. Short-circuit Risk of a Power Station.—The operating engineer often uses the phrase "short-circuit risk of a station"; by this term is means the danger of burning up the apparatus of a station when a short-circuit occurs on the bus-bars. In case the short-circuit occurs in a place where the protective apparatus cannot operate, there is much danger of burning up the synchronous generators in the station. But when the short-circuit occurs the voltage on the bus-bars drops, and so whatever induction generators are connected to the bus-bars stop generating and so do themselves no harm.

A synchronous generator (i.e., an ordinary alternator), on the other hand, if short-circuited, would be very quickly damaged, either by excessive mechanical strains or else by overheating.

A station having 50,000 kw. capacity of synchronous generators and 50,000 kw. capacity of induction generators in parallel with them has a short-circuit risk of only 50,000 kw. while its operating capacity is nearly 100,000 kw. It is thus advantageous to have a good share of the generators in a large station of the induction, or asynchronous type, but too many induction generators would result in a low power factor in the station and hence it is not advisable to have much more than half of the generators of this type.

Induction generators are not extensively used at present, however, the advantages to be gained by their short-circuit characteristics being equally well obtained by suitably placed reactors in bus-bars and generator leads. (See Fig. 280.)

The regenerative action of an induction motor running above synchronous speed is used to some extent in electric railway installations; a car running down hill may drive its induction motors at higher than synchronous speed and thus furnish power to the rest of the system. In addition to the saving in power of such a scheme, there is the advantage that the motors are acting as brakes, thus saving the braking equipment. Induction motors installed for hoisting purposes may be used as induction generators in the same way.

PROBLEMS

1. Construct a table showing synchronous speeds for 2, 4, 6, 8, 10, and 12-pole, three-phase motors, for 60-cycle and 25-cycle power supply.

2. An 8-pole, 60-cycle, 3-phase motor has what frequency currents in the rotor when running at 898 r.p.m.? 850 r.p.m.? 7 per cent slip? 100 per cent slip?

3. The stator and rotor of a 220-volt induction motor are each Y-connected. When the stator current is 50 amperes (full load) the rotor current is 126 amperes. At 5 per cent slip, what is the magnitude and frequency of the generated voltage per phase in the rotor? (Use transformer turn-ratio relations.)

4. The flux density and volume of iron used in the rotor and stator of an induction motor are the same. How do the eddy-current losses, and hysteresis losses, compare when the rotor is running with 2 per cent slip? 10 per cent slip? Standstill?

5. A stator has 48 inductors. Give a winding diagram to make a 4-pole, 3-phase, Y-connected stator.

6. A 150-h.p., 220-volt, 60-cycle, 8-pole motor has a delta-connected stator and a Y-connected rotor. With full load the efficiency is 93 per cent and the power factor is 86 per cent. There are 288 stator inductors and 224 rotor inductors. Neglecting the magnetizing current, what are the stator and rotor currents at full load? At standstill what e.m.f. is induced in each phase of the rotor? At what frequency?

7. If the standstill reactance (per phase) of the rotor of Prob. 6 is 0.26 ohm, at what speed will the rotor turn, if generating maximum possible torque, with a Y-connected resistor of 0.072 ohm per phase used to complete the rotor circuits, the rotor resistance being 0.016 ohm per phase?

8. The total no-load losses in a 440-volt, 30-h.p., Y-connected motor are 1220 watts. The maximum magnetizing ampere-turns required for the air gap are 220, and for the iron are 44. There are 10 turns per pole per phase. What is the no-load current (effective value), and what is the power factor. If the turn ratio is one to one, what is the approximate value of the stator current and its power factor, if the rotor, lightly loaded, is carrying 10 amperes?

9. A 60-cycle, 4-pole, 3-phase induction motor, having a rotor resistance of 0.062 ohm per phase, shows maximum torque with a speed of 1560 r.p.m. What is the self-inductance per phase of the rotor?

10. How much resistance is to be used in each phase of a Y-connected resistor, if the motor of Prob. 9 is to give its maximum possible torque at standstill?

CHAPTER IX

THE SYNCHRONOUS MOTOR

278. Feasibility of Running an Alternator as a Motor.—If two alternating-current generators are operating in parallel on the same bus-bars and the driving power is taken away from one of them, *it will (in general) continue to run at exactly the same speed it had before the driving power was taken off.*

Suppose two engine-driven generators, operating in parallel, are running at 720 r.p.m. An accurate speed-indicating device is put on one of them, so that its speed can be read, then the steam is shut off from the engine to which this generator is connected. We should naturally expect the generator to slow down and stop when its driving source is removed, but, by watching the speed indicator while the steam is being shut off from the engine, we may see that the alternator not only does not stop but does not perceptibly change its speed while the steam is being shut off. The speed does not even drop to 719.9 r.p.m. but remains at 720 r.p.m., provided the speed of the other alternator is held constant at 720 r.p.m. during the operation.

279. Reversal of Operation.—Now, a machine cannot rotate unless it is being supplied with power, and as the steam engine is delivering no mechanical power it is evident that the machine must be receiving electrical power. Suppose that two single-phase alternators are operating in parallel as shown in Fig. 350 and that the power output of No. 2 is indicated by the wattmeter W. It is supposed that W is so connected that when alternator No. 2 is helping No. 1 to carry the load it reads *positive power* or *power output.* Now when the steam is shut off from the prime mover of No. *2* it will be noticed that W indicates *negative power* or *power input;* that is, *generator No. 2 is running as a motor* and when so running it is styled a *synchronous motor.*

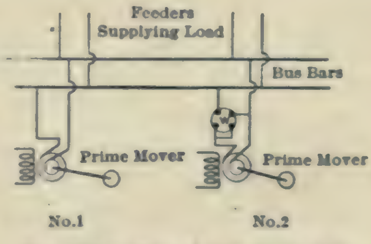


FIG. 350.—Two single-phase alternators operating in parallel.

280. Speed of a Synchronous Motor.—An alternator which is to be used as a synchronous motor differs, in some features of design, from a generator in order that it may be better adapted to the particular purpose for which it is to be used, but essentially a synchronous motor is nothing but an alternating-current generator reversing its normal manner of operation. In two respects the synchronous motor and generator are identical; they both have separately excited fields (generally rotating) and they both run at synchronous speed, no matter what the load may be. A 10-pole synchronous motor supplied with 25-cycle power would run 300 r.p.m. no-load, 300 r.p.m. half-load, and 300 r.p.m. full-load, and the same speed even if overloaded. But a 10-pole generator running 300 r.p.m. would generate an e.m.f. of 25 cycles per second; hence the significance of the term " synchronous " motor.

281. Operation of a Synchronous Motor.—As a synchronous motor, of itself, has no starting torque, some auxiliary device must be used to bring it up to speed. Consider a four-pole field structure excited by continuous current, with an armature carrying a single coil, the coil-sides of which are *A* and *B*, as in Fig. 351*a*. With the armature stationary, when an alternating current of, say, 60 cycles, is passed through the coil, there will be 60 impulses per second tending to cause the armature to move in a clockwise direction, and 60 impulses per second tending to make the armature move in a counter-clockwise direction. Whenever the current is flowing into the paper in coil-side *A*, the coil will be urged to the left; and at instants when the current in *A* is flowing out of the paper, the coil will be urged to the right; the net torque acting on the coil will therefore be zero.

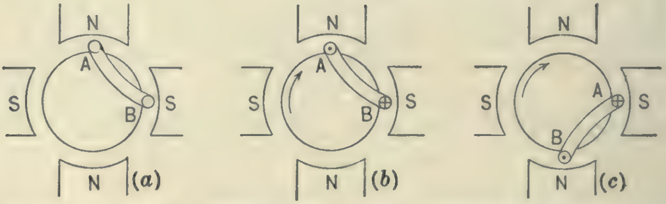


Fig. 351.—To show why a synchronous motor, operating at synchronous speed, develops unidirectional torque.

Consider, however, that the armature is rotating in clockwise direction at synchronous speed and that, at the instant shown in Fig. 351*b*, the current is at its maximum value and is flowing out of the paper in *A* and into the paper in *B*. Each coil-side will at this instant exert a torque tending to rotate the armature in clockwise direction, the direction with which the armature was assumed as turning at synchronous speed.

One-half cycle of the current later, the direction of the current in both coil-sides will be reversed; in *A* it will be flowing into the paper with maximum value and in *B* it will be flowing out of the paper with the same value. Since the armature is turning at synchronous speed, during the time the current in coil-side A has been decreasing from maximum value out of the paper to maximum value into the paper, coil-side A will have moved 180 electrical degrees from its position as shown in Fig. 351*b*, and will be in the position shown in Fig. 351*c*. The torque exerted by both coil-sides will again be such as to urge the armature in clockwise direction, since both the direction of the current flowing in a coil-side and that of the flux in which it is situated are now reversed.

The above discussion shows that the torque exerted by a single coil, moving at synchronous speed as in Fig. 351, is pulsating, but always in the same direction; with a polyphase winding the torque exerted is constant. The discussion also indicates why the armature can maintain its rotation only when it is exactly at synchronous speed.

282. Starting Characteristics.—As explained above, the synchronous motor may be started by some auxiliary driver and, after being brought up to synchronous speed and proper voltage, it may be connected to the line just as though it were an "incoming" alternator. (A description of the operations necessary to synchronize an incoming alternator was given on page 282.) If a separate starting device is used it is generally a small induction motor (of 5 to 10 per cent of the rating of the synchronous motor) mounted on the same shaft with the armature of the synchronous motor.

If a source of c.c. current is available, and the exciter of the motor (to be discussed later) is mounted on the same shaft, the exciter is sometimes used to start the synchronous motor.

If an induction motor is used it must have at least one pair of poles less than the synchronous motor, otherwise the synchronous motor could not be brought up to synchronous speed. This is because the speed of an induction motor is from 5 to 10 per cent less than synchronous speed. If a ten-pole synchronous motor is to be started by an induction motor the latter will have only eight poles. Suppose the power supply is 60 cycles; then the synchronous speed is 720 r.p.m. for a ten-pole machine and 900 r.p.m. for an eight-pole machine. The induction motor would therefore be designed to run at a speed 20 per cent (180 r.p.m.) less than synchronous when it is supplying a load just equal to the *stray power losses* of the synchronous motor. It is always wound for the same voltage and number of phases as the synchronous motor, so that the same bus-bars may be used to feed both motors.

283. Induction Motor Method of Starting.—Another method of starting is called *the induction motor method*. In this method no extra starting motor is necessary as the synchronous motor itself is made to act as an induction motor. We have seen that when an armature, wound with a polyphase winding, is supplied with polyphase currents, it generates a rotating magnetic field. This rotating magnetic field produces eddy currents in the pole faces and the *starting winding* (see next paragraph), and these eddy currents react on the armature to make it revolve. By this action the armature is accelerated until synchronous speed is reached. During the period of acceleration the field of the synchronous motor is left without excitation; after synchronous speed has been reached it is gradually excited and, if the *armature current decreases, the excitation is increased until the normal value is obtained.* It sometimes happens that the armature pulls into synchronism with improper polarity, in which case the *armature current will increase as the field current is increased.* If the c.c. field of the motor is strengthened somewhat, the motor will "slip a pole and come into step," the armature current increasing momentarily during the operation.

284. High Starting Current and Low Torque.—The current taken from the line for starting a synchronous motor in this manner is generally two or three times the full-load current, but in spite of this large current the starting torque is not high unless the machine has been properly designed. It was said that this method of starting depended upon the reaction of the eddy currents in the pole faces, but with a laminated pole we know these currents cannot be high because of the subdivision of the path for the eddy currents. To remedy this defect a bar winding is generally fitted in the pole faces, thus permitting the necessary currents to flow in the pole faces when starting; this is generally called the *starting winding.*

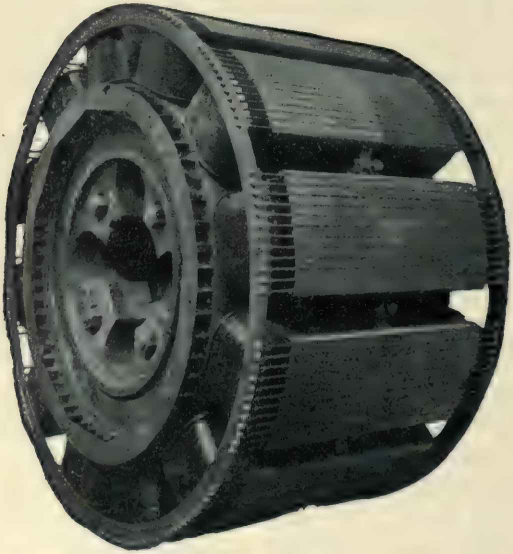


Fig. 352.—Showing the heavy bar winding in the pole faces of a synchronous motor.

285. Starting Winding in Pole Faces.—This consists of heavy bars of brass or copper imbedded in slots in the pole faces (the direction of the slots is parallel to the armature shaft) and short-circuited at their ends by brass end rings, as in Fig. 352. It will be seen that the bars and end rings really constitute the equivalent of the rotor of a squirrel-cage induction motor. The cage winding also tends to damp out oscillations of the armature (called "hunting") and because of this action the winding is sometimes called a "damping winding."

286. Low-voltage Taps for Starting.—In starting a motor by the induction method, it is not feasible to connect it, when stationary, to the line of normal voltage, because the starting current would be so excessive (from 5 to 10 times the rated current) that the armature winding might be injured. Therefore the transformers feeding the motor are usually fitted with half-voltage taps and power is taken from these taps to the lower sides of a double-throw switch, to the blades of which the motor armature is connected as in Fig. 353. The starting switch is thrown down at first and held there until the motor reaches synchronous speed, when it is quickly thrown to its upper position, which is the running position.

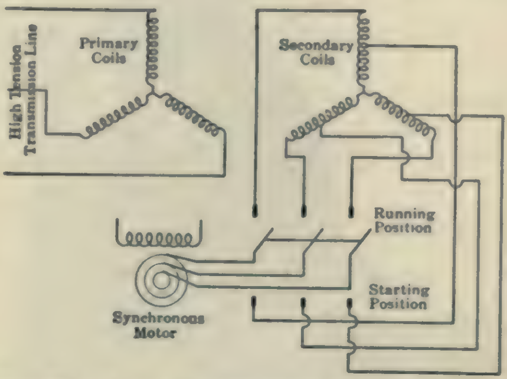


FIG. 353.—A synchronous motor is generally started by means of a compensator and two-throw switch, so as to limit the starting current to a reasonable value.

287. Excitation.—Of course, the field circuit of a synchronous motor requires continuous current. As the a.c. line feeding the motor cannot furnish the current for excitation some separate source of c.c. power is required. Sometimes a small c.c. self-exciting generator is mounted on the shaft of the synchronous motor, as shown in Fig. 354; the output of this small c.c. generator is just sufficient to supply the power for the field circuit of the synchronous motor, perhaps 3 per cent of the rating of the motor.

288. Speed-load Curves.—The speed of a synchronous motor is absolutely constant throughout its range of operation. If an excessive overload is put on the motor, it will pull out of synchronism with the line and a very heavy current will rush through the armature. This causes the circuit breakers to open and the motor is cut off from the supply line and stops. It must then be re-started and synchronized as described in a previous paragraph. That overload at which the motor falls out of synchronism is called the " pull-out " point; generally this is between 75 and 150 per cent overload.

289. Use of Synchronous Motors.—Speed-load curves for two motors are given in Fig. 355. The shape of these curves shows that the synchronous motor is entirely unsuited for loads requiring a variable speed such as railway work, or for driving machine tools. It has been used chiefly in frequency-changing motor-generator sets and more recently is being used in ship-propulsion.

The motor-generator sets are generally used in connection with 25-cycle transmission lines. A 25-cycle synchronous motor is direct-connected to a 60-cycle generator which furnishes power to local lighting circuits. This transformation is necessary because 25 cycles per second is too low to use for lamps, as bad flickering results. The synchronous motors of these sets are also used to regulate the power factor of the transmission line, as described in a later paragraph.

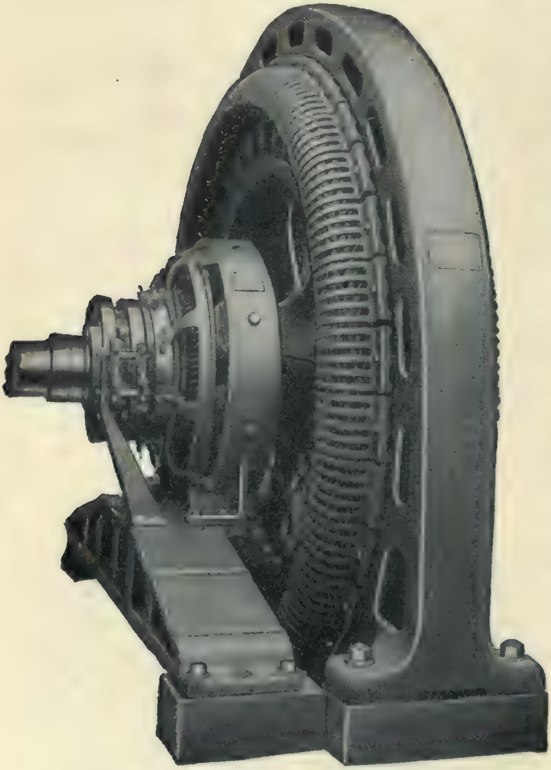


FIG. 354.—The exciter for the synchronous motor is frequently mounted on the shaft of the motor itself.

290. Phase Characteristics or " V " Curves.—Suppose a synchronous motor is running light and the field current is at its normal value; the armature current will be quite small, in fact, just enough to supply the no-load losses. *If, now, the field current is altered, either above or below its normal value, it will be noticed that the armature current rapidly increases;* for very low or very high values of the field current the armature current may be much greater than the full-load current although the motor is running light.

If the locus of the armature current per line is plotted, it forms a V-shaped curve, as shown in the full-line curve 3 of Fig. 356. If, now, the motor is loaded (mechanically) up to its rating and the field current again carried through as wide a range as possible, curve 1 will be obtained. For half load, a curve similar to the one numbered 2 would be obtained. These curves, showing the relation between the armature current and the field current for a fixed load, are called the *phase characteristics*, or " V " curves, of the synchronous motor.

291. Variation of Power Factor with Field Current.—If the power input were measured as well as the armature current, the power factor could be calculated, the line voltage being known. For the different loads the power-factor curves would resemble those shown in dotted lines in Fig. 356. From these curves it may be seen that although a variation in the field current (from normal) is accompanied by an increase of the armature current, the *power input* to the motor is practically unaltered because of the decreased power factor.

We must conclude that field currents other than normal cause, in the armature circuit, either a leading or a lagging current, the amount of lead or lag depending upon how much the field has been changed from its normal value. If a power-factor meter had been used in the V-curve test, it would have indicated that, with fields *greater* than normal, the motor took a *leading* current from the line, while with *less* than normal excitation the motor took a *lagging* current from the line. This idea is generally expressed by saying that *an overexcited synchronous motor draws a leading current and an underexcited motor a lagging current.*

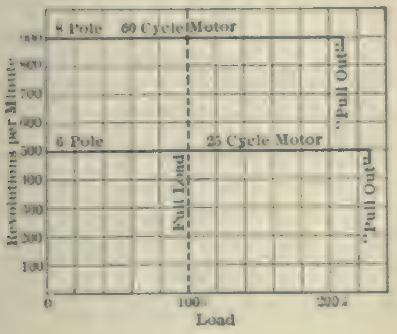


FIG. 355.

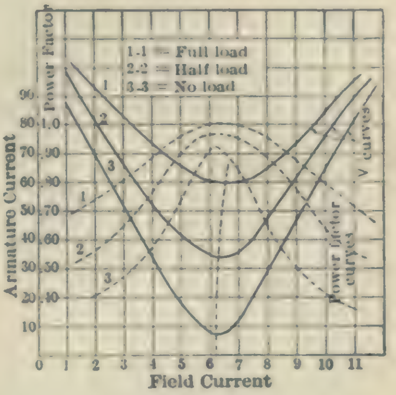


FIG. 356.

FIG. 355.—Speed-load curves for synchronous motors.

FIG. 356.—If, with the load held constant, the field current of a synchronous motor is varied, the armature current changes as shown by these "V" curves.

292. Reason for Reactive Current.—The reason for these facts becomes apparent when we consider the circuit made up of the synchronous motor armature, the line, and the armature of the generator supplying the line, as shown in Fig. 357. Let E_g represent the generator voltage per phase, and E_m the e.m.f. *generated in the armature of the motor per phase*; any current which flows in this circuit must then be caused by the resultant of E_m and E_g.

In Fig. 358 are shown the possible relations of these vectors. First, assuming no load and neglecting the stray power, we see that when E_m and E_g are equal and opposite, the resultant e.m.f. in the circuit in zero, hence no current will flow. But now suppose the motor is overexcited so that the motor voltage is shown by OE'_m in Fig. 358. The resultant of OE'_m and OE_g is OR'. This resultant voltage will cause a current to flow (through the two armatures and line) which will lag nearly 90° behind OR', as the armatures are highly inductive. This current is shown at OI' in Fig. 358. This current is mostly reactive and it *leads the line voltage,* OE_g. Now if the motor voltage is decreased to OE''_m the resultant voltage becomes OR'' and the current through the circuit OI''. This again is reactive current and it *lags behind the line voltage,* OE_g.

293. Power Factor Depends upon Load and Field Current.—When the motor is already drawing from the line an active current (to supply the power for whatever load it is carrying) and the field current is varied, then the total armature current is made up of the active current and the reactive current; the resultant power factor depends upon the relative magnitudes of the two. This fact accounts for the results given in Fig. 356 which show higher power factors for the full-load run and half-load run than for the no-load run.

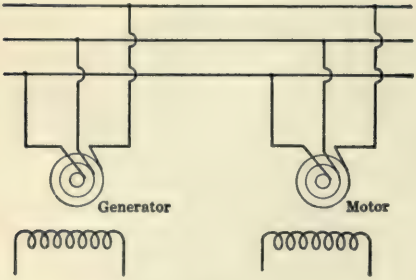


FIG. 357.

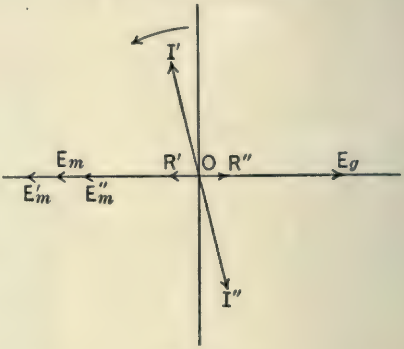


FIG. 358.

FIG. 357.—The circulating current may be obtained by considering the motor and generator armatures in series with the line.

FIG. 358.—Showing why an over-excited synchronous motor draws a leading current from the line to which it is connected, and an under-excited motor, a lagging current.

At light loads it is sometimes impossible to make the power factor equal to unity, no matter how the field current is varied. There are likely to be upper harmonics in the current wave form at light loads, and when such is the case with a three-phase motor, the sum of the wattmeter readings is never equal to the product of $\sqrt{3}\times$volts$\times$amperes, and hence the power factor, as calculated from $\cos\phi=\frac{\text{watts}}{\sqrt{3}\,EI}$ (where E is the line voltage), cannot be equal to unity. This is shown in the power-factor curves of Fig. 356.

294. Effect of Changing Load and Excitation.—We have just seen that the over-excited synchronous motor takes a leading current from its supply line and an under-excited motor a lagging current. Of course, the motor always takes a component of current, in phase with its supply voltage, sufficient to supply the power required by the motor to overcome its losses and to furnish power to its load.

Figure 359 shows the possible current and voltage relations in an over-excited synchronous motor. The voltage per phase is given by the vector *OE* and the motor line current, for certain conditions, by the vector *OI*. Now this current *OI* may be regarded as made up of two components, *OA*, in phase with *OE*, and *OB*, leading the voltage *OE* by 90°. The product *OE*×*OA* represents the electrical power per phase required by the motor to supply its load and losses.

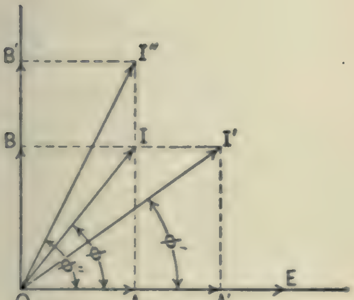


FIG. 359.

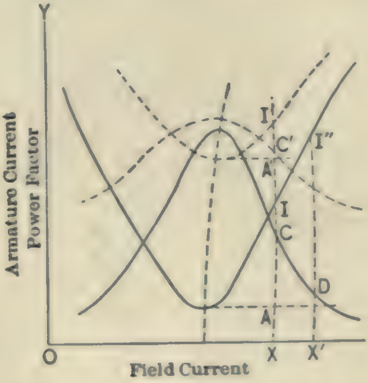


FIG. 360.

FIG. 359.—Showing why the power factor of a synchronous motor changes with its excitation; the changes are relatively greater with the smaller loads.

FIG. 360.—Two V curves of the synchronous motor, to be considered in connection with the vector diagram of Fig. 359.

Now suppose that the load on the motor is increased and its excitation left at its previous value. The reactive component, *OB*, will remain as before but the product *OE*×*OA* must increase to supply the increased load. As *OE* is assumed constant, *OA* must increase to some value *OA'* and the total motor current per phase is therefore given by *OI'*. The effect of this increased load is evidently to increase the power factor of the motor, as $\cos \phi'$ is greater than $\cos \phi$.

Now suppose the load on the motor had remained at its first value but the field excitation had been increased; the active component *OA* would remain constant but the reactive component *OB* would have been increased from *OB* to *OB'*. The motor current per phase would have changed from

OI to OI'' and the power factor would have decreased from $\cos\phi$ to $\cos\phi''$.

In Fig. 360 are shown the two V curves for the loads assumed in Fig. 359. For the load OA the current and power factor are shown by the full-line curves, and for the load OA' they are shown by the dotted curves. Evidently, when the load is changed from $OA \times OE$ to $OA' \times OE$ (Fig. 359), the current increasing from XI to XI' (Fig. 360), the power factor increases from XC to XC' (Fig. 360). But if the load current OA is left constant and the excitation is increased from OX to OX' (Fig. 360), the motor current is changed from XI to $X'I''$ and the power factor from XC to $X'D$.

295. Equivalent Circuit of Synchronous Motor.—It is evident that for any condition of load and excitation the armature current of the synchronous motor can be considered as made up of two components, one in phase with the line voltage and one 90° out of phase.

A synchronous motor is never run in practice with an underexcited field but practically always with an overexcited field, so that we may always consider the reactive current to be *leading* with respect to the line voltage.

Now let us suppose a circuit made up of a resistance R, paralleled by a condenser C in each phase, and we evidently have a circuit which, by properly proportioning R and C, may be used to represent the synchronous motor. In Fig. 361 the line current I is evidently equal to the vector sum of I' and I''. But I'' corresponds to the energy current of the motor and I' corresponds to the leading component of the motor current.

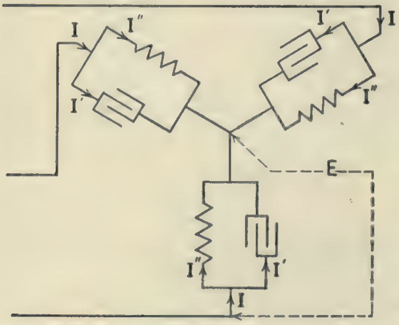


Fig. 361.—A circuit which is electrically equivalent to an over-excited three-phase synchronous motor.

Also, as we know $I''=E/R$ and $I'=E2\pi fC$, it is seen that, by choosing proper values of R and C, I'' and I' may be made to take any values we like. Hence the circuit of Fig. 361 is the exact equivalent of the synchronous motor, *provided we so select R that the energy dissipated in it is equal to the power supplied to the motor and the condenser has such a capacity that its charging current is just equal to the reactive component of the motor current.*

An increase in motor excitation (above normal) would be represented in Fig. 361 by an increase in the capacity of the condenser C, and an increase of the motor load would be represented by a decrease in the resistance R.

296. Use of a Synchronous Motor as a Synchronous Condenser.—The carrying capacity of any electrical circuit, in power, depends upon the power factor of the receiving circuit. A certain transmission line, for example, is rated to carry 100 amperes per wire at 60,000 volts between wires, and hence, *when the current and voltage are in phase with each other*, the carrying capacity of this line is $100 \times 60{,}000 \times \sqrt{3} = 10{,}932$ kw. But if the receiving circuit is made up mostly of induction motors and lightly loaded transformers, the current in the transmission line may lag as much as 45° behind the voltage. Under these conditions the carrying capacity of the line is equal to $60{,}000 \times 100 \times \sqrt{3} \times \cos 45° = 7335$ kw. Now the *carrying capacity* of the transmission line in this case is only 70.7 per cent of what it is when the carrying capacity is 10,932 kw. For we must bear in mind the fact that the customer pays only for the *actual power* he uses, i.e., $\sqrt{3}EI \cos \phi$, and not for the *apparent power*, $\sqrt{3}EI$.

297. Improvement of the Line Power Factor.—It is therefore advantageous for the transmission-line owners to contract with customers in

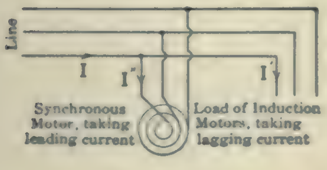

Line
I
I″
I′
Synchronous Motor, taking leading current
Load of Induction Motors, taking lagging current

FIG. 362.

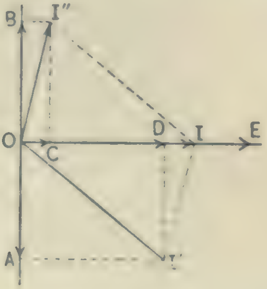


FIG. 363.

FIG. 362.—A suitably over-excited synchronous motor, running unloaded on the end of a transmission line, may be used to improve the power factor of an ordinary induction motor load.

FIG. 363.—The vector relations of the currents of Fig. 362.

such a way as to bring about a high power factor in the line. This can be done if a customer, using a synchronous motor, is persuaded to run the motor with an overexcited field. When such an arrangement is not possible, it generally pays, on long, expensive, transmission systems, for the owners to install at the end of the line a synchronous motor. In this case it is likely that the motor will not supply any mechanical power but will merely "float" on the line, with its field considerably overexcited. In such case it is called a *synchronous condenser*.

Figure 362 illustrates the operation of such a scheme. The current I' lags behind the line voltage and the current I'' is ahead of the line voltage. Evidently under proper conditions the line current I may be in phase with the line voltage.

The vector relations in such a case are shown in Fig. 363. The current supplied to the induction motors per phase is shown at OI', having the active component OD and the reactive component, OA. The synchronous condenser current per phase is shown at OI'', having an active component OC, just sufficient to supply its own losses. The field has been over-excited to such an extent that its leading component OB is just equal to OA. Evidently the line current OI will be in phase with the phase voltage OE, hence the line power factor will be unity and the line carrying capacity as great as is possible.

It is not generally attempted to bring the power factor of the line up to unity; this scheme of using a synchronous condenser generally results in a power factor of perhaps 0.90 or 0.95. (See section 370.)

298. Phase Shifting of a Synchronous Motor, as the Load is Varied.—It has been shown that varying the excitation of the synchronous motor does not affect its load, and it will be remembered that the same thing was shown in the case of one alternator operating in parallel with another.

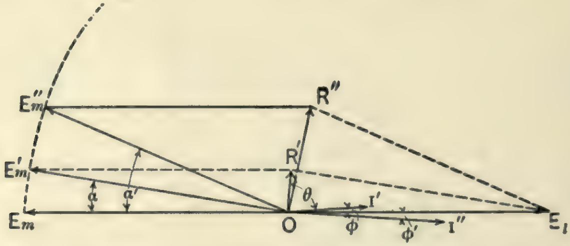


FIG. 364.—Showing how the phase shifting of the synchronous motor, with respect to the impressed voltage, will bring about an increased input to the motor armature.

In discussing the parallel operation of alternators it was proved that a *change in the relative phases of the machine e.m.fs.* was required in order to change the load distribution.

We can show that the same holds good for a synchronous motor; *the motor c.e.m.f. must change its phase with respect to the line e.m.f. if a change of load is to be accomplished.* Actually, of course, the change in the load is what happens first, then the motor e.m.f. changes its phase in order to produce a corresponding change in the electrical power input.

Figure 364 shows how this phase shifting produces a change in the power input. It is supposed that the line voltage and the motor voltage are just equal and opposite, as shown at OE_l and OE_m. The resultant of these two is zero and hence this is the condition for zero input to the motor.

Now suppose that, owing to the fact that a load has been put on the motor, it has been retarded for a fraction of a second until its voltage vector has taken a position not quite opposite to that of the line voltage; this is shown at OE'_m. The resultant of OE'_m and OE_l is shown at OR';

this voltage OR' causes a current to flow almost 90° behind the voltage because of the high ratio of inductance to resistance in the motor armature. The angle θ is determined by the relation $\tan \theta = \frac{X_m}{R_m}$, where $X_m = 2\pi f L_m =$ the reactance of the motor armature, and R_m is the resistance of the motor armature.

The magnitude of the current is determined by the relation $IZ_m = OR'$, where Z_m is the impedance of the motor armature. As Z_m is generally small, it requires only a small voltage to force the full-load current through the armature, and hence only a small phase shift, α, is required. On some machines tested a phase shift of 20° (electrical) was sufficient to make the motor vary its load from zero to full load.

The current is shown at OI' and evidently this represents a motor input of $OE_t \times OI' \times \cos \phi$. If more mechanical load is put on the motor a further shift in the phase of OE_m occurs. Suppose it moves back to OE''_m, producing a resultant voltage, OR''. The current which this voltage causes is shown by the vector OI'', much larger than OI' and in a slightly different phase. The input $= OE_t \times OI'' \times \cos \phi'$, which will be just sufficient to give a mechanical output equal to the assumed increased load.

It will be noticed from this diagram that ϕ may change from a leading to a lagging angle as the load is increased. If it is desired to maintain a power factor of unity for the motor as the load is varied, it is evident that the motor voltage must be slightly increased as the load is increased. This is also shown in the form of the V curves of Fig. 356. The value of field current which gives $\cos \phi = 1$ for light loads is not quite sufficient to give $\cos \phi = 1$ for the full load.

299. Hunting of a Synchronous Motor.—Suppose two a.c. machines, one running as a generator and the other as a synchronous motor, are placed close together with their shafts co-axial, so that by suitable experimental apparatus we can compare the relative positions of the two armatures as they both revolve at synchronous speed.

Let us suppose that we can stand at some fixed point on the periphery of the generator armature, as it revolves, and look at some point on the armature of the motor. Of course, this point would seem to us stationary, for although it would really be revolving at high speed, we (supposedly on the periphery of the generator armature) would be revolving at the same speed. If a load were now put on the motor, *we would see the armature of the motor slow up a little, for a fraction of a second, until the point we were observing on the motor armature had moved backward a few degrees,* and then the motor armature would regain its former speed. The point on the motor armature, on which our gaze was fixed, would again appear stationary, *but it would be behind its former position.* This change in its position would correspond to the angle α, in Fig. 364.

A synchronous motor which has been properly designed, and which operates well, acts as we have just described. For a given load any point on the motor armature has a definite position when referred to a fixed point on the generator armature. But it might be that the point on the motor armature continually oscillated back and forth about its proper mean position, so that at one instant the motor armature would be ahead of its proper position, and the next instant it would be behind its proper position. While these oscillations were taking place, the armature would be revolving on the average at synchronous speed. This oscillation of the armature of a synchronous motor, about its proper mean position with respect to the generator armature, is called *hunting.*

300. Natural Period.—Under special conditions this hunting becomes so violent that the motor will not stay in synchronism, but pulls itself out of step with the line and opens the circuit breakers. It cannot be proved here, but must be taken as a fact, that the armature of a synchronous motor has *a natural period of oscillation* just as a pendulum has. If a pendulum bob is displaced and allowed to swing, we know that it always swings with a definite time for one oscillation; the time taken for the pendulum, swinging freely, to complete one oscillation is called the *natural period* of the pendulum. The synchronous motor armature has a natural period just as the pendulum has, and if any irregularity occurs either in the line voltage or load, with a periodicity which corresponds to the natural period of the motor, violent oscillations are very likely to be set up, and as a result the motor is likely to be pulled out of synchronism.

When a motor is hunting, the electrical instruments showing the input oscillate back and forth, and the normal regular hum of the motor is changed into a hum broken by rhythmic beats.

301. Prevention of Hunting.—There are several ways to stop the hunting of a synchronous machine. One method consists in changing the natural period of the motor armature by the addition of a heavy flywheel; another way is to fit the pole faces with very heavy damping grids. If the hunting is started by some irregularity of the load, such as would be produced by the flapping of a belt, this trouble should first be removed and the hunting may cease without further change.

302. Mechanical Analogy of the Synchronous Motor.—Some of the operating characteristics of the synchronous motor can easily be remembered by bearing in mind a suitable mechanical analogue; figure 365 gives such an analogue. A source of power, such as the steam engine shown at A, represents the power supply of the synchronous motor. A clutch BB' represents the synchronizing switch; a flywheel D represents the armature of the motor which is connected by a thin flexible shaft, C, to the clutch $B'B$. A pulley E, carrying a belt driving some load, represents the load of the synchronous motor.

The half of the clutch *B* is running all the time, as this represents the live line to which the motor is to be connected. Now evidently the clutch *BB′* could not be thrown together with the half *B′* stationary; something would break. So *B′* must first be driven by some outside source of power until it reaches the same speed as *B*, and then when the pins on *B′* are opposite to the holes of *B*, the clutch may be thrown together. This corresponds to closing the synchronizing switch of the motor.

When the clutch is thrown in, suppose a certain point *P′* on *D* is just in line with a point marked *P* on the driver *A*. We know that when load is put on the belt and pulley *E*, the thin shaft *C* will twist, the amount of twist depending upon the load on *E*. But if the shaft *C* twists backward, the point *P′* must fall behind the point *P* by a certain angle; this angle we called α in Fig. 364.

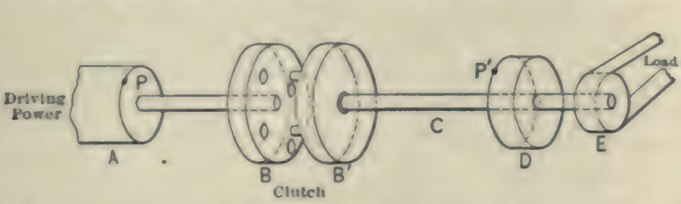


FIG. 365.—A mechanical system which closely resembles, in its action, a synchronous motor running on a power line.

Now, no matter how great the twist of *C* may be, *P′* and *P* revolve at the same speed, *P′* falling farther behind *P* with increasing load on *E*. But if so much load is put on *E* that the strength of *C* is exceeded, then it will twist into two pieces and the set will become inoperative. This corresponds to the opening of the circuit breakers when a synchronous motor is overloaded.

The hunting of a motor has an exact analogy in the arrangement of Fig. 365. *D*, in combination with *C*, has a certain natural period of oscillation. While *D* is revolving with the same r.p.m. as *A*, it may also be executing natural oscillations, so that *P′* is oscillating back and forth with respect to *P*. If the belt on *E* flaps with a period equal to the natural period of the system *D*–*C*, *D* will be quite likely to oscillate more and more violently until the shaft *C* breaks.

CHAPTER X

RECTIFYING DEVICES

THE SYNCHRONOUS CONVERTER; MERCURY-ARC RECTIFIER; THERMIONIC RECTIFIER

303. Need of Rectifying Devices.—We have already mentioned the fact that it is generally best to generate electrical power in the form of alternating-current power because of the ease with which the voltage may be " stepped up " for transmission and then " stepped down " again for use in motors, lamps, etc. There are many installations in which the power is desired as continuous-current power at the place where it is used; it must generally be transmitted from the main generating station as high-voltage a.c. power and then changed into c.c. power at the place where it is used. This is the function of a rectifying device—to change a.c. power into c.c. power or *vice versa.*

Most of the electric railway and trolley systems of the country are operated by c.c. power, and in many of the large cities power is distributed to the customers as c.c. power, being originally generated in both cases in large generating stations as a.c. power. For these purposes *synchronous converters* (also called *rotary converters*) are nearly always used. For charging storage batteries from a.c. lines, *mercury-arc,* and *thermionic rectifiers* are used.

304. Principle of Operation of the Synchronous Converter.—The synchronous converter is really a combined synchronous motor and continous-current generator; it receives alternating-current power as a synchronous motor running at synchronous speed and delivers continuous-current power. In appearance and construction it is practically identical with a c.c. generator with this addition: on that end of the armature opposite to the commutator is mounted a set of slip rings (the number of rings depending upon the number of phases desired) which are connected by taps to the armature winding just as though this winding had been intended for use as a synchronous motor.

In Fig. 366 is shown a small, three-phase synchronous converter; it is seen to be identical in appearance with a c.c. generator, with, however, the slip rings added on one end of the armature.

Suppose that such a machine is running as a synchronous motor; the action of the commutator and revolving armature winding will be just the same as if the armature were being driven by a belt and pulley instead of by the synchronous motor action, and hence there will be, on the brushes, a uni-directional e.m.f. If a load circuit is connected to the brushes, c.c. power will be supplied to this load. As the amount of c.c. power output increases, the a.c. power input will correspondingly increase—in fact, the input on the a.c. end must always equal the c.c. output plus the losses in the machine. The speed remains constant (at synchronous speed for the machine considered as a synchronous motor), irrespective of the load.

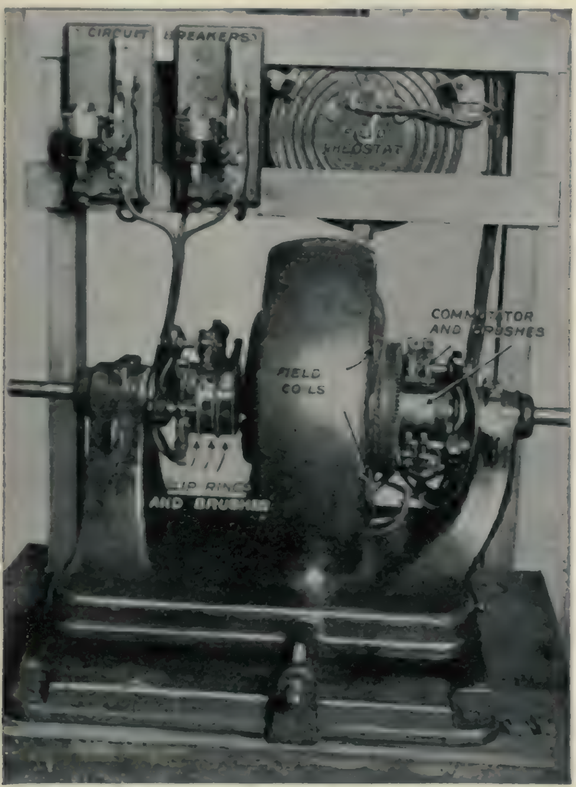


FIG. 366.—A small three-phase synchronous converter suitably arranged for experimental testing.

305. Continuous Voltage Independent of Load.—The value of the continuous voltage is nearly independent of the load, *provided the alternating voltage is maintained constant;* this point is covered more completely by saying that, for any converter, the ratio of the alternating voltage to the continuous voltage is a constant. Moreover, this ratio is nearly independent of the field current, the frequency of the a.c. supply, and the size of the machine; it depends only on the number of phases for which the winding is tapped on the a.c. end.

A three-phase converter has one ratio and a single-phase converter has another, but the ratio for all three-phase converters is the same, independent of size or frequency. The value of this ratio for different phases will now be derived.

306. Voltage Ratio and its Dependence upon Number of Phases.—Suppose that a converter winding consists of only twelve coils in a bipolar field, as shown in Fig. 367, and that each coil has the same number of turns and each is spaced $\frac{1}{12} \times 360° = 30°$ from its neighbor. The derivation of the voltage ratio depends upon the assumption that *each coil generates a sine wave e.m.f.* and hence, that the e.m.f. of each coil may be represented by a rotating vector.

The vector diagram for the e.m.f. generated by each coil is given in Fig. 368; all the vectors are of the same length and each is spaced 30° from its neighbor. Now, we know that alternating e.m.fs. must be added vectorially (instead of arithmetically) to give the resultant e.m.f., hence, if we tap the winding of Fig. 367 at A and B (diametrical points), the vector giving the voltage A–B will be found by adding *vectorially* the e.m.fs. of coils 1 to 6 inclusive. But this vector will evidently be the diameter of a regular polygon having for its sides the vectors given in Fig. 368.

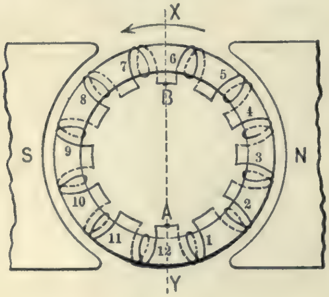


FIG. 367.

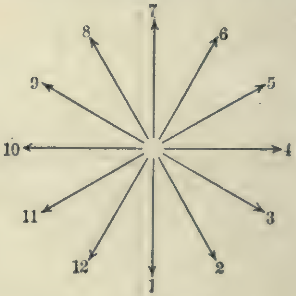


FIG. 368.

FIG. 367.—A 12-coil, closed-circuit winding in a bipolar field.

FIG. 368.—If each coil of the machine of Fig. 367 generates a sine-wave voltage this may be represented by a rotating vector. As the coils are 30° (electrical) apart their respective vectors will have this separation.

307. Voltage for Single-phase Taps.—This polygon is shown in Fig. 369, and the voltage between the points A and B, Fig. 367, is shown by the line AB in Fig. 369. The c.c. brushes, by means of the commutator, are continually maintained at a difference of potential equal to AB, Fig. 369, because the brushes continually make contact with the winding on the line XY (Fig. 367), and so there are continually six coils adding their e.m.fs. between the c.c. brushes.

But suppose that the points A and B are connected to a pair of slip rings; the maximum e.m.f. between the rings will be AB, Fig. 369, and this is the same as the e.m.f. between the c.c. brushes.

Hence we have the relation that, on a single-phase converter, the *maximum alternating voltage is the same as the continuous voltage*, and hence *the effective alternating voltage of a single-phase converter is equal to the continuous voltage* $\times \frac{1}{\sqrt{2}}$. This ratio is independent of the size of the machine, the speed, the number of poles, or anything else; for a single-phase converter this is a fixed quantity and cannot be varied.

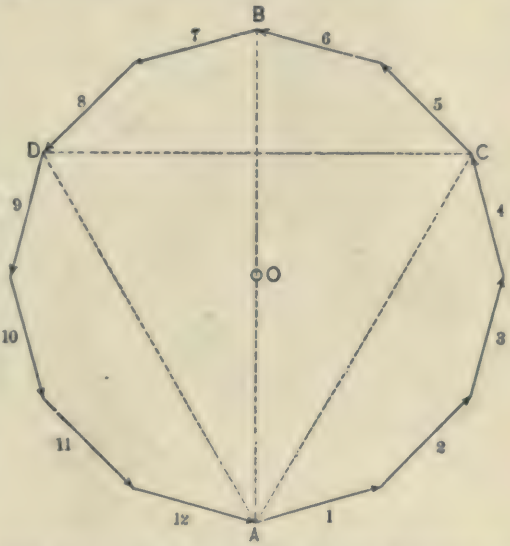


Fig. 369.—The e.m.f. polygon for the armature of Fig. 367.

308. Voltage for Three-phase Taps.—Now, suppose the winding shown in Fig. 367 is to be tapped for a three-phase a.c. supply; the taps would be at A, C, and D, Fig. 369. Between each of these taps there are four coils, and the vectors giving the different e.m.fs. are shown in Fig. 369 by AC, CD, and DA. These vectors are equal to each other and are 120° apart, as they should be for a three-phase winding.

The length of the vector AC can be found in terms of the vector AB in the following manner. In Fig. 370 the construction of Fig. 369 is duplicated to some extent, but instead of the vector polygon we have the circumscribing circle. The angle α, Fig. 370, is equal to 120° for the three-phase taps, AC; hence COD $(=\alpha/2)$ is equal to 60°. Therefore, we have

$$AC = 2AD = 2(OA \sin \alpha/2) = AB \sin \alpha/2 = AB \sin 60°.$$

But $\qquad AB =$ the continuous voltage $\div \sqrt{2}$.

Hence

$$\text{Three-phase voltage (effective)} = \frac{\text{the continuous voltage}}{\sqrt{2}} \times \sin 60°. \qquad (136)$$

$$= \text{the continuous voltage} \times 0.612.$$

Therefore, the alternating voltage of a three-phase synchronous converter is equal to 61.2 per cent of the continuous voltage, and this ratio is independent of the size of the converter, etc.

The voltage ratio for any other number of taps can be found by substituting the proper angle for $\alpha/2$.

The actual ratio of voltages on a synchronous converter is somewhat different from that obtained from this theoretically derived formula. There is an impedance drop in the armature, which, of course, varies with the load, and this makes the ratio vary slightly as the machine is loaded. A three-phase converter having a ratio of 0.615 at no-load might have a ratio of perhaps 0.63 or more when carrying full-load. If the wave of alternating e.m.f. impressed is not a true sine curve the ratio will, of course, be different from that obtained on the assumption of sine curve of e.m.f.

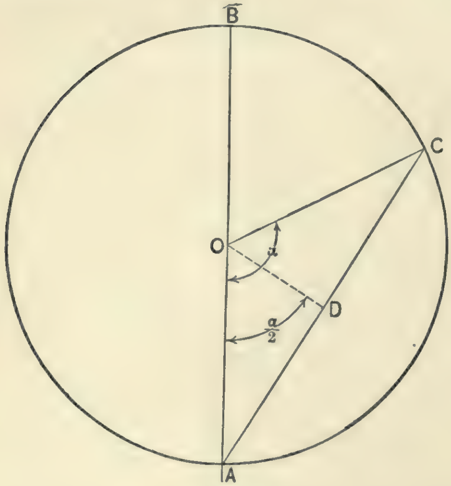


FIG. 370.

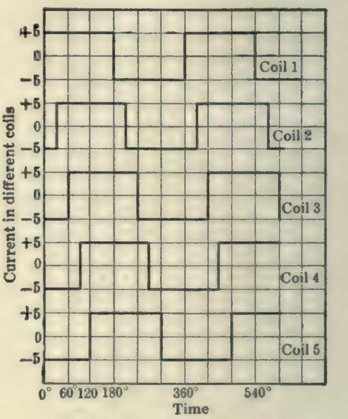


FIG. 371.

FIG. 370.—Showing how to find the voltage ratio for a converter of any number of phases.

FIG. 371.—In so far as the continuous-current output is concerned the current in each coil is of rectangular shape.

309. Current Forms in the Coils of a Synchronous Converter.—The rated capacity of a synchronous converter is generally fixed by the safe rise in temperature, as for any other electrical machine. Of course, the heating increases with increase of load due to the increased I^2R loss in the armature coils.

Now the currents flowing in the different coils of a synchronous converter armature are of very peculiar shape. An alternating current is flowing into the armature at the a.c. taps and a continuous current is flowing away from the c.c. brushes. The actual current in any coil is the difference between these two currents.

Consider the armature shown in Fig. 367, connected to a commutator and supplying a c.c. load, i.e., acting merely as a c.c. generator. The currents in the different coils would be rectangular waves, as shown in Fig. 371. If the load current were 10 amperes, the current in each coil would be 5 amperes, because the armature has two paths.

As each coil moves under a brush its current is changed from one direction to the opposite direction, but while the coil is moving from one brush to the other its current is constant in magnitude and direction. Each coil would have its current commutated, or changed in direction, 30° (one-twelfth of 360°) later than its neighbor and so the current forms of Fig. 371 are seen to be correct.

Now consider the current in each coil, the machine to be running as a single-phase synchronous motor. The current in each coil will be a sine wave (supposing a sine wave of e.m.f. to be impressed) and the maximum value of the current in any one coil is equal to one-half the maximum of the current in the single-phase line supplying the power; this is true because there are two paths in the armature between which the line current divides equally.

310. Ratio of Alternating Current to Continuous Current in a Single-phase Converter.—Neglecting the losses in the machine, to make the equation simple, we may now write for the single-phase converter,

$$\text{a.c. input} = \text{c.c. output}$$

But $$E_{\text{a.c.}} = 0.707 E_{\text{c.c.}}$$

Hence, $$I_{\text{a.c.}} = 1.41 I_{\text{c.c.}}$$

which gives the relative values of the continuous current and the effective alternating current. We know that the maximum value of an alternating current is equal to 1.41 × the effective value, so that

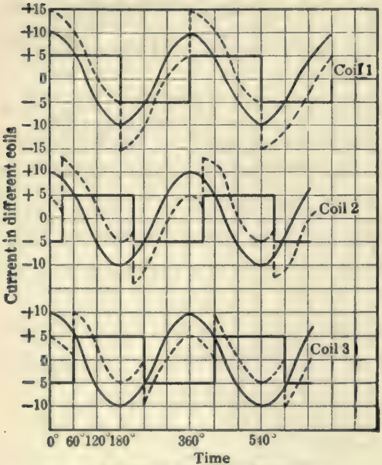
$$I_{\text{a.c.}}(\text{maximum}) = 1.41 I_{\text{a.c.}}(\text{effective}) = 2 \times I_{\text{c.c.}}$$

Therefore, if the machine, running as a single-phase synchronous converter, is supplying 10 amperes to the c.c. line (5 amperes in the individual coils, as shown in Fig. 371), the *maximum* value of the current in the a.c. supply line is 20 amperes and the maximum value of the alternating current in a coil is half of this value, or 10 amperes.

311. Actual Current in a Coil.—The actual current in any coil of the converter is the resultant of a sine wave of current (maximum value equal to 10 amperes) and the rectangular current shown in Fig. 371. Figure 372 shows the result of subtracting the continuous current in a coil from the alternating current, on the supposition that the alternating current has its maximum value when coil 1 is undergoing commutation.

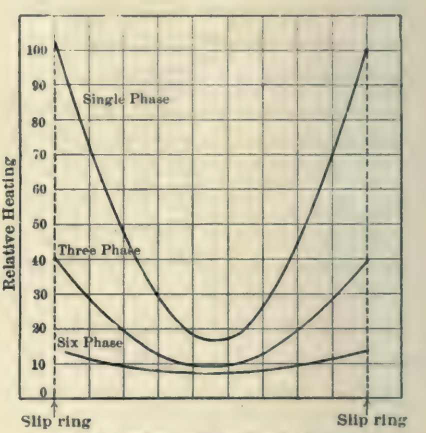
From Fig. 372 it is seen that the currents in the individual coils are very different in magnitude and form; this makes the heating of the various coils different, it being much greater in those coils near the a.c. taps. In case the converter is operating with a power factor other than $\cos \phi = 1$, that coil having the greatest current will be close to the a.c. tap but on one side of the tap only; the adjacent coil, connected on the other side of the a.c. tap, will be much cooler.

312. Heating of Coils.—The heating of the different coils will evidently be different, because the heating depends upon the magnitude of the current; it follows that some coils will be hot when the converter is operating and others will be comparatively cool. *In those coils which run coolest the alternating current flowing into the coil and the continuous current flowing out of the coil nearly neutralize each other;* the more completely this neutralization takes place the cooler the coil will be.



Resultant current in dotted lines

FIG. 372.



Relative heating in coils of a rotary converter. D. C. load same for all curves.

FIG. 373.

FIG. 372.—The actual current in any coil is obtained by considering both the sine wave of input and the rectangular curve of current output; it is shown in the figure by the dashed line curves.

FIG. 373.—Due to the fact that the neutralization of the alternating and continuous currents is more complete in some coils than in others, different coils will heat to different degrees. In this figure the relative heating of various coils is shown for single-phase, three-phase, and six-phase converters.

313. Neutralization of Currents in Various Coils.—In a polyphase synchronous converter, this neutralization of currents is much more complete than in a single-phase converter; also the heating of the different coils becomes more nearly alike as the number of phases is increased. It is possible to develop an expression for the heating in the various coils for different numbers of phases;[1] such an analysis yields results as shown in Fig. 373.

The base line of this set of curves represents the distance between consecutive taps; it would correspond to half the circumference for a single-phase, bipolar converter, one third for a three-phase, one-sixth for a six-phase, etc. The ordinates represent the relative heating in the different coils for a given c.c. output of the converter. These curves show how the heating of the coils decreases and becomes more nearly alike in the different coils as the number of phases is increased.

314. Heating of Coils When the Power Factor is not Unity.—Figure 373 shows the conditions for a converter having normal excitation, that is, with the power factor of the machine unity. If the excitation is changed so that the converter is operating at a power factor less than unity (due either to a lagging or a leading current) the relative heating in the different coils is still more varied; there is not only more difference between the hottest and the coolest coil than when the machine has normal excitation, but, in addition, nearly all coils are hotter. This is shown in Fig. 374, where the full-line curve shows the relative heating in the different coils for $\cos\phi = 1$ and the dotted curve shows the same results when the converter is run with an under-excited field.

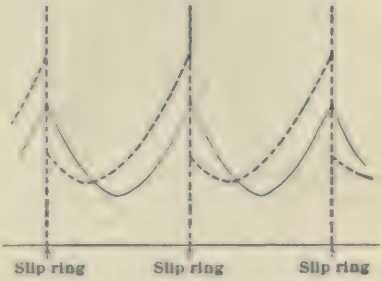


FIG. 374.—If the converter is operated with field current other than normal, the total heating of the armature increases (for given output) and the difference in heating of the different coils increases.

315. Ratio of Alternating Current to Continuous Current in a Three-phase Converter.—Again neglecting the losses in the machine, we may write for the three-phase converter,

$$\text{a.c. input} = \text{c.c. output}$$

or

$$\sqrt{3}E_{\text{a.c.}}I_{\text{a.c.}} = E_{\text{c.c.}}I_{\text{c.c.}}$$

But

$$E_{\text{a.c.}} = 0.612E_{\text{c.c.}}$$

Hence

$$I_{\text{a.c.}} = 0.943I_{\text{c.c.}}$$

which gives the relative values of the continuous current and the effective alternating current per line.

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[1] Alternating Currents, Hay, p. 241 et seq.

Therefore, if a three-phase converter is supplying 10 amperes to the c.c. line, the current per line on the a.c. side will be 9.43 amperes.

316. Capacity of a Synchronous Converter.—Figure 373 shows how the heating of the coils is diminished as the number of phases is increased and makes it evident that a *given armature* will have *more capacity as a six-phase converter than as a three-phase, or greater as a three-phase than as a single-phase, etc.* If we have a certain armature which has a safe capacity of 100 kw. used as a continuous-current machine, it will have more or less capacity used as a synchronous converter, depending on the number of phases, as given below.

Number of phases	1	3	4	6	12
Safe capacity in kw., P. F. = 1.0	85.5	134	165	197	224
Safe capacity in kw., P. F. = 0.87	....	99	115	129	135

These comparative theoretical figures are based on the assumption of equal total I^2R loss and not on the safe temperature of the hottest coil.

If the number of a.c. taps were very greatly increased, the safe capacity of the armature, with proper field excitation, would increase to 230 kw. Used as a single-phase alternator it would have a safe capacity of 70.7 kw. on a load of unity power factor.

317. Converters Always Polyphase.—Single-phase converters are practically never used because they have such a low capacity and have, in addition, a marked tendency to "hunt," as explained in the chapter on synchronous motors. Converters are built three-phase in sizes up to 500 kw.; in sizes of 500 kw. and larger, they are generally wound six-phase because of the increased capacity obtained. They are practically never used twelve-phase because the increase in capacity is obtained only at the expense of complication and increased cost of the brush rigging, rings, etc.

318. Methods of Starting Synchronous Converters.—The synchronous converter is essentially a synchronous motor when considered from the input side; some method therefore must be used for starting and synchronizing it with the line. There are three different methods in use to-day: starting as a c.c. motor from the commutator end; starting as an induction motor from the slip-ring end; or having some auxiliary starter, such as a small induction motor.

In order to start a converter from the c.c. end it is necessary to have available a c.c. supply of the same voltage as that which the converter gives when in normal operation. If, for example, the converter is only one of several in a sub-station and one or more are in operation, the c.c. power for starting may be taken from the c.c. end of those already operating. A starting rheostat is put in the armature circuit and gradually cut out as the machine (starting as a shunt-wound c.c. motor) speeds up; the speed is adjusted by the field rheostat to bring the converter into synchronism with the line, and then the converter is switched to the line after the synchronizing device (lamps or synchroscope) shows the proper phase.

The induction-motor method of starting is becoming more and more common. Half-voltage taps are supplied on the transformers which normally supply the converter, and with the field circuit of the machine open, the a.c. end of the converter is switched to the half-voltage line. When it has accelerated to nearly synchronous speed, the starting switch (which is double throw) is thrown over to the normal voltage line and then the field circuit is closed (connected to the c.c. end of the converter), and the field current gradually increased to its normal value.

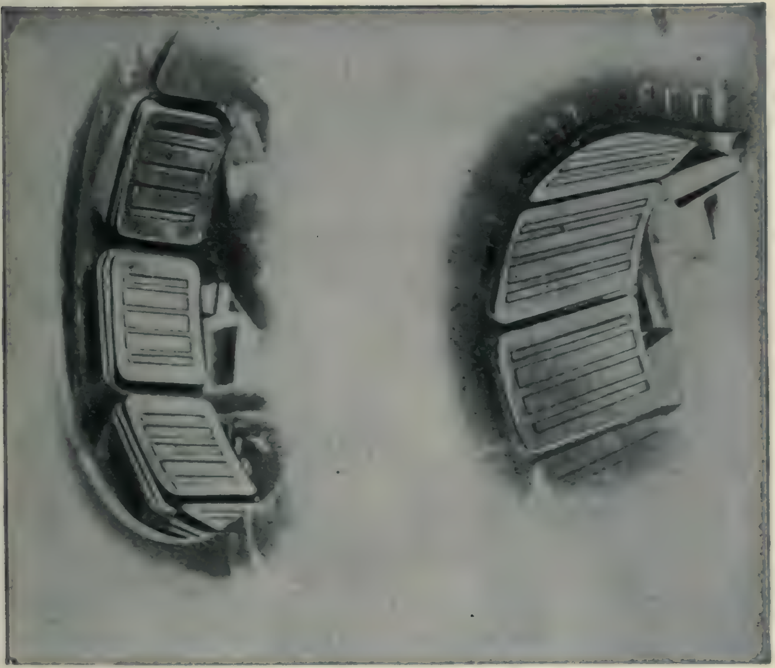

FIG. 375.—Synchronous converters are nearly always fitted with "starting windings" (*amortisseurs*) fitted into the pole faces.

The starting winding used to start synchronous converters by the induction-motor method consists of heavy bars of copper imbedded in slots in the pole face and short-circuited at their ends by a copper band surrounding the pole, as illustrated in Fig. 375. In fact, these grids, cross-bars and band, are sometimes made in one piece, a copper casting.

With commutating-pole converters no connections between the grids upon adjacent poles are made, in as much as such connections would cause the commutating poles, which are placed between the main poles, to be surrounded by a band of copper and so prevent the commutating flux from changing as rapidly as the c.c. load current with sudden changes of load.

319. Field "Break-up" Switch.—In using the induction-motor method of starting, a very high voltage may be induced in the field coils, because these coils act like the secondary of a transformer, the armature being the primary; the field circuit must be subdivided to avoid puncturing the insulation of the field coils. This is done by a field "break-up switch" generally mounted on the frame of the machine.

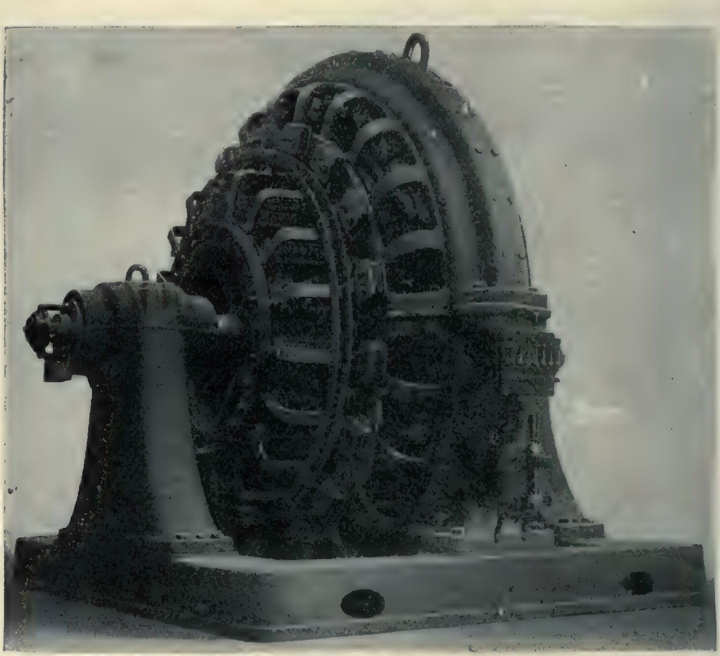

FIG. 376.—A 2000-kw. converter, showing a field "break-up" switch mounted on the yoke.

Figure 376 shows a 2000-kw. synchronous converter on the frame of which may be seen the field "break-up" switch; on this machine a seven-pole switch is used to break the field circuit in several places. The large single-pole switch on the field frame is connected in the equalizer bus-bar circuit.

320. Reversed Polarity.—When starting by the induction-motor method, the field winding being open, the armature comes into step, each pole of the rotating field of the armature setting itself opposite one of the main field poles. The field poles are thus magnetized by the armature, but they may be either north or south poles, depending upon which way the armature comes into step. There being a definite relation between the polarity of the poles, the direction of rotation of the armature, and the polarity of the c.c. brushes, it follows that the converter may come into synchronism with either correct or reversed polarity. In practice, the connections between the converter and the sub-station bus-bars are permanent; it is therefore necessary that the polarity be correct, or always the same, before the converter is connected to the c.c. bus-bars.

The field " break-up " switch is a double-throw switch, so wired that in one position the field is properly connected to the brushes, and in the other position the field is connected reversed. If the converter comes into synchronism with reversed polarity (as indicated by the switchboard voltmeter), the operator closes the field switch in the reversed position, which tends to magnetize the field poles in a direction opposite to that in which they are magnetized by the armature. This causes the converter armature to slip back one pole; in this new position the polarity is correct. If the break-up switch is left in the reversed position, the converter will continue slipping, one pole at a time, because each time that a pole is slipped the polarity of the converter changes, and each time the c.c. field will build up in the reverse direction. Therefore, as soon as the polarity of the converter is correct, the break-up switch must be thrown to the position of proper field connection.

It is to be remembered that with any self-excited c.c. generator there is a correct and an incorrect connection of the shunt field, as was explained on page 248 of Volume I.

321. Sparking at the C.C. Brushes.—If a converter is equipped with commutating poles the induction-motor method of starting is likely to cause bad sparking at the c.c. brushes while the machine is speeding up. Each time a pole of the rotating field of the armature passes a commutating pole, the flux is momentarily increased and the field sweeping by the armature turns short-circuited by the brushes, induces large circulating currents which are ruptured as the commutator bars move from under the brushes. To prevent this, the brush holders are equipped with a simple lever arrangement whereby all brushes except one pair may easily be lifted off the commutator while the armature is accelerating, and then dropped back in place on the commutator when synchronous speed has been reached and the danger of serious sparking is over. One pair of narrow brushes must be left down, to energize the switchboard voltmeter, and the shunt field if necessary to cause the converter to slip a pole.

322. Use of Auxiliary Starter.—When the third method of starting is used, a small induction motor is mounted on one of the bearing pedestals, its rotor being mounted directly on the shaft of the converter. The stator is wound for the same voltage and number of phases as the converter armature, and the motor has just sufficient capacity to run the converter at synchronous speed when it is supplying no load. The induction motor always has one pair of poles less than the converter (why?) and has a capacity rating of from 5 to 10 per cent of that of the converter.

323. Large Commutator Required on a Polyphase Converter.—A cut of a large six-phase converter is given in Fig. 376. It is to be noticed that on this converter the commutator is much larger than it would be on a c.c. generator of the same size. This is because of the fact brought out in a previous paragraph: the capacity of the six-phase converter is practically twice as much as that of a c.c. generator of the same size (*size—not capacity*); and hence the c.c. brush rigging and the commutator must have twice the current capacity required for a c.c. generator with the same size armature.

324. Compounding a Synchronous Converter by a Series Field and Line Inductors.—We have proved that the ratio of continuous voltage to alternating voltage is fixed for a given synchronous converter. It is not, therefore, at once evident how a converter may be designed to give a voltage on the c.c. end increasing with load; yet this is generally desired. For use in railway and lighting installations, a converter with about 10 per cent compounding is generally desired. For example, the specifications for a railway converter might call for 550 volts at no load, 600 volts at full-load; and those for a converter for a lamp load, 225 volts at no load, 250 volts at full load.

But the ratio of the continuous voltage to the alternating voltage is fixed, and it is therefore evident that if it is desired that the continuous voltage should increase with the load *it can be accomplished only by causing the impressed alternating voltage to increase correspondingly with load.* A three-phase railway converter to give 550–600 volts would require an impressed alternating voltage of $550 \times 0.612 = 337$ volts at no load, and an alternating voltage of $600 \times 0.612 = 367$ volts at full load. However, slightly more than this voltage would be required to overcome the effect of armature impedance drop, which increases with the load.

The ordinary method of compounding a converter is to have *inductors inserted in the a.c. lines supplying the converter* and to equip the converter field with *a series winding* as well as a shunt winding. By use of this series winding, the excitation of the machine is increased with an increase of load and this increased excitation causes an automatic increase in the a.c. voltage impressed on the machine.

Now, in so far as its effect on the a.c. supply line is concerned, the synchronous converter acts exactly like a synchronous motor, and hence, if it is over-excited, it draws from the line a *leading reactive current,* besides whatever active current it may be using. The magnitude of the reactive current depends upon how much the machine is over-excited; the more the over-excitation the greater is the leading current.

The armature circuit of the converter may be represented by a condenser and resistance in parallel per phase, as shown in Fig. 377. The current I_R, through R, represents the active current that the converter requires, and the current I_C flowing into the condenser represents the reactive component of the converter current. As the over-excitation of the machine is increased, the capacity of the condenser C must be imagined to increase proportionately in order that the circuit of Fig. 377 may correctly represent the conditions in the actual synchronous converter circuit.

In Fig. 378 the converter is again imagined as consisting of a resistance and a condenser in parallel per phase, and there is an inductance L in each supply line. The voltage of the line is constant (with a value equal to E, per phase, irrespective of the load and the excitation of the converter) and we wish to find the effect on E_R, the voltage impressed on the converter per phase, as the excitation of the machine is increased. In Fig. 378 this increase of excitation is to be represented by an increase in the capacity of C.

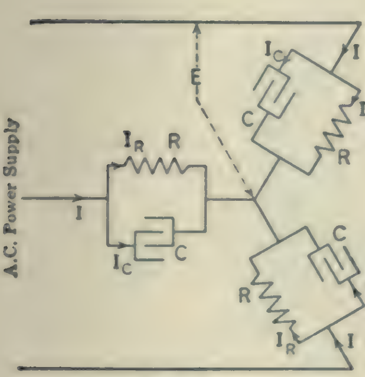

FIG. 377.

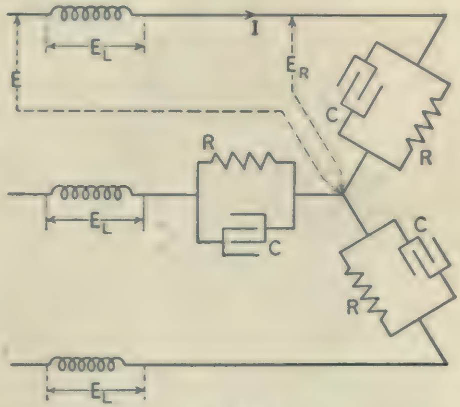

FIG. 378.

FIG. 377.—An electrical circuit equivalent to an over-excited three-phase converter.
FIG. 378.—This diagram, in connection with that of Fig. 379, shows why an over-excited synchronous converter tends to compound itself.

325. Vector Diagram of the Circuit.—The vector diagram of the current and e.m.fs. is shown in Fig. 379. Suppose that at first there is no superexcitation on the machine, so that the current and phase voltage, E_R, are in phase. A circular arc is constructed with E as a radius; the e.m.f. to overcome the drop in the inductance L is shown at E_L and hence the voltage across the armature of the converter is found by vectorially subtracting E_L from E. The result, E_R, is found to be slightly less than E.

If some superexcitation is now put on the converter, giving a condenser current I'_c, the line current is found at I'; the e.m.f. to overcome the inductance drop is proportional to this current and 90° ahead of it and is shown at E'_L. By subtracting this E'_L from E we find the converter voltage at E'_R, which gives a voltage somewhat greater than the impressed phase voltage E. (It is to be noticed that in constructing this diagram E_L and E must always so combine that E_R is found in phase with I_R; I_R and the converter voltage E_R must be in the same phase, and on the vector diagram this is shown to be possible by the phase shifting of E.)

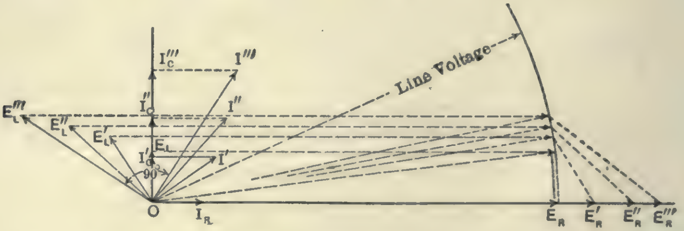

FIG. 379.—Vector diagram of the various voltages of the circuit in Fig. 378, showing the compounding action of the line inductors.

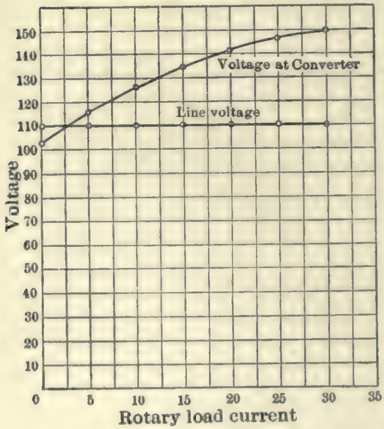

FIG. 380.—Showing the compounding action of the circuit of Fig. 378; this amount of compounding is greater than employed in practice.

For further increases in the field excitation of the converter, the currents are shown by I'' and I''' and the corresponding voltages are shown by E''_R and E'''_R. It is thus seen that, when there is inductance drop in the line supplying the a.c. power to the machine, *it is possible to bring about an increase in the voltage impressed on the converter by sufficiently over-exciting the field, even though the line voltage, E, remains constant.*

326. Compounding Accomplished Automatically.—As the ratio of the alternating voltage to the continuous voltage is practically constant, this increase brought about in the impressed voltage means a corresponding increase in the continuous voltage of the converter. The over-excitation of the converter field is automatically increased as the load increases, by means of a *series-field winding* in addition to the shunt field; the shunt field has enough m.m.f. to give the machine normal excitation, and the over-excitation of the converter is brought about entirely by the series field. By introducing sufficient series field it is possible to compound the converter to any degree desired. There is, however, a practical limit to the amount of compounding possible, for if too much inductance is introduced the converter is likely to hunt badly. This method of compounding is accompanied by undue heating in some of the coils, as shown in Fig. 374.

Figure 380 represents the effect of operating a small converter, having a strong series field, on a line having a correspondingly high inductance. These results were obtained in the laboratory; such heavy compounding is generally not found in practice.

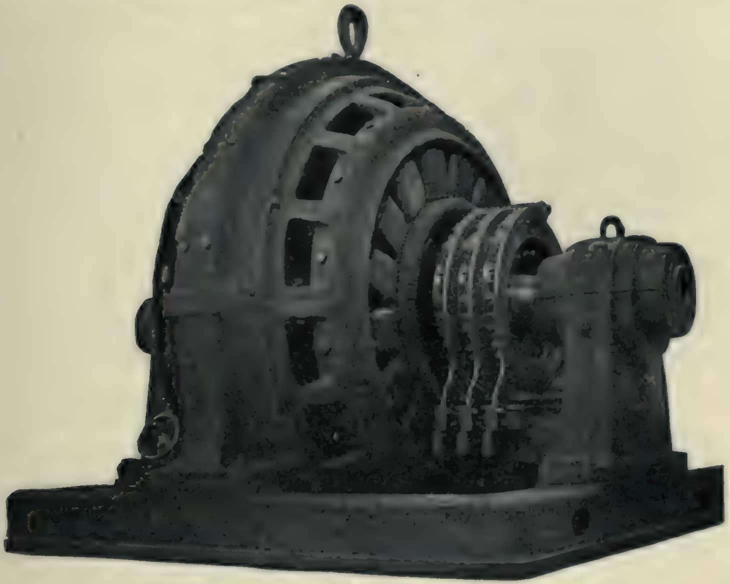

FIG. 381.—A converter fitted with a synchronous booster.

327. Compounding by Synchronous Booster.—When this method of compounding is to be used, the armature of a small stationary-field alternator is sometimes built up on the same shaft with the armature of the synchronous converter, and the field frame for the alternator is mounted on the side of the converter field frame. In another type the armature of the booster is stationary and the field revolves.

The alternator armature winding must be of the same number of phases as the converter armature winding; its current capacity must be equal to that of the converter armature and its normal generated voltage must be at least equal to one-half the desired amount of compounding. The different phases of the booster armature winding are kept separate. The beginning of each phase is connected to a slip ring and the end is connected to one tap of the converter armature.

The a.c. line supplying power to the converter is connected (through brushes) to the alternator slip rings. Hence the converter is connected to the a.c. line through the armature of the alternator, which, by suitable excitation of its field, may be made either to raise or lower the voltage impressed on the a.c. taps of the converter. By suitably winding the alternator field coils, and connecting them in series with the continuous-current load, the compounding may be made automatic.

A synchronous converter fitted with a synchronous booster is shown in Fig. 381. This machine was built for 10 per cent compounding; therefore, the booster is somewhat more than 10 per cent of the capacity of the converter itself.

328. Motor-generator Sets.—One disadvantage of using a synchronous converter to obtain c.c. power from an a.c. supply is that the continuous voltage is fixed; in many cases (notably in mine installations) c.c. power of variable voltage is required. For this purpose a motor-generator set is used instead of a synchronous converter. An induction, or synchronous, motor takes power from the a.c. line and drives a continuous-current generator, the field excitation of which may be varied. Of course, a motor-generator set costs more to install than a synchronous converter of equal capacity, and in addition it is not as efficient in operation as the converter; but in some cases the advantages gained by an adjustable, continuous voltage offset these disadvantages. In case a synchronous motor is used as the driver of the motor-generator set, it may be used to correct the line power factor, as was shown to be possible in section 296.

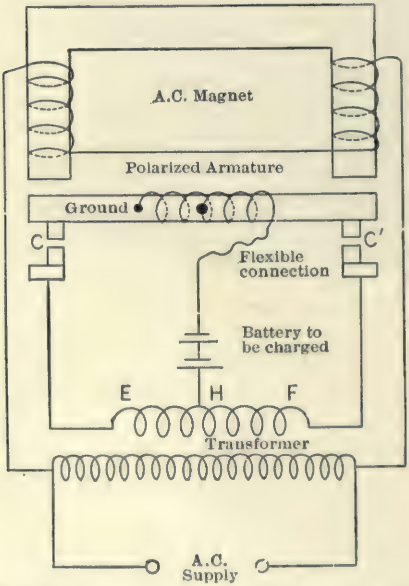

FIG. 382.—Connection scheme of the vibrating rectifier.

329. Vibrating Rectifiers. Rotating Commutator Rectifiers.—It frequently happens that a small amount of c.c. power is desired where only a.c. power is at hand; the charging of automobile storage batteries is the principal instance of this kind. Generally the amount of power desired is not sufficient to warrant the installation of a motor-generator set or a synchronous converter, and some simple, cheap device must be used.

A vibrating rectifier which is on the market furnishes a few amperes of uni-directional current at 10 volts or less. It consists of a polarized armature which is so pivoted that when acted on by an a.c. magnet it vibrates back and forth. A diagram of the connections is given in Fig. 382, and a cut of the device in Fig. 383. When *E* is positive (with respect to *H*) the battery is charged through the contact *C*, and when *F* is positive (with respect to *H*) the battery is charged through the contact *C′*. These contacts must be adjustable, so that sparking may be eliminated. This rectifier is small and simple to operate and has proved very useful. The battery cannot be connected improperly to the rectifier and cannot discharge if the alternating voltage fails; this makes the rectifier a very reliable piece of apparatus.

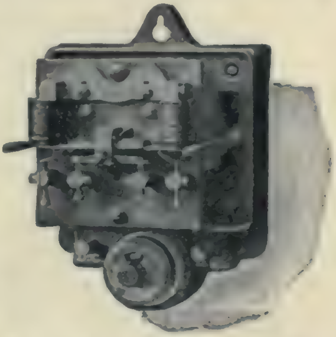

FIG. 383.—A commercial vibrating rectifier, for getting a few watts of power with uni-directional current.

Another scheme for obtaining a small amount of c.c. power employs a commutator which is rotated by a small synchronous motor. The commutator has as many segments as the motor has poles and every other segment is connected together. Then each set of segments is connected through slip rings and brushes to the a.c. supply line. Brushes are placed on the commutator, and the battery to be charged is connected to these brushes. Careful adjustment of the brush position is necessary to eliminate sparking in this rectifying device.

330. Mercury-arc Rectifier.—Neither of the devices just described will operate well on circuits where much c.c. power is desired, because sparking would take place at the contacts and the device would require constant attention when in operation.

But it is found that a tube filled with ionized [2] mercury vapor (the air having been exhausted from the tube), having mercury for one terminal and some such metal as iron for the other terminal, *permits of the passage*

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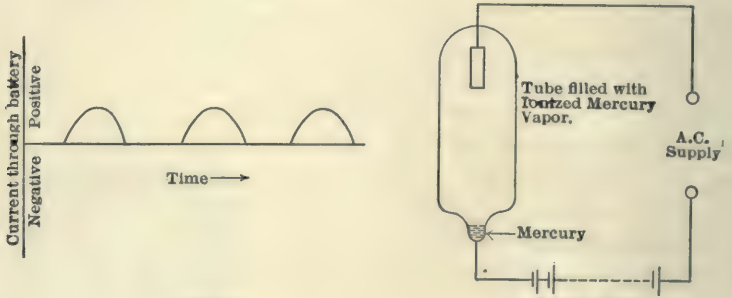
[2] The term *ionized* is used to characterize a gas in which the normal atom has been somewhat changed; a minute quantity of negative electricity (called an *electron*) has been freed from the atom and can move away from it. A gas in which this change has occurred conducts an electric current quite freely and is said to be *ionized*; a gas in its normal condition conducts very poorly, in fact it is a good insulator. A gas may be ionized by the passage through it of an electric spark, by the influence of radium, etc.

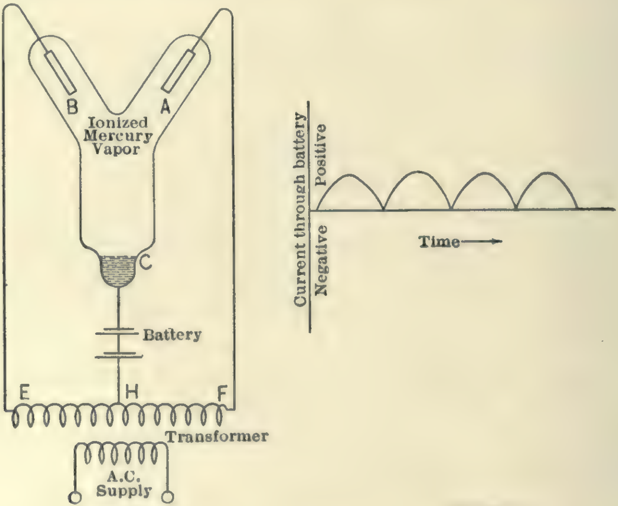
of current in one direction only. If an alternating voltage is impressed on the two terminals of the tube, current will flow through the tube only when the mercury terminal is negative and the iron terminal positive. *In the tube itself, the current can flow from the iron to the mercury but cannot flow from the mercury to the iron.* This is the same as saying that electrons can leave the mercury and flow to the iron through the tube, but electrons cannot leave the iron electrode and flow in the opposite direction.



Current

Connections

Fig. 384.—In a bulb of this type current will be conducted in one direction only, with ordinary voltages. A battery in series with the a.c. supply will therefore receive pulses of uni-directional current.



Connections

Current

Fig. 385.—By adding another anode to the bulb of Fig. 384, and changing the connections to those given here, the current through the battery becomes more nearly uniform.

If a mercury tube is kept filled with ionized mercury vapor it will therefore act as a *rectifying valve;* the current which flows through it will be pulsating, as shown in Fig. 384. Although such a pulsating current is not suitable for motors or lamps, it does very well for charging storage batteries. By the addition of another positive terminal of iron, and by using connections as shown in Fig. 385, it is possible to rectify every alternation, instead of every other one. The mercury terminal, called the *cathode,* is shown at C, and the two iron terminals, called the *anodes,* are shown at A and B. The battery is connected between the cathode and a center tap, H, on the transformer. When F is positive with respect to H, current will flow through the tube and battery by the anode A, and when E is positive with respect to H, current will flow through the battery and tube by the anode B. The current through the battery will therefore have the form given in Fig. 385.

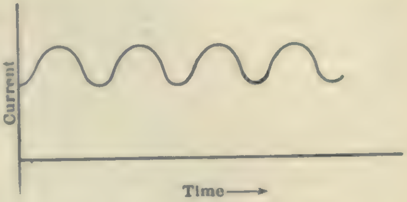


FIG. 386.—If the transformer of Fig. 385 has sufficient magnetic leakage the shape of the uni-directional current is changed to that given here.

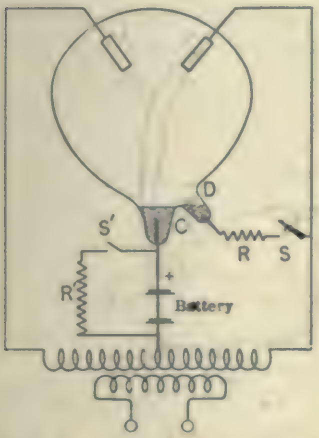


FIG. 387.—To give the initial ionization in the mercury rectifier, an auxiliary starting electrode, D, is used; after the tube starts to function, switch, S, is opened.

331. Ionization of Gas Necessary for the Operation.—In order to operate as described above, it is necessary that the tube be filled with ionized mercury vapor. If the mercury vapor is once ionized the current flowing through the tube will maintain the ionization, but if the current ceases for a fraction of a second the vapor immediately becomes non-ionized and will not carry current. Hence a tube as represented in Fig. 385 would not maintain its ionization because the current repeatedly reaches zero value.

If, however, the transformer supplying the power to the tube is designed with considerable leakage (giving it self-induction) the current is changed from the form given in Fig. 385 to that given in Fig. 386; this current never reaches a zero value after the tube starts to operate; hence if the ionization in the tube is started by some method, the current itself will maintain the ionization.

332. Use of Starting Anode.—To start the ionization an additional anode, called the *starting anode*, is provided. This consists of a pool of mercury close to the cathode as shown at D in Fig. 387. When the tube is tipped slightly the mercury of D runs over and makes contact with C. If the starting switch S is closed, a current will flow from D to C, its magnitude being limited by the resistance R. Now, if the tube is tipped back, the mercury bridge connecting C and D is broken and the rupturing of the current causes a slight arc, *which is sufficient to produce the initial ionization of the mercury vapor.*

To make the tube start readily, a starting resistance R', controlled by the switch S', is first used as a load. After the tube has operated through this artificial load for a few seconds, it becomes warmed and the battery may then be connected in the circuit and the starting resistance cut out. Of course, the switch S, controlling the starting anode D, is also opened as soon as the tube is operating properly.

333. Application of the Mercury-arc Rectifier.—The principal application of the mercury-arc rectifier is for charging storage batteries; for this purpose the tubes are made in various sizes with capacities of 3 to 50 amperes. For greater current capacities, the glass tubes are likely to overheat, and therefore attempts have been made to utilize metal tubes immersed in oil to get proper cooling. However, the metal used must be one which does not amalgamate with mercury, and iron seems to be the only suitable substance. As an iron tube will not hold a proper vacuum, tubes suitable for heavy currents are as yet in the experimental stage. If this problem is solved, it may be possible to substitute mercury tubes for synchronous converters in railway operation.

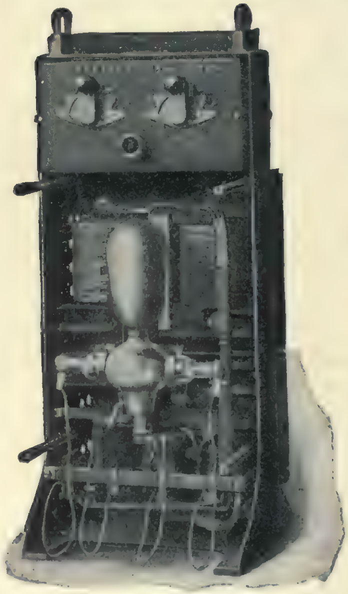


FIG. 388.—A commercial form of mercury rectifier, for charging storage batteries.

Flaming arc lamps, similar to the magnetite lamps, are still used to some extent, and these require continuous current for their operation. Series arc-lamp systems are most conveniently operated from constant-current transformers, which, of course, require an alternating-current power supply. The secondary of the transformer, instead of being connected to the lamp circuit directly, is connected through a mercury tube so that the arc lamps operate on a rectified current. The constant-current transformer operates to give practically constant current, just as if the mercury rectifier were not in the circuit.

A view of a mercury tube outfit for charging storage batteries is shown in Fig. 388; this tube is designed to carry 30 amperes, and the tube is generally guaranteed to give 400 hours' life. As a matter of fact, these tubes generally give more than 1000 hours' life in actual operation; their failure is due to a change in the vacuum.

334. Thermionic Rectifiers.—For rectifying small amounts of power, a new type of rectifier is now available which utilizes the idea of the mercury-arc rectifier in addition to electron evaporation from hot metals. In one form of this rectifier, called the tungar rectifier, a small bulb is filled with some inert gas, such as argon, at a low pressure. An anode is placed close to a tungsten filament, which can be heated to incandescence by passing current through it. If a source of alternating voltage is connected to the anode and hot filament, current will flow through the tube only when the filament is negative with respect to the anode. When the filament is negative, electrons which evaporate from its hot surface are attracted to the anode and on their way across they ionize the argon. This ionized gas conducts current quite freely. When the anode is negative with respect to the hot filament, the evaporated electrons are not attracted to it and so the gas is not ionized and does not conduct. The rectifying action of the tube is due to one terminal being sufficiently hot to evaporate electrons, and the other not so hot.

CHAPTER XI

SINGLE-PHASE MOTORS

335. Single-phase Induction Motor.—All of the analyses of induction-motor action given in Chapter VIII were carried out with a polyphase motor in mind. In some respects the single-phase induction motor differs from the polyphase motor. Unless supplied with some additional starting device, such as a commutator, or extra stator winding with inductances or condensers, etc., *the single-phase induction motor has no starting torque.* But when the motor is operating near synchronous speed (with not much more than 20 per cent slip) its characteristics are exactly like those of a polyphase induction motor. There are some differences, such as a lower power factor and less slip in the single-phase motor than in the polyphase, but these differences are slight.

The absence of starting torque is due to the fact that the stator has *only one set of coils* and is supplied with *single-phase current* only; *it can, therefore, produce only an oscillating magnetic field and not a rotating magnetic field.* To develop torque it is necessary to have a rotating magnetic field, as was shown for the polyphase motor. When the motor is running at nearly synchronous speed, the rotor currents, acting in conjunction with the single-phase stator winding, actually do produce a rotating magnetic field, and for this reason the single-phase and polyphase motors act alike near synchronous speed.

336. Split-phase Method for Starting a Single-phase Motor.—One way of designing a single-phase motor to give it a starting torque is known as the split-phase method. Although there is only one winding on the stator which can carry much current, there is an additional winding at 90 degrees to this, which is of sufficient size to carry a magnetizing current during the starting period only. After the rotor approaches synchronous speed this additional winding is automatically cut out of the circuit. Fig. 389 shows the connection for such a split-phase motor; the

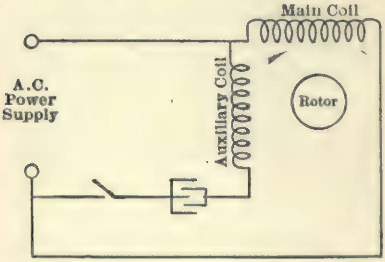


FIG. 389.—One scheme for starting a split-phase induction motor.

auxiliary coil is connected to the single-phase supply line *through a condenser*. This condenser, if large enough, will make the current in the auxiliary coil lead the current in the main coil by perhaps 45°, hence the two coils act like the two coils of a two-phase motor and produce a rotating magnetic field. After a suitable speed has been reached, the auxiliary circuit is opened to prevent overheating. An inductance may be used instead of a condenser to bring about the necessary phase difference of the currents in the two windings.

337. Single-phase Commutator Type of Motor.—Another type of single-phase induction motor which is much used employs a rotor fitted with a commutator and brushes, quite similar to a continuous-current motor. After the motor reaches the proper speed, all of the commutator bars are automatically short-circuited and the brushes lifted from the commutator. This action changes the rotor to an ordinary squirrel-cage rotor. This motor starts on the same principle as the repulsion motor, which is discussed in a succeeding section. A view of the commutator end of the motor is shown in Fig. 390.

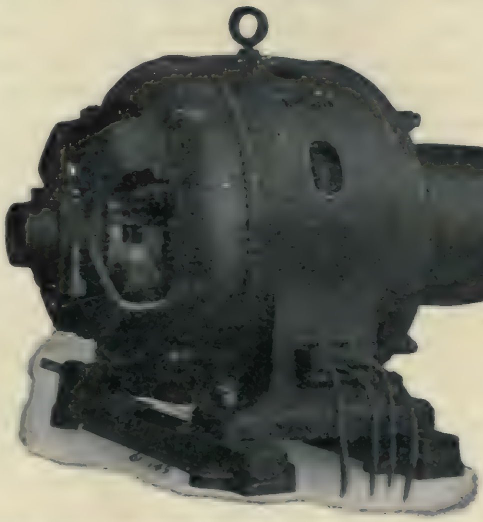


FIG. 390.—A single-phase motor which starts in the repulsion-motor principle.

338. Polyphase Motor Superior to Single-phase Motor.—It is evident that the polyphase motor is much more suitable for ordinary installations than the single-phase motor; in the first place the polyphase motor costs less per h.p. than the single-phase and is more efficient in operation. In addition, no extra devices are necessary with the polyphase motor to produce a starting torque. Because of these facts the single-phase motor is used only in small sizes, and where polyphase power is not available.

339. Application of the Single-phase Series Motor.—As was explained in Chapter IX of Vol. I, a motor suitable for electric traction must have a high starting torque, a variable speed-load curve and good efficiency when operating at widely differing speeds.

The *induction motor* runs at nearly constant speed irrespective of load. Any attempt to regulate the speed necessarily results in decreased efficiency, unless complex connections are introduced. The cascade connection of two motors gives only a *definite change in speed,* and the same is true of the scheme wherein the number of poles is varied. Instead of having one speed of efficient operation, the motor has two speeds, but neither of these schemes gives a variable-speed motor, and in addition the switching and wiring become complicated. If rotor resistance is relied upon for speed variation a low efficiency is obtained at the same time.

The *synchronous motor* has a rather small starting torque (when started as an induction motor) and operates at constant speed, irrespective of load; it is evidently not suited for railway purposes.

The single-phase series motor, however, has starting and running characteristics very similar to those of the c.c. series motor and is thus suited for railway work. This type of a.c. motor has been developed as a railway motor and is used to a limited extent for this purpose. The most notable example is the New York, New Haven & Hartford R.R., which has been operating for several years with single-phase series-motor locomotives. It seems to be fairly successful in this installation, but because of its inferiority to the c.c. series motor, it is not likely to come into extensive use.

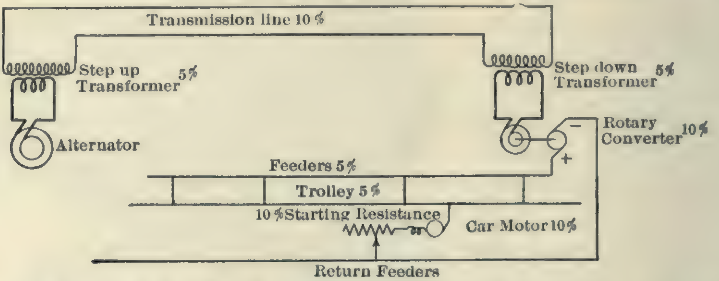


Fig. 391.—A simplified diagram of the electrical distribution, and power losses, in a railway system. Somewhat less than 50 per cent of the power leaving the station gets to the car wheel.

340. A.C. Series Motor vs. C.C. Series Motor.—At present, nearly all electric railways in this country use the continuous-current series motor as their motive power. The distribution of the electric power from the generating station to the car wheel is inefficient because of the many steps involved.

The power is generated as alternating-current, goes through step-up transformers at the station to the high-tension transmission line and so to the sub-station; there it goes through step-down transformers to the synchronous converter, where it is changed to c.c. power and is sent out through the c.c. feeders and trolley to the car. The speed control of the series motor requires the use of rheostats, and considerable power is used up in these as well as in all the other parts of the system just enumerated; probably not more than 45 per cent of the power generated in the main station is delivered to the car wheel. Of course, this figure will vary widely in different installations, depending upon the type of apparatus installed, schedule of cars, etc.

The system of distribution and probable losses in each part of the system are given in Fig. 391; the different percentages given show the approximate loss of power in that part of the system. These figures will vary widely for different installations.

In case the alternating-current series motor is used on the car, the power distribution is much simpler. At the station there may be a step-up transformer connected directly to the trolley and feeders and then through an auto-transformer on the car to the motor. As speed control is obtained by varying the ratio of the transformer, no rheostat losses occur at the motor; evidently the losses in such a distributing system will be much less than those of a c.c. system. If the a.c. motor were as efficient and reliable as the c.c. series motor, probably all railway installations would use alternating-current power.

341. Principle of Operation.—*A continuous-current series motor will run in the same direction, whichever way it is connected to the c.c. line*, provided *the relative connection* of its armature and field is left undisturbed. Hence, it follows that a c.c. series motor, connected to an a.c. line, would exert a *uni-directional torque*, and would revolve continuously in the same direction and not oscillate back and forth as might be expected.

Such an application of the c.c. motor is not feasible because of the *low torque* which it would give, the *heating of the yoke and poles*, and the *heavy sparking* which would occur at the brushes. The low torque would result from the high impedance of the field winding, which would permit but little current to flow through the motor; the heating of the poles and yoke would result from the excessive eddy-current losses due to the alternating magnetic field through this part of the magnetic circuit; and the sparking would result from certain causes taken up in a succeeding paragraph.

342. Construction.—In a series c.c. motor the flux through the field frame is unidirectional and constant, and therefore this part of the magnetic circuit may be made solid, of cast steel. It is only the armature, in which the flux reverses, owing to the rotation of the armature, which is necessarily made of laminated iron. But in the series a.c. motor the current through the field coils is alternating and hence the *flux through all of the magnetic circuit is alternating;* therefore, the entire field structure, as well as the armature core, must be made of laminated iron. Because of this fact, the series a.c. motor cannot be constructed as cheaply nor of such rigid mechanical design as the c.c. motor.

The field coils of the a.c. series motor consist of but very few turns as compared to those of the c.c. motor, and it is found that the field frame is best made without projecting poles, such as are used on the c.c. motor. The field frame resembles very much the stator of an induction motor, the field coils being imbedded in slots.

It would not be well to fill all the slots with field coils, as such a design would not be efficient. But it is found necessary to put on the field frame an additional set of coils called the *compensating winding*. The two sets of coils (main field and compensating field) completely fill the slots of the field so that it resembles very closely the stator of an induction motor.

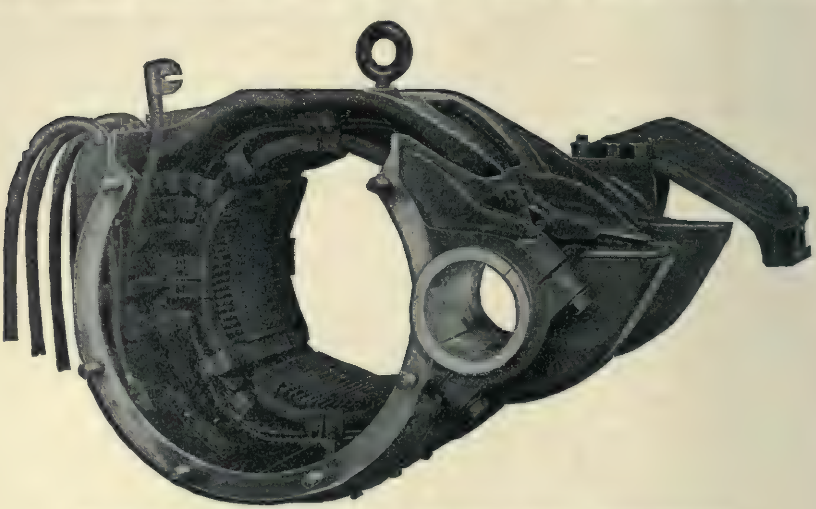


FIG. 392.—The field structure of a single-phase series, railway motor.

The field of a series a.c. motor is shown in Fig. 392; this field is completely assembled and ready for the armature.

The armature of an a.c. motor resembles that of a c.c. motor very closely; the only difference is due to the manner of winding. *The number of turns per coil* is very low in the a.c. motor and these coils are not connected directly to the commutator bars but through *resistance leads,* to be explained later.

343. Power Factor—Compensating Winding.—If a c.c. series motor were operated on an a.c. circuit, the power factor would be very low, probably not more than 0.30 or 0.40. This low power factor would be due to the high inductance of the field and armature windings and could be increased only by decreasing the self-induction of these two windings.

We know that an a.c. system with a low power factor is very inefficient, both from the standpoint of first cost and from that of operating cost; the series a.c. motor must therefore be designed with low self-induction if it is to be successful. The self-induction of the field winding is kept low by using *fewer turns* than would be used for a similar c.c. motor, and the self-induction of the armature winding is reduced to practically zero by the use of a *compensating winding.*

This winding consists of a set of coils, wound in slots in the field frame in such a manner that their magnetomotive force is just equal and opposite to that of the armature coils. Hence the armature cannot build up a magnetic field because of these compensating coils, and so it has practically no self-induction. The compensating winding is connected in series with the armature so that the magnetic effects of the armature coils are neutralized at all loads.

Even though the self-induction of the armature is reduced to nearly zero by the compensating winding, the motor cannot be successfully used on 60-cycle circuits. The self-induction of the field coils is so large that the power factor of the motor is small if used on a circuit of frequency greater than 25 cycles. The advocates of this type of motor have agitated for 15-cycle power for railway operation, but this frequency is not likely to become standard in America. Practically all alternating-current series motors are designed for 25-cycle power.

The flux density in an a.c. motor must be kept low in order to prevent heavy hysteresis and eddy-current losses in the field frame; thus, for a given rating the a.c. motor is larger than the c.c. motor. Also, to produce the required torque, the armature must have a larger number of conductors in the a.c. motor because of the low flux density used; this larger number of armature turns does not increase the self-induction of the motor because of the action of the compensating winding.

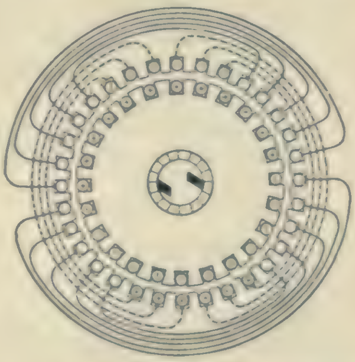


FIG. 393.—Conventional cross-section of a series a.c. motor, showing how the main and compensating fields are disposed.

In Fig. 393 is shown a cross-section of an a.c. series motor showing the relative positions of the main field, compensating field, and brushes. As the compensating coils carry twice as much current as the armature coils, it is necessary to have only half as many turns in the compensating coils as in the armature. In Fig. 393 the compensating coils are indicated by dotted lines and the main field coils by full lines.

344. Commutation.—Sparkless commutation is a very difficult thing to obtain on the a.c. series motor; of course, in the c.c. motor commutation is the most difficult problem for the designer to solve, but in the a.c. motor it is a much harder problem.

The difficulties met in the c.c. motor are present in the a.c. motor and, in addition, *there is a transformer effect of the field on the armature coils which may produce a large current in those coils which are short-circuited by the brushes.* In Fig. 394 is shown a diagram of the field and armature coils, a ring armature being shown for simplicity of representation.

The coils marked X and Y are short-circuited by the brushes and these coils are threaded by the field flux, which is alternating. Hence an e.m.f. will be induced in coils X and Y just as though they were the secondary winding of a transformer of which the field coils represent the primary. This *transformer e.m.f.* will set up a large current in the short-circuited coils and the rupturing of this current as the coil moves from under the brushes causes heavy sparking at the commutator.

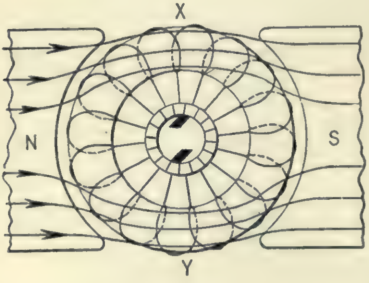


Fig. 394.

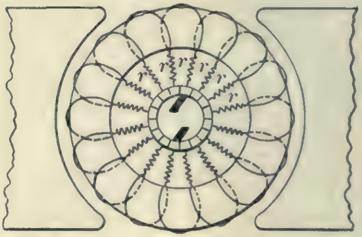


Fig. 395.

Fig. 394.—Showing how a transformer e.m.f. is induced in the coils short-circuited by the brushes.

Fig. 395.—Resistance leads are connected between the coil connections and the commutator bars, to limit the current set up in the short-circuited coil by the transformer e.m.f.

345. Commutating Poles of no Use on an A.C. Motor.—Commutating poles cannot be used to overcome the difficulty as in the c.c. motor, because *this transformer e.m.f. in the short-circuited coils is greatest when the motor current is passing through its zero value.* Hence, if commutating poles were used in series with the armature as in the c.c. motor, the flux from the commutating poles *would be zero when it should be a maximum.* The method of eliminating the trouble is not by commutating poles but by increasing the resistance of the circuit through which the transformer current flows.

346. Resistance Leads.—The resistance of the coil itself is not increased, but an extra resistance is introduced in the leads connecting the coils to the commutator bars. These leads (sometimes called *preventive* *leads*) generally consist of a piece of resistance wire, which is placed in the bottom of the same armature slot in which the coil is to be placed. These leads have a resistance which is about twice that of the coil itself, and are introduced into each commutator connection as shown by *r*,*r*,*r*, in Fig. 395.

The path for the transformer current is made up of the coil, two resistance leads, two commutator bars, and the brush, so that the use of these leads cuts down the transformer current in a coil to about one-fifth the value it would have without them.

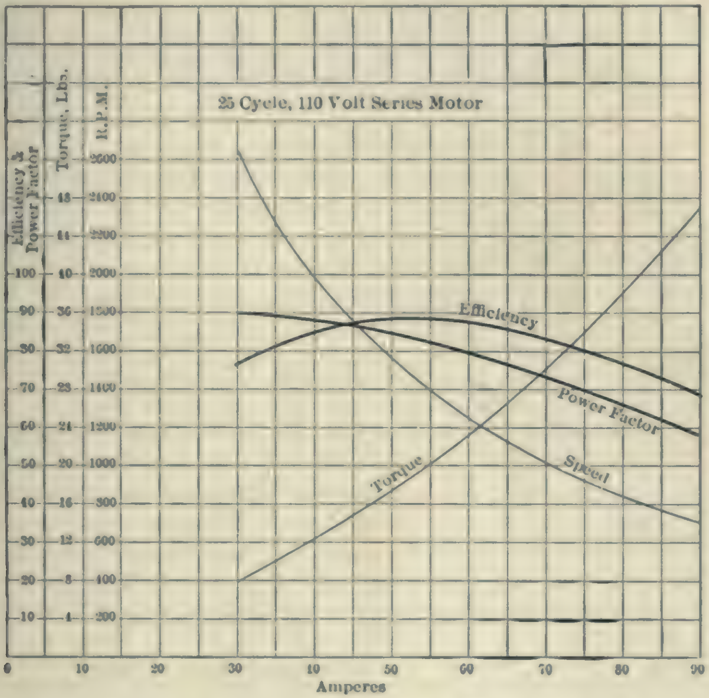


FIG. 396.—Operating characteristics of a 5 h.p. single-phase series motor.

The addition of these resistance leads does not materially affect the resistance of the armature as a whole, because the total armature resistance between brushes is many times that of one coil and the load current has to flow through only two of the resistance leads in addition to the coils. Thus, if the armature had 100 coils and each resistance lead had twice the resistance of a coil, the resistance of the armature between brushes would be 54/50 as great as though no resistance leads were used, while the resistance of the path of the transformer current would be increased five times. It is claimed that the use of such leads makes sparkless commutation possible at all loads on the a.c. series motor, and also that the leads show no deterioration after much service.

347. Operating Characteristics.—The operating, or running, characteristics of the motor resemble very much those of the c.c. series motor. A small series motor tested in the laboratory gave results as shown in Fig. 396.

The torque, or tractive effort, increases nearly as the square of the current, being a maximum for the lowest speeds; this is one of the characteristics that make the motor suitable for tractive work. The efficiency is fairly high for all loads in the working range of the motor, but is lower for all loads than that of a corresponding c.c. motor. The extra iron and copper losses in the a.c. motor account for this fact.

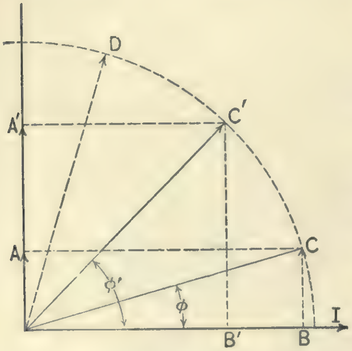


FIG. 397.—Vector diagram for the single-phase series motor.

The power factor is a maximum for the high speeds and gradually decreases with increase of load. The reason for this may be seen by reference to the vector diagram shown in Fig. 397. The impressed voltage on the motor is constant, and in Fig. 397 is given by the radius *OD* of the circular arc *CC′D*. The phase of the current is assumed as *OI*. There are three reactions balancing the impressed e.m.f. of the motor, viz., the inductance reaction, which is proportional to the current, the resistance reaction, also proportional to the current, and the c.e.m.f. due to the rotation of the armature conductors.

For a current of 10 amperes the inductance drop of the motor tested was *OA*, the power factor was cos ϕ and the vector *OB* is the arithmetical sum of the $\dot{I}R$ drop and c.e.m.f. of the motor. For 40 amperes the inductance drop was *OA′* and the power factor was cos ϕ', which is evidently less than cos ϕ. As the vector *OB′* is less than *OB*, and as the *IR* drop for 40 amperes is evidently greater than for 10 amperes, it follows that the c.e.m.f. is less for 40 amperes than for 10 amperes. Furthermore as the field flux must be greater with the 40 amperes, it follows that the speed must have decreased rapidly with the increase in armature current. These characteristics are just those shown in Fig. 396.

348. The Repulsion Motor.—This motor is one employing a wound armature with commutator and brushes, but the armature circuit is not connected electrically to the power-supply line. It was first developed by Elihu Thomson in 1887, but has not yet come into very extensive use. It has operating characteristics very similar to those of the series a.c.

motor. It has an advantage over the series motor in that its armature is not connected to the power line and so its power supply, which is fed into the field circuit only, may be of much higher voltage than can well be used on the series motor.

The principle on which the motor operates may be understood by reference to Fig. 398. The field circuit is connected to the power supply and so an alternating flux is set up in the armature. The armature brushes are short-circuited and the plane of the brushes, A–B, is at an angle with the direction of the magnetic field. An e.m.f. is induced in the armature coils by the alternating field flux, and this e.m.f. causes a current to circulate through the armature coils and the path connecting the brushes together. The armature conductors carrying current react on the magnetic field set up by the field coil and so produce a torque. The magnitude of this torque depends upon the value of the angle α, Fig. 398, and the direction of the torque is always the same as that in which the brushes A–B have been moved away from the center line of the field.

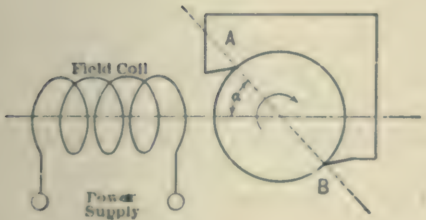


Fig. 398.

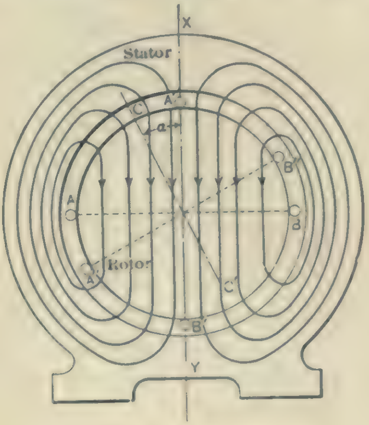


Fig. 399.

Fig. 398.—A simplified diagram of a repulsion motor, to bring out the principle involved in its operation.

Fig. 399.—To show how the torque and current of the short-circuited coil vary with its position in the field.

349. Torque Varies with the Position of the Brushes.—The dependence of the torque upon the angle α may be seen by reference to Fig. 399. *Any armature having short-circuited brushes may be represented by one short-circuited turn, this turn being in a plane 90° away from the plane of the brushes.* In Fig. 399, for example, the short-circuited coil, AB, is equivalent to an armature having its brushes in the plane XY, parallel to the field. Now such a short-circuited turn as AB (Fig. 399) would have a large short-circuit current flowing in it as soon as the field of the motor was excited, but this large current could produce no turning effort because the conductors A and B, in which the current is supposed to be circulating, lie in a zero magnetic field.

Suppose, now, that the short-circuited brushes of the repulsion motor are advanced 90° so that the one armature coil, representing all the armature winding, is in the position $A'B'$, Fig. 399. Evidently the coil could produce no turning effort because no current would circulate in it. The plane of the coil is parallel to that of the flux, and hence there is no e.m.f. induced in it by the alternating field flux.

If the brushes are now placed in such a position that the equivalent armature coil is shown by $A''B''$ in Fig. 399, it is evident that there will be a current flowing in the conductors of the short-circuited armature and that these conductors will lie in a magnetic field. Hence there will be a force exerted between the field and the armature conductors tending to make the armature turn. As the flux alternates, this torque will continue to be in the same direction, because *the current in the armature and the field flux both reverse together*, so that the force between them is constantly in the same direction.

To discuss this point more completely, it would be necessary to consider the relative phases of the field flux and armature current and this would complicate the question too much. We have given enough analysis to show that the torque of such a motor (like that of the series a.c. motor) is uni-directional and pulsating.

350. Direction of Rotation.—When the armature conductors are represented by the equivalent coil $A''B''$, the brushes must be at right angles to the plane of this coil, as shown by CC' in Fig. 399. When the brushes are placed back of the center line of the magnetic field, the armature will rotate in a counter-clockwise direction and *vice versa*. Hence, to vary the torque of a repulsion motor in magnitude and direction, it is only necessary to change the position of the brushes on the commutator. The variation of the torque and current with the position of the brushes is shown in the curves of Fig. 400. It is seen that the current, as well as the torque, varies greatly with the position of the brushes.

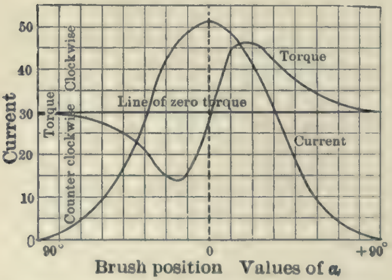


FIG. 400.—As the position of the brushes of the repulsion motor is changed the torque and stator current go through the variations shown here.

351. Compensated Repulsion Motor.—The power factor of the repulsion motor described in the preceding sections is rather low, and this is, of course, objectionable. Several schemes have been developed for increasing the power factor; a motor in which this is attempted is called a *compensated repulsion motor*.

The general scheme is to have a winding so placed on either the field or armature that the effect of the field inductance is practically neutralized. One of the simplest of these schemes is shown in Fig. 401. This type of motor was developed by Winter and Eichberg; it gives the same characteristics as the ordinary repulsion motor, with the exception that the power factor is much higher. The short-circuited brushes are placed in the plane of the magnetic field, and an extra set of brushes, at right angles to the first pair, is connected to the secondary of a transformer, the primary of which is placed in the power-supply line, in series with the motor field winding. The secondary has several taps, so that the power factor may be maintained high at various speeds. Other motors of this general type are available which maintain a power factor close to unity for all loads. In general, the compensated motor is more bulky and costly than one of the non-compensated type.

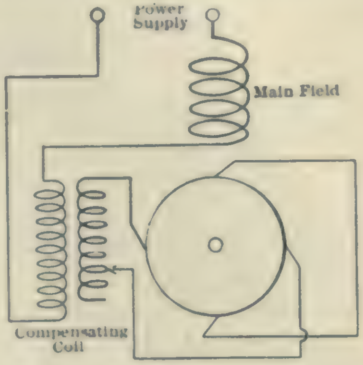


Fig. 401.—Showing the general idea of a compensated repulsion motor.

CHAPTER XII

THE TRANSMISSION OF POWER

352. The General Problem.—It is evident that the transmission of power must constitute one of the chief problems of the electrical engineer. In the first small electric plants, the customers were all situated within a short distance of the power house, and the transmission of the power was therefore a very simple problem. With the more general use of electrical power and the combining of many small power plants into a few large ones (with resultant economy) the distance over which the power must be distributed has continually increased. Moreover, as the operation of coal-fed power houses involves an ever-mounting fuel cost, the power stations of the country must gravitate to the coal fields themselves (thus saving coal haulage) and to the water-power regions. This tendency further emphasizes the problems of transmitting large blocks of power over great distances, the large industrial centers generally being far distant from the coal mines and water falls.

353. Voltages Used in Transmission Lines.—The power lost in a given transmission line varies with the square of the current; hence the current must be kept low to make the transmission an economical one. Of course, even if the current is high it is theoretically possible to keep the loss small by keeping the resistance low, but this can be done only by greatly increasing the investment for copper. As such increased cost of conductor soon becomes prohibitive, recourse must be had to raising the voltage, thus transmitting a given amount of power with a lower current. As the current varies inversely with the voltage, for a given power, it follows that the loss in heating the transmission line varies inversely with the square of the voltage; hence the very high voltages used in long lines.

In distributing power around the streets of a suburban community, 2200 is the standard voltage; this is stepped down by transformers (generally located on the poles in the street) to 110 or 220 volts, at which voltage the power is delivered to the customer. Frequently, the 2200-volt distribution network is itself supplied from a large transformer which receives power from a transmission line of much higher voltage. In large communities, the voltage generated in the power stations, and sent through the streets in lead-covered cables in subway ducts, is either 6600 or 11,000 volts. Large amounts of power can be economically sent 10 or 20 miles at such voltages, but even such voltages are too low to be classed as transmission line voltages.

As a general rule, we may say that for reasonably large amounts of power (say 25,000 kw. or over) the voltage used increases directly with the distance, and that this voltage is from 500 to 1000 volts per mile of transmission. Thus, a line intended to transmit 25,000 kw. a distance of 150 miles would be designed for 100,000 to 150,000 volts.

Although a saving in cost of conductor might be effected by going to even higher voltages, the increasing cost of insulators and transformers would more than offset this advantage. The upper limit of voltage, for economical operation over a given distance, mounts somewhat with the amount of power to be carried; a 100,000-kw. line would be of considerably higher voltage than a 5000-kw. line of the same length. This useful upper limit of voltage is intimately connected with the electric strength of the air, which is to be discussed in section 361.

354. Material for Conductors.—The only two materials used for transmission-line conductors are aluminum and copper. Although copper is a much better conductor than aluminum, for the same cross-section, aluminum is so much lighter that for equal resistances the aluminum weighs less than half as much as the copper. The breaking strength of aluminum is however much less than that of copper for a given resistance; therefore the factor of safety of aluminum, when loaded with sleet and ice, may not be as great as that of copper. Assuming the copper to be of 98 per cent conductivity, 55,000 pounds per square inch tensile strength, density of 8.89, and a price of P dollars per pound, and the aluminum of 61 per cent conductivity, 25,000 pounds tensile strength, density of 2.70, and price p, the relative merits of the two, for equal resistances, are shown by the following table:

	Copper	Aluminum
Cost	1	$0.488\ p/P$
Cross-section	1	1.63
Diameter	1	1.28
Weight	1	0.488
Breaking strength	1	0.731

Sufficient data are not yet available to make it possible to say which material is the more reliable under severe weather conditions, as it seems that the ice-collecting tendency of aluminum may depend upon the length of time it has been exposed to the weather. When new, it seems less likely than copper to permit the accumulation of ice.

On one typical line consisting of two three-phase circuits, each using three No. 0000 cables, aluminum for conductor cost only $1450 per mile of line, as compared with $2050 for copper. These prices were for copper at $0.235 and aluminum at $0.16 a pound. This apparent advantage of aluminum is lessened when it is considered that the poles must be made higher, or put closer together for the aluminum, because of its inferior strength and greater expansion under temperature changes. In an actual line, the cost of the conductor is only about 25 per cent of the total cost of the line.

At present, copper is used much more than aluminum; but with greater voltage of transmission, and the consequent increase in the importance of the corona limit (see section 361) aluminum may come into greater favor, because with the greater cross-section which its use necessitates a greater voltage may be used without serious corona loss.

355. Transmission Lines Universally Three-phase.—For a given amount of power, a given maximum voltage between any two wires of the line, and a given allowed I^2R loss in transmission, the three-phase three-wire system uses less copper than any other, and would always be used even if the machinery connected to it were not practically always three-phase.

Comparing a single-phase line with a three-phase line, for example, we shall suppose that a certain power, P, is to be transmitted at unity power factor with a voltage between wires not to exceed E volts. Let R_1 be the resistance of each of the wires of the single-phase system, and R_3 be the resistance of each wire of the three-phase system. Likewise, let I_1 and I_3 be the single-phase and three-phase line currents. The allowable loss is a certain fraction of the transmitted power, say aP.

Then

$$P = EI_1 = \sqrt{3}EI_3 \quad \text{or} \quad I_1 = \sqrt{3}I_3,$$

and

$$aP = 2I_1^2R_1 = 3I_3^2R_3,$$

or, if we substitute $\sqrt{3}I_3$ for I_1,

$$6I_3^2R_1 = 3I_3^2R_3,$$

or

$$R_1 = \tfrac{1}{2}R_3.$$

From this it is seen that the cross-section of the single-phase wire must be twice that of the three-phase wire; hence the weight of one single wire of the single-phase line will be twice that of one wire of the three-phase line. As there are three wires in the three-phase line, as against two wires for the single-phase line, the weight of copper for the three-phase line is seen to be only 75 per cent of that required for the single-phase line.

If we assume that there is an equal strain on the insulators of the two lines, and that each line is grounded at its neutral point, the voltage between the wires of the single-phase line could be higher than that for the three-phase line. Working out the problem on this basis, the weight of copper required is just the same for the two; in fact, it is the same no matter what the number of phases may be.

356. Weight of Copper Required for a Given Transmission.—For the transmission of a load of P kilowatts at power factor $\cos\phi$, three-phase, the voltage and current required are fixed by the relation, $P=\sqrt{3}EI\cos\phi$. Let us suppose a certain percentage of the transmitted power, aP, to be allowed as permissible I^2R loss in the line. How will the required copper vary with the different factors, such as length, voltage, etc.?

Let R = resistance per wire;
W = total weight of wire;
l = length of one wire;
k = a constant depending upon the conductivity of the copper;

Evidently $W=\frac{kl^2}{R}$, as we know that for a given resistance the weight must vary as the square of the length, and for a given length the cross-section must vary inversely as the resistance.

Also as

$$P=\sqrt{3}EI\cos\phi;$$

$$I=P/\sqrt{3}E\cos\phi.$$

Now

$$3I^2R=3\left(\frac{P}{\sqrt{3}E\cos\phi}\right)^2R=aP,$$

and substituting for R its equivalent $\frac{kl^2}{R}$ we get

$$\frac{\sqrt{3}Pk}{a}\left(\frac{l}{E\cos\phi}\right)^2=W,$$

and substituting the proper value of k to give W in pounds when E is in kilovolts, l is in miles, and P is in kilowatts, we get

$$W=\frac{280P}{a}\left(\frac{l}{E\cos\phi}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (137)$$

It is thus seen that for a given weight of copper, to transmit a given amount of power with the same efficiency (same I^2R loss), the voltage of the line must rise in direct proportion to the distance to be covered. It is also seen that a low power factor demands a correspondingly greater weight of copper; a load of $\cos\phi=0.7$ requires twice as much copper as an equal load (in kilowatts) at unity power factor.

From Eq. (137) it is possible to calculate the proper size of wire necessary for given conditions. If the total weight of copper, the length of

TABLE IV †

Copper Cables, Concentric-lay

Cir. Mils and A. W. G. or B. & S. Gage; English Units
100 Per Cent Conductivity; Density 8.89 at 20° C.

Circular Mils and A.W.G.	Resistance at 25° C. or 77° F.*		Weight in Pounds, Bare		Standard Strands			Flexible Strands		
	Ohms per 1000 Feet	Ohms per Mile	Per 1000 Feet	Per Mile	Number of Wires	Diameter of Wires in Mils	Outside Diameter, in Mils	Number of Wires	Diameter of Wires in Mils	Outside Diameter, in Mils
2,000,000	0.00539	0.0285	6180	32,600	127	125.5	1631	169	108.8	1632
1,900,000	0.00568	0.0300	5870	31,000	127	122.3	1590	169	106.0	1590
1,800,000	0.00599	0.0316	5560	29,300	127	119.1	1548	169	103.2	1548
1,700,000	0.00634	0.0335	5250	27,700	127	115.7	1504	169	100.3	1504
1,600,000	0.00674	0.0356	4940	26,100	127	112.2	1459	169	97.3	1460
1,500,000	0.00719	0.0380	4630	24,500	91	128.4	1412	127	108.7	1413
1,400,000	0.00770	0.0407	4320	22,800	91	124.0	1364	127	105.0	1365
1,300,000	0.00830	0.0438	4010	21,200	91	119.5	1315	127	101.2	1315
1,200,000	0.00899	0.0475	3710	19,600	91	114.8	1263	127	97.2	1264
1,100,000	0.00981	0.0518	3400	17,900	91	109.9	1209	127	93.1	1210
1,000,000	0.0108	0.0570	3090	16,300	61	128.0	1152	91	104.8	1153
950,000	0.0114	0.0600	2930	15,490	61	124.8	1123	91	102.2	1124
900,000	0.0120	0.0633	2780	14,670	61	121.5	1093	91	99.4	1094
850,000	0.0127	0.0670	2620	13,860	61	118.0	1062	91	96.6	1063
800,000	0.0135	0.0712	2470	13,040	61	114.5	1031	91	93.8	103
750,000	0.0144	0.0759	2320	12,230	61	110.9	998	91	90.8	999
700.000	0.0154	0.0814	2160	11,410	61	107.1	964	91	87.7	965
650,000	0.0166	0.0876	2010	10,600	61	103.2	929	91	84.5	930
600,000	0.0180	0.0949	1850	9,780	61	99.2	893	91	81.2	893
550,000	0.0196	0.1036	1700	8,970	61	95.0	855	91	77.7	855
500,000	0.0216	0.1139	1540	8,150	37	116.2	814	61	90.5	815
450,000	0.0240	0.1266	1390	7,340	37	110.3	772	61	85.9	773
400,000	0.0270	0.1424	1240	6,520	37	104.0	728	61	81.0	729
350,000	0.0308	0.1627	1080	5,710	37	97.3	681	61	75.7	682
300,000	0.0360	0.1899	926	4,890	37	90.0	630	61	70.1	631
250,000	0.0431	0.228	772	4,080	37	82.2	575	61	64.0	576
0000	0.0509	0.269	653	3,450	19	105.5	528	37	75.6	533
000	0.0642	0.339	518	2,735	19	94.0	470	37	67.3	471
00	0.0811	0.428	411	2,170	19	83.7	418	37	60.0	420
0	0.102	0.540	326	1,720	19	74.5	373	37	53.4	374
1	0.129	0.681	258	1,364	19	66.4	332	37	47.6	333
2	0.162	0.858	205	1,082	7	97.4	292	19	59.1	296
3	0.205	1.082	163	858	7	86.7	260	19	52.6	263
4	0.259	1.365	129	680	7	77.2	232	19	46.9	234
5	0.326	1.721	102	540	7	68.8	206	19	41.7	209
6	0.410	2.170	81.0	428	7	61.2	184	19	37.2	186
7	0.519	2.74	64.3	339	7	54.5	164	19	33.1	166
8	0.654	3.45	51.0	269	7	48.6	146	19	29.5	147

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* Let C = per cent conductivity, R_{25} = resistance of 100 per cent conductivity cable at 25° C. (from table), R_t = resistance of cable of conductivity C at any temperature $t°$ C., then

$$R_t = R_{25}\left[\frac{100}{C} + 0.00385(t - 25)\right].$$

† This and the following tables are taken from Pender's Handbook for Electrical Engineers.

line, and the number of wires are known, the weight per mile of conductor in pounds, w, can be at once calculated. Then if r is the resistance of the proper wire, in ohms per mile of conductor, we have

$$r=\frac{10a}{lP}(E\cos\phi)^2. \qquad (138)$$

Of course, wires and cables are available only in certain definite sizes; the proper wire to use is the available one that has a resistance equal to, or slightly less than, that called for by Eq. (138). The constants of copper cables available for use in transmission lines are given in Table IV.

357. Regulation of a Single-phase Transmission Line.—As a transmission line evidently has resistance and inductance, it follows that if the voltage is held constant at the beginning of the line (generator end) the voltage at the load end will vary in some manner as the load is changed. In general, the voltage at the load end will fall as the current carried over the line is increased; of course, when no current is being carried there is no drop in the line, and the voltage at the load end must therefore be the same as that at the generator end.[1] If E_g is the voltage at the generator end of the line and E_l is the voltage of the line at the load end, the regulation of the line is defined by the equation

$$\text{Regulation (in per cent)}=\frac{E_g-E_l}{E_l}\times 100. \qquad (139)$$

If R is the total resistance of a single-phase line (resistance of both wires), X is the total reactance of the line, and I is the load current, the regulation may be calculated by the help of the voltage diagram shown in Fig. 402. Here a non-inductive load is assumed and the load current is therefore shown in phase with the load voltage E_l. The IR drop of the line is plotted in phase with I and the IX drop is plotted 90° ahead of I, giving the IZ drop in the phase shown. By vectorially combining the IZ drop with the load voltage, the generator voltage is obtained and hence the regulation of the line may be calculated by substituting the proper values in Eq. (139).

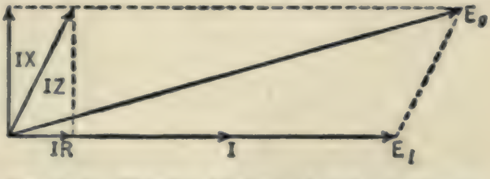

FIG. 402.—Showing how to calculate the regulation of a single-phase transmission line with a load of unity power factor.

In Fig. 403 the conditions are shown for getting the generator voltage when the load current lags behind the load voltage by the angle ϕ. The IR drop is again shown in phase with I and the IX drop 90° ahead of I, giving the IZ drop now almost in phase with the load voltage. The generator voltage is again obtained by adding vectorially the IZ drop and the load voltage, and it is seen that the regulation is much higher (worse) for the inductive than for the non-inductive load.

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[1] See Section 364 for exceptions to this statement.

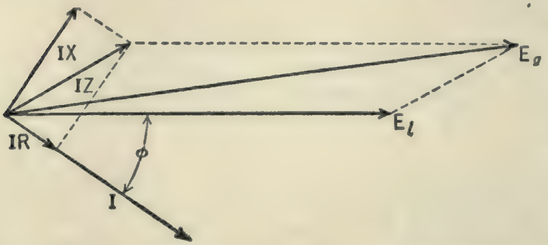

FIG. 403.—Showing how the diagram of Fig. 402 is changed if the load is inductive.

By inspection of Fig. 403, it is seen that the generated voltage may be computed from the relation

$$E_g=\sqrt{(E_l+IR\cos\phi+IX\sin\phi)^2+(-IR\sin\phi+IX\cos\phi)^2}. \quad . \quad (140)$$

In case the voltage of the generator end of the line is given, and the voltage at the load end is to be found, when the line is carrying a certain load at a specified power factor, the problem is much more involved. From Eq. (140) it is evident that E_l can be obtained in terms of the impressed voltage E_g, the line current, and the line and load constants, but that the solution will involve the solution of a rather complex equation. If the symbols have the same significance as for Eq. (137) we can derive the relation

$$E_l=A\sqrt{1\pm\sqrt{1-\frac{(R^2+X^2)P^2\times10^6}{A^4\cos^2\phi}}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (141)$$

in which

$$A=E_g\sqrt{\frac{1}{2}-\frac{(R\cos\phi+X\sin\phi)P\times10^3}{E_g^2\cos\phi}}. \quad . \quad . \quad . \quad . \quad (142)$$

The combination of plus and minus signs in Eq. (141) shows that there are two possible solutions to the problem, both solutions satisfying the conditions specified. Actually, the plus sign always gives the real solution called for in practice, the other solution being for a heavily overloaded line.

In case the R and X of the load circuit are given, instead of the load power, the solution is very simple. The resistances of the load and line are added arithmetically, to give the total resistance of the circuit; in the same way the total reactance of the circuit is obtained and hence the total impedance can be calculated. The current is then calculable, and this, multiplied by the impedance of the load circuit, gives the voltage of the load.

358. Regulation of a Three-phase Transmission Line.—The regulation of a three-phase line is best analyzed by thinking of the three-phase line as made up of three single-phase lines, with the hypothetical neutral of the three-phase line forming the common return path for the three separate single-phase lines. We have previously shown (section 83) that in a balanced three-phase line a neutral wire, even if installed, would carry no current and that therefore *there would be no drop in it.* Hence, each imaginary single-phase line would have an IZ drop in one side only, this one side being one of the wires of the actual three-phase system. This idea is indicated in Fig. 404; each of the three-phase wires has a resistance R and a reactance X. The imaginary neutral, which has been shown, will have no drop if the three-phase load is balanced, as we are assuming it to be. We may now consider the three-phase circuit as three independent single-phase circuits. One of these single-phase circuits has a generator O–A, which supplies power to the load circuit o–a, over the single-phase line consisting of the wire A–a and the wire O–o. In the latter wire there is no drop to consider. If E_g is the voltage of O–A, the voltage at the load o–a will evidently be equal to this minus (vectorially) the IZ drop in wire A–a. Owing to the symmetry of the connections, the magnitude of the voltage across o–a will be the same as that across o–b and o–c. Furthermore, as the voltages O–A, O–B, and O–C are 120° apart in phase, the voltages o–a, o–b, and o–c will be similarly separated in phase. Hence the voltage across points a–b of the load will be equal to the voltage across o–a multiplied by $\sqrt{3}$, the same ratio as has been previously proved true for three-phase Y-connected systems.

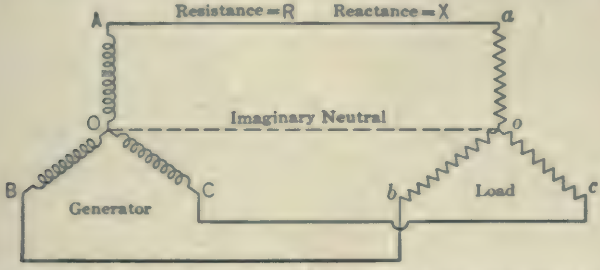

FIG. 404.—To calculate the regulation of a three-phase line, the circuit is considered as three single-phase circuits, the neutral serving as a common wire for all phases.

In solving the three-phase line, therefore, we always treat the generator as a Y-connected machine and the load as a Y-connected load. The resistance of one wire of the three-phase line is calculated, as is its reactance. This reactance of the wire is calculated *to neutral*, that is, as the reactance which would exist between the wire considered and a neutral wire if there were one present. (Tables of reactance to neutral of various wires are given on pp. 403–404.) The problem is then solved as a single-phase line, with no drop in the hypothetical neutral return, and the line voltages are then obtained by multiplying the result by $\sqrt{3}$.

As an example we shall calculate the regulation of a three-phase line having a Y-connected load of $R = 1000$ ohms and $X = 400$ ohms per phase. The voltage between lines at the load is 100,000, and the three-phase transmission line has a resistance of 50 ohms per wire and an inductive reactance, to neutral, of 40 ohms per wire. The problem is indicated in Fig. 405.

The voltage impressed on each phase of the load is $100{,}000/\sqrt{3}$ or 57,800 volts. The impedance of each phase of the load is $\sqrt{1000^2 + 400^2} =$ 1077 ohms. The current in each wire of the transmission line is therefore 57,800/1077 = 53.6 amperes. The power factor of the load, $\cos\phi =$ $1000/1077 = 0.928$. By calculation, or from tables, we find $\sin\phi = 0.37$. The IR drop per wire is 2680 volts, and the IX drop per wire is 2145 volts. Then, using Eq. (140) we find the voltage across one phase of the generator

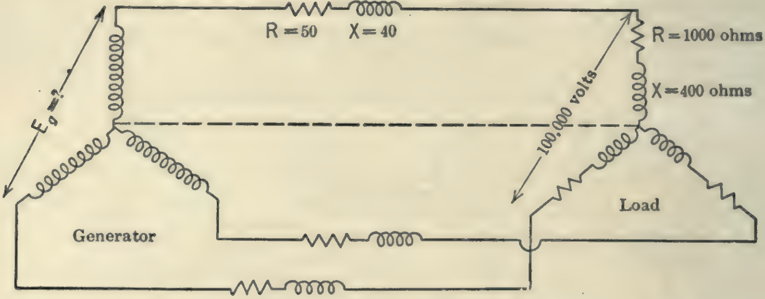

FIG. 405.—A typical three-phase circuit, to have its regulation calculated.

$$E_g = \sqrt{\{57{,}800 + (2680 \times .928) + (2145 \times .37)^2\} + \{(-2680 \times .37) + (2145. \times 928)\}^2}$$

$$= \sqrt{61{,}085^2 + 1000^2} = 61{,}100 \text{ volts.}$$

The voltage between lines at the generator is therefore $61{,}100\sqrt{3} = 105{,}700$ volts. The regulation of the line is therefore

$$\text{Regulation} = \frac{105{,}700 - 100{,}000}{100{,}000} \times 100 = 5.7 \text{ per cent.}$$

In case the power to be transmitted is given, the voltage at the generator is given, and that at the load is to be found, the problem must be solved by the help of Eqs. (141) and (142). All quantities are again to be calculated "to neutral." For this three-phase problem it is to be noted,

TABLE V

25-Cycle Reactance of Solid Non-magnetic Wires *

Ohms per mile of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, Cir. Mils. or A.W.G.	Diam. of Wire, Inches	Inches between Wires, Center to Center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.04770	0.1032	0.1383	0.1588	0.1733	0.1939	0.2085	0.2196
750,000	0.8660	0.05497	0.1105	0.1456	0.1661	0.1807	0.2011	0.2157	0.2270
500,000	0.7071	0.06523	0.1208	0.1558	0.1763	0.1909	0.2115	0.2259	0.2372
350,000	0.5916	0.07425	0.1298	0.1648	0.1854	0.1998	0.2204	0.2350	0.2462
250,000	0.5000	0.08274	0.1383	0.1733	0.1939	0.2085	0.2289	0.2435	0.2548
0000	0.4600	0.08697	0.1425	0.1775	0.1981	0.2126	0.2331	0.2477	0.2589
000	0.4096	0.09283	0.1484	0.1835	0.2039	0.2185	0.2389	0.2536	0.2649
00	0.3648	0.09869	0.1542	0.1893	0.2097	0.2243	0.2449	0.2594	0.2707
0	0.3249	0.1045	0.1601	0.1951	0.2157	0.2302	0.2507	0.2652	0.2765
1	0.2893	0.1101	0.1661	0.2011	0.2215	0.2361	0.2565	0.2712	0.2825
2	0.2576	0.1163	0.1719	0.2069	0.2273	0.2419	0.2625	0.2770	0.2883
4	0.2043	0.1280	0.1835	0.2187	0.2391	0.2537	0.2741	0.2887	0.3001
6	0.1620	0.1397	0.1953	0.2303	0.2509	0.2653	0.2859	0.3004	0.3117
8	0.1285	0.1515	0.2071	0.2421	0.2625	0.2771	0.2975	0.3122	0.3235
10	0.1019	0.1632	0.2187	0.2537	0.2743	0.2889	0.3093	0.3239	0.3353
12	0.08081	0.1749	0.2305	0.2655	0.2861	0.3005	0.3211	0.3356	0.3469
14	0.06408	0.1866	0.2422	0.2773	0.2977	0.3123	0.3327	0.3473	0.3587
16	0.05082	0.1984	0.2539	0.2889	0.3095	0.3239	0.3445	0.3591	0.3703

Size of Wire, Cir. Mils. or A.W.G.	Feet between Wires, Center to Center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.2289	0.2435	0.2548	0.2639	0.2785	0.2898	0.3103	0.3249	0.3362
750,000	0.2361	0.2507	0.2620	0.2712	0.2858	0.2971	0.3175	0.3321	0.3434
500,000	0.2465	0.2609	0.2723	0.2815	0.2960	0.3073	0.3279	0.3423	0.3536
350,000	0.2554	0.2701	0.2814	0.2905	0.3051	0.3164	0.3368	0.3514	0.3627
250,000	0.2639	0.2785	0.2898	0.2990	0.3136	0.3249	0.3453	0.3599	0.3712
0000	0.2682	0.2828	0.2941	0.3032	0.3178	0.3291	0.3495	0.3642	0.3755
000	0.2740	0.2886	0.2999	0.3090	0.3236	0.3349	0.3554	0.3700	0.3813
00	0.2800	0.2944	0.3057	0.3150	0.3294	0.3407	0.3613	0.3758	0.3871
0	0.2858	0.3002	0.3115	0.3208	0.3354	0.3466	0.3671	0.3818	0.3929
1	0.2916	0.3062	0.3175	0.3266	0.3412	0.3525	0.3730	0.3876	0.3989
2	0.2975	0.3120	0.3233	0.3326	0.3470	0.3583	0.3789	0.3934	0.4047
4	0.3092	0.3238	0.3351	0.3442	0.3588	0.3701	0.3906	0.4052	0.4165
6	0.3210	0.3354	0.3467	0.3560	0.3706	0.3818	0.4023	0.4169	0.4281
8	0.3327	0.3472	0.3585	0.3678	0.3822	0.3935	0.4141	0.4286	0.4399
10	0.3444	0.3590	0.3703	0.3794	0.3940	0.4053	0.4257	0.4404	0.4517
12	0.3561	0.3706	0.3819	0.3912	0.4056	0.4169	0.4375	0.4520	0.4633
14	0.3678	0.3824	0.3937	0.4030	0.4174	0.4287	0.4493	0.4638	0.4751
16	0.3796	0.3942	0.4055	0.4146	0.4292	0.4405	0.4609	0.4755	0.4869

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* The reactances given in this table also apply, with a practically negligible error (about 1 per cent) to ordinary stranded wires of the *same cross-section.*

TABLE VI

60-CYCLE REACTANCE OF SOLID NON-MAGNETIC WIRES *

Ohms per mile of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, Cir. Mils. or A.W.G.	Diam. of Wire, Inches	Inches between Wires, Center to Center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.1145	0.2478	0.3319	0.3811	0.4158	0.4652	0.5003	0.5270
750,000	0.8660	0.1319	0.2652	0.3493	0.3985	0.4336	0.4826	0.5176	0.5448
500,000	0.7071	0.1565	0.2898	0.3739	0.4230	0.4581	0.5074	0.5421	0.5693
350,000	0.5916	0.1782	0.3115	0.3955	0.4449	0.4795	0.5289	0.5640	0.5908
250,000	0.5000	0.1986	0.3319	0.4158	0.4652	0.5003	0.5493	0.5844	0.6115
0000	0.4600	0.2087	0.3420	0.4260	0.4754	0.5101	0.5595	0.5945	0.6213
000	0.4096	0.2228	0.3561	0.4403	0.4893	0.5244	0.5734	0.6085	0.6356
00	0.3648	0.2368	0.3701	0.4543	0.5033	0.5384	0.5877	0.6224	0.6496
0	0.3249	0.2509	0.3842	0.4682	0.5176	0.5523	0.6017	0.6364	0.6635
1	0.2893	0.2650	0.3985	0.4826	0.5316	0.5666	0.6156	0.6507	0.6778
2	0.2576	0.2791	0.4124	0.4965	0.5455	0.5806	0.6300	0.6647	0.6918
4	0.2043	0.3072	0.4403	0.5248	0.5738	0.6089	0.6579	0.6929	0.7201
6	0.1620	0.3353	0.4686	0.5527	0.6021	0.6368	0.6861	0.7208	0.7480
8	0.1285	0.3635	0.4969	0.5810	0.6300	0.6650	0.7140	0.7491	0.7762
10	0.1019	0.3917	0.5248	0.6089	0.6582	0.6933	0.7423	0.7774	0.8045
12	0.08081	0.4196	0.5531	0.6371	0.6865	0.7212	0.7706	0.8053	0.8324
14	0.06408	0.4479	0.5813	0.6654	0.7144	0.7495	0.7985	0.8335	0.8607
16	0.05082	0.4762	0.6092	0.6933	0.7427	0.7774	0.8268	0.8618	0.8886

Size of Wire, Cir. Mils. or A.W.G.	Feet between Wires, Center to Center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.5493	0.5844	0.6115	0.6334	0.6684	0.6956	0.7446	0.7796	0.8068
750,000	0.5666	0.6017	0.6288	0.6507	0.6858	0.7129	0.7619	0.7970	0.8241
500,000	0.5915	0.6262	0.6533	0.6756	0.7103	0.7374	0.7868	0.8215	0.8486
350,000	0.6130	0.6481	0.6752	0.6971	0.7321	0.7593	0.8083	0.8433	0.8705
250,000	0.6334	0.6684	0.6956	0.7174	0.7525	0.7796	0.8286	0.8637	0.8909
0000	0.6435	0.6786	0.7057	0.7276	0.7627	0.7898	0.8388	0.8739	0.9010
000	0.6575	0.6925	0.7196	0.7416	0.7766	0.8030	0.8528	0.8878	0.9150
00	0.6718	0.7065	0.7336	0.7559	0.7906	0.8177	0.8671	0.9018	0.9289
0	0.6858	0.7204	0.7476	0.7698	0.8049	0.8317	0.8810	0.9161	0.9429
1	0.6997	0.7348	0.7619	0.7838	0.8188	0.8460	0.8950	0.9301	0.9572
2	0.7140	0.7487	0.7759	0.7981	0.8328	0.8599	0.9093	0.9440	0.9712
4	0.7419	0.7770	0.8041	0.8260	0.8611	0.8882	0.9372	0.9723	0.9994
6	0.7702	0.8049	0.8320	0.8543	0.8893	0.9161	0.9655	1.001	1.027
8	0.7985	0.8332	0.8603	0.8826	0.9172	0.9444	0.9938	1.028	1.056
10	0.8264	0.8614	0.8886	0.9105	0.9455	0.9727	1.022	1.057	1.084
12	0.8547	0.8893	0.9165	0.9387	0.9734	1.001	1.050	1.085	1.112
14	0.8826	0.9176	0.9448	0.9670	1.002	1.029	1.078	1.113	1.140
16	0.9108	0.9459	0.9730	0.9949	1.030	1.057	1.106	1.141	1.168

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* The reactances given in this table also apply, with a practically negligible error (about 1 per cent) to ordinary stranded wires of the *same cross-section*.

in using Eqs. (141) and (142), that P is the power per phase, that is, one-third of the specified load, and E is the voltage to neutral.

359. Inductive Reactance of Transmission Lines.—For a given spacing and size of wires, there are formulae available which permit the calculation of the self-induction (to neutral) for any length of line. As the self-induction itself is never used in calculations—$2\pi fL$, the inductive reactance, being used instead—this quantity has been calculated and tabulated for all sizes of wires and spacings that may be used in practice.

Tables V and VI give the inductive reactances, to neutral, for a single-phase, or a symmetrically arranged three-phase line. The symmetry assumed for the three-phase line, in making these calculations, is that the three wires are arranged in the form of an equilateral triangle. If the wires are otherwise disposed, the reactance cannot be obtained from the tables; for unsymmetrical arrangements more detailed texts on transmission lines must be consulted.

As 25 cycles and 60 cycles are the only frequencies extensively used in the United States, the reactances have been given for these two frequencies only. For others the reactance may be readily obtained by direct proportion.

360. Graphical Determination of the Regulation.—Various schemes have been designed for solving the regulation of alternating-current lines without going through the arithmetical work involved. One of these schemes makes use of a chart originated by R. D. Mershon, reproduced in Fig. 406. To use the chart we first calculate the per cent resistance drop and reactance drop in the line (using one wire only).

Let l = length of line, in miles (length of one wire);
I = amperes per wire demanded by the load;
E_l = volts to neutral, at the load;
r = resistance of each wire in ohms per mile;
x = reactance (to neutral) of each wire, in ohms per mile.

Then

$$\text{Per cent resistance drop} = \frac{rlI}{E_l} \times 100,$$

$$\text{Per cent reactance drop} = \frac{xlI}{E_l} \times 100.$$

To use the chart, follow the circular arc marked 0, from the bottom of the chart, until it intersects the vertical giving the power factor of the load. From this point of intersection lay off to the right the per cent resistance drop, and from the point so obtained lay off vertically the per cent reactance drop. From the point so obtained read off, from the circular arc scale, the regulation of the line.

As an illustration of the use of this chart, we shall solve again the problem given in section 358.

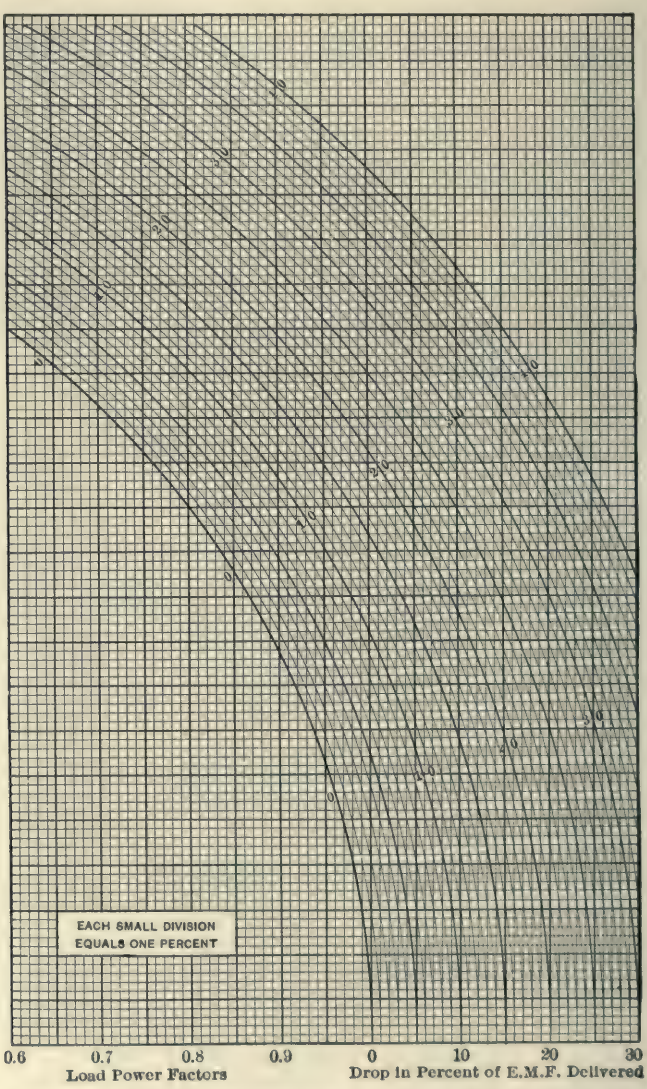

FIG. 406.—A convenient chart for graphically determining the regulation of transmission lines.

$$\text{Per cent resistance drop} = \frac{50 \times 53.6}{57,800} \times 100 = 4.64.$$

$$\text{Per cent reactance drop} = \frac{40 \times 53.6}{57,800} \times 100 = 3.71.$$

Entering the chart at the bottom of the arc marked 0, we follow it until it intersects the power-factor line 0.928, move to the right 4.64 squares and then up 3.71 squares. This point lies on the circle numbered 6; hence the regulation is 6 per cent, as previously obtained by calculation.

361. Corona.—The principal limitation on the voltage which can be efficiently used on transmission lines is the disruptive strength of the air. If the voltage gradient around a conductor exceeds about 76 kilovolts per inch, the air is actually broken down electrically and becomes partially conducting. This breakdown of the air is accompanied by a bluish haze around the wire, together with a hissing sound and the characteristic odor of ozone.

The electric field between two conductors of a transmission line is distributed about as shown in Fig. 407, being most intense near the wire.

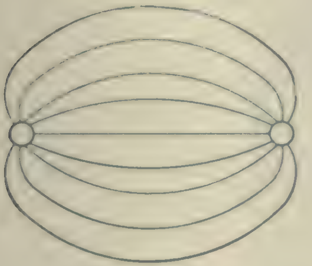

FIG. 407.

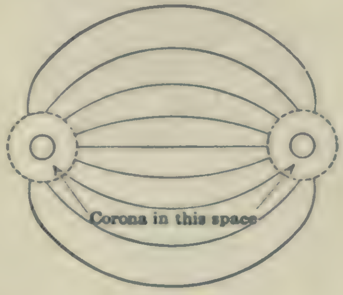

FIG. 408.

FIG. 407.—Showing approximately the electric field distribution between the wires of a single-phase transmission line.

FIG. 408.—The presence of corona around a wire acts to increase the apparent diameter of the wire, in so far as the electric field is considered.

As the voltage on such a pair is gradually raised, the gradient at the surface of the wire reaches the critical value of 76 kilovolts per inch and the corona starts. This makes the air immediately around the wire a conductor, and, in so far as the electric field is concerned, has the effect of increasing the diameter of the wire, as suggested in Fig. 408. But, with a given voltage between wires, the gradient at the surface of the wires decreases as the size of the wires increases, for the wire separations ordinarily used in transmission lines. Hence, the corona spreads for a definite distance into the air and then stops, as indicated by the dotted circle of Fig. 408. All the space inside the dotted circle of this figure acts, to some extent, like a conductor and is filled with the blue haze. As the voltage impressed on the pair of wires is still further increased, the corona zone around the wires spreads out farther and farther from the wire, until finally a spark jumps from one wire to the other, an arc follows, and the line becomes short-circuited.

It would naturally be thought that, with a given spacing between centers of wires, the air would be more likely to break down if the wires were made larger and the distance between their closest surfaces thus decreased. Such is not the fact, however; if the wires of an ordinary transmission line were replaced by very small ones, thus increasing the distance between the surfaces of the wires, the air would be broken down all around the wires, showing that the gradient in the air had been *increased* by thus *increasing the distance between the nearest surfaces of the wires.* Increasing the distance does decrease the *average gradient;* but decreasing the size of the wire (thus increasing its curvature) tends to increase the gradient at the surface of the wire, and this effect is more than enough to overcome the decrease in the average gradient due to increased distance.

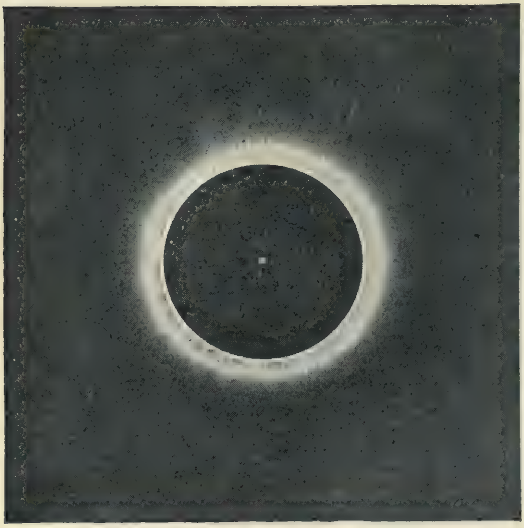

FIG. 409.—Photograph showing the corona surrounding one wire of a transmission line operated at too high a voltage.

In Fig. 409 is shown a photograph of one wire of a transmission line on which high voltage has been impressed; the brush-like discharge of the corona is plainly visible.

The general laws governing corona formation have been experimentally determined as follows:

For a given density of air, and a given spacing between wires, the voltage at which corona starts is lowered by using wires of smaller diameter.

The critical voltage at which corona starts can be increased somewhat by increasing the spacing of a given pair of wires, but this increase is not as effective in preventing corona as an increase in the diameter of the wires.

Decreasing barometric pressure and increasing the temperature of the air around the conductor both decrease somewhat the critical voltage at which corona starts.

The corona starts much more readily on dirty wires than on clean ones, because the bits of dirt on the surface give local concentrations of the electric field, acting like metallic points.

The formation of corona around the wires of a transmission line represents a power loss, which may be very considerable if the critical voltage is much exceeded. The corona power loss, in a three-phase line, in kilowatts per mile of single conductor, in fair weather, is given by the formula

$$P=130(f+25)(e-e_0)^2\sqrt{\frac{r}{s}}\times 10^{-5}. \quad . \quad . \quad . \quad . \quad . \quad (143)$$

In this formula f = frequency, in cycles per second;
e = effective kilovolts between wires;
e_0 = disruptive critical voltage, in kilovolts, for the wires and spacing used (obtained from tables);
r = radius of the conductor;
s = distance between conductor centers, in same unit as r.

The above formula is for average temperature and pressure (77° F. and 29.9 inches); a slight correction is necessary for other conditions.

The critical disruptive voltage for wires of various sizes and spacing is given in Table VII; this table is for seven-strand cable, and gives the critical voltage *between wires* as required for Eq. (143).

As an example, it is desired to know the corona loss under average weather conditions of a three-phase 60-cycle line 150 miles long, wires No. 0000, spaced 10 feet, the line voltage being 150,000 volts.

The critical pressure e_0, for this line, is found in Table VII to be 130 kilovolts, hence by Eq. (143) the loss is

$$P=130(60+25)(150-130)^2\sqrt{\frac{0.265}{120}}\times 10^{-5}$$
$$=2.08 \text{ kw. per wire per mile.}$$

As there are 450 miles of wire, the total loss would be 938 kw., a prohibitive figure. A pressure of 150 kv. is much too high for a line of No. 0000 wires spaced 10 feet.

Under stormy conditions the disruptive voltage may drop as low as 80 per cent of the fair-weather figure, given in Table VII, so that the highest advisable voltage for the line in question would not be greater than 110 kv.

362. Effect of Line Capacity.—A long transmission line evidently possesses considerable electrostatic capacity; the different wires of the line act like the plates of a condenser, the dielectric of the condenser being the air between the wires. This capacity may require a large charging current if the line is long and the voltage is high.

The charging current is required whether the load is connected or not;

TABLE VII

Corona Limit of Voltage–7–Strand Cables

Kilovolts between lines, three-phase, sea level, 25° C.

Size A.W.G. or Cir. Mils	Diameter Inches	Diameter Cm.	Spacings in Feet									
			3	4	5	6	8	10	12	14	16	20
			Spacings in Centimeters									
			91	122	152	183	245	305	366	426	490	610
4	0.230	0.584	....	56	58	60	62	64	66	68	69	71
3	0.261	0.663	....	62	65	67	70	72	74	76	77	80
2	0.290	0.736	....	....	71	73	76	79	81	83	85	87
1	0.330	0.839	....	....	79	81	85	88	91	93	95	97
0	0.374	0.950	....	....	....	90	95	98	102	104	108	109
00	0.420	1.07	....	....	....	98	104	108	111	114	117	121
000	0.470	1.19	....	....	....	....	114	118	121	124	127	132
0000	0.530	1.35	....	....	....	....	125	130	135	138	141	146
250,000	0.590	1.50	....	....	....	....	138	144	149	152	156	161
300,000	0.620	1.57	....	....	....	....	....	151	156	161	165	171
350,000	0.679	1.73	....	....	....	....	....	161	166	170	175	180
400,000	0.728	1.85	....	....	....	....	....	171	176	180	185	192
450,000	0.770	1.96	....	....	....	....	....	178	184	190	194	200
500,000	0.818	2.04	....	....	....	....	....	188	194	199	205	210
800,000	1.034	2.62	....	....	....	....	....	....	234	241	244	256
1,000,000	1.152	2.92	....	....	....	....	....	....	256	264	270	281

as long as the line is alive it must be supplied with the charging current that its capacity requires, at the voltage and frequency used. If much load is being carried, the presence of the charging current is not noticeable as the actual current is the vector sum of the load current and the charging current; with the ordinary inductive load, the charging current required from the power house may be very small, the lagging current of the load being just sufficient to neutralize it. When there is no load connected to the transmission line, however, the charging current is very evident; it is a maximum at the generator end of the line and decreases gradually to zero at the load end.

The charging current of a condenser is given by the equation $I=2\pi fCE$. The quantity $2\pi fC$ is called the *susceptance* of the condenser (the resistance of the condenser being negligible); it is the quantity by which the voltage must be multiplied to obtain the charging current. The susceptance of transmission lines has been calculated, for wires of various sizes and spacings used in practice, for both 25 and 60 cycles; it is given in Tables VIII and IX.

TABLE VIII

25-Cycle Capacity Susceptance to Neutral * Standard Strands

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) $\times 10^{-6}$.

Approximate micromhos per mile of each conductor of a single-phase or of a symmetrical three-phase line.

Size of Cable, C.M. or A.W.G.	Diameter of Strand, Inches	Inches between Conductors, Center to Center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	8.70	6.01	5.10	4.62	4.08	3.77	3.55
750,000	0.998	.....	7.94	5.67	4.85	4.41	3.91	3.63	3.42
500,000	0.814	21.0	7.10	5.23	4.54	4.14	3.71	3.44	3.27
350,000	0.681	15.0	6.48	4.90	4.29	3.94	3.53	3.30	3.14
250,000	0.575	12.2	6.01	4.63	4.08	3.77	3.39	3.17	3.02
0000	0.528	11.2	5.79	4.49	3.97	3.67	3.33	3.11	2.97
000	0.470	10.1	5.53	4.33	3.85	3.57	3.24	3.03	2.89
00	0.418	9.26	5.28	4.18	3.74	3.47	3.16	2.97	2.83
0	0.373	8.54	5.06	4.05	3.63	3.36	3.08	2.89	2.76
1	0.332	7.94	4.85	3.91	3.52	3.28	3.00	2.83	2.70
2	0.292	7.38	4.65	3.79	3.41	3.19	2.92	2.75	2.64
4	0.232	6.55	4.32	3.57	3.22	3.03	2.78	2.64	2.53
6	0.184	5.90	4.02	3.36	3.06	2.89	2.65	2.53	2.42

Size of Cable, C.M. or A.W.G.	Feet between Conductors, Center to Center								
	3	4	5	6	8	10	15	20	25
1,000,000	3.39	3.17	3.03	2.90	2.75	2.64	2.45	2.32	2.25
750,000	3.28	3.08	2.94	2.83	2.67	2.56	2.39	2.28	2.20
500,000	3.14	2.95	2.81	2.72	2.57	2.47	2.31	2.20	2.12
350,000	3.02	2.84	2.72	2.62	2.50	2.40	2.25	2.14	2.07
250,000	2.90	2.75	2.64	2.54	2.42	2.32	2.18	2.09	2.03
0000	2.86	2.70	2.59	2.51	2.39	2.29	2.15	2.06	1.99
000	2.79	2.64	2.53	2.45	2.34	2.25	2.12	2.03	1.96
00	2.73	2.59	2.48	2.40	2.29	2.22	2.07	1.99	1.93
0	2.67	2.53	2.43	2.36	2.25	2.17	2.04	1.96	1.90
1	2.61	2.48	2.39	2.31	2.22	2.14	2.01	1.93	1.87
2	2.54	2.42	2.34	2.26	2.17	2.09	1.98	1.90	1.84
4	2.45	2.32	2.25	2.18	2.09	2.03	1.92	1.84	1.79
6	2.36	2.25	2.17	2.10	2.03	1.96	1.85	1.79	1.74

---

* The susceptance *between* conductors equals one-half the values given in this table.

TABLE IX

60-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL * STANDARD STRANDS

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) $\times 10^{-6}$.

Approximate micromhos per mile of each conductor of a single-phase or of a symmetrical three-phase line.

Size of Cable, C.M. or A.W.G.	Diameter of Strand, Inches	Inches between Cables, Center to Center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	20.9	14.4	12.3	11.1	9.80	9.05	8.52
750,000	0.998	.....	19.1	13.6	11.6	10.6	9.39	8.71	8.22
500,000	0.814	50.5	17.0	12.6	10.9	9.95	8.90	8.26	7.84
350,000	0.681	36.0	15.6	11.8	10.3	9.46	8.48	7.92	7.54
250,000	0.575	29.3	14.4	11.1	9.80	9.05	8.14	7.62	7.24
0000	0.528	26.9	13.9	10.8	9.54	8.82	7.99	7.46	7.13
000	0.470	24.3	13.3	10.4	9.24	8.56	7.77	7.28	6.94
00	0.418	22.2	12.7	10.0	8.97	8.33	7.58	7.13	6.79
0	0.373	20.5	12.1	9.73	8.71	8.07	7.39	6.94	6.63
1	0.332	19.1	11.6	9.39	8.44	7.88	7.20	6.79	6.48
2	0.292	17.7	11.2	9.09	8.81	7.65	7.01	6.60	6.33
4	0.232	15.7	10.4	8.56	7.73	7.28	6.67	6.33	6.07
6	0.184	14.2	9.65	8.07	7.35	6.94	6.37	6.07	5.81

Size of Cable, C.M. or A.W.G.	Feet between Cables, Center to Center								
	3	4	5	6	8	10	15	20	25
1,000,000	8.14	7.62	7.28	6.97	6.60	6.33	5.88	5.58	5.39
750,000	7.88	7.39	7.05	6.79	6.41	6.15	5.73	5.47	5.28
500,000	7.54	7.09	6.75	6.52	6.18	5.92	5.54	5.28	5.09
350,000	7.24	6.82	6.52	6.30	5.99	5.77	5.39	5.13	4.98
250,000	6.97	6.60	6.33	6.11	5.81	5.58	5.24	5.01	4.86
0000	6.86	6.48	6.22	6.03	5.73	5.50	5.16	4.94	4.79
000	6.71	6.33	6.07	5.88	5.62	5.39	5.09	4.86	4.71
00	6.56	6.22	5.96	5.77	5.50	5.32	4.98	4.79	4.64
0	6.40	6.07	5.84	5.66	5.39	5.20	4.90	4.71	4.56
1	6.26	5.96	5.73	5.54	5.32	5.13	4.83	4.64	4.49
2	6.11	5.81	5.62	5.43	5.20	5.01	4.75	4.56	4.41
4	5.88	5.58	5.39	5.24	5.01	4.86	4.60	4.41	4.30
6	5.66	5.39	5.20	5.05	4.86	4.71	4.45	4.30	4.18

* The susceptance *between* conductors equals one-half the values given in this table.

The susceptance is given in these tables *to the neutral;* in case the line is three-phase, the susceptance obtained from the tables is to be multiplied by the voltage to neutral of the line, that is, by the line voltage divided by $\sqrt{3}$.

Let us calculate the charging current of the line for which the corona loss was calculated, 150 miles long, 60 cycles, No. 0000 wires, 10 feet spacing, 150,000 volts between wires. The voltage to neutral is $150{,}000 \div \sqrt{3} = 86{,}600$ volts. In Table IX the susceptance, to neutral, of No. 0000 cable spaced 10 feet, is 5.50 micromhos per mile. The charging current per mile is therefore $I = 5.50 \times 10^{-6} \times 86{,}600 = 0.476$ ampere.

As the line is 150 miles long, the charging current entering each wire at the generator end is $150 \times 0.476 = 71.4$ amperes. It would require a very large generator, therefore, just to keep such a transmission line alive. A current of 71.4 amperes at 150,000 volts in a three-phase system requires a machine with a kilovolt-ampere rating of

$$\sqrt{3} \times 150{,}000 \times 71.4 \div 1000 = 18{,}500 \text{ kv.a.}$$

In case the line was run at 25 cycles instead of 60 cycles, the charging current would be cut down proportionately, and the rating of the machine just large enough to keep the line alive would be 7720 kv.a.

It is to be noticed that although a very large alternator would be required to keep the line alive, the steam turbine driving the generator would have to do practically no work, because the only loss this large kv.a. input represents is the I^2R loss in the line itself. This will generally be only about 1 per cent of the kv.a. supplied to the unloaded line.

363. Most Suitable Frequency for Transmission Lines.—In section 358, dealing with the regulation of transmission lines, it was shown that the reactance of the line is very important in determining the regulation of the line, especially when the load is inductive, which is generally the case. Also it was shown that the reactance was much greater at 60 cycles than at 25 cycles, in fact 60 25 as much, so that from the standpoint of regulation 25 cycles is much more suitable than 60 cycles.

It now appears that from the standpoint of charging current, and amount of machinery required to keep the line alive, 25 cycles is also much preferable to 60 cycles. On account of these two factors, regulation and charging current, practically all long transmission lines have been installed for 25-cycle operation. In case an incandescent-lamp load is to be supplied from the line, frequency changers should be installed, because 25-cycle current is not suitable for use with low-power tungsten lamps. On 25-cycle current a very noticeable and irritating flickering occurs in all lamps with thin filaments, that is, with all 110-volt lamps of low candle-power.

It might be thought that frequencies even lower than 25 cycles would be preferable for transmission-line operation; but if the frequency is lowered too much, objections arise with regard to the cost and weight of machinery. If the frequency is lowered to less than 25 cycles, it will be found that the cost of apparatus, per kv.a., will be in general considerably higher than it would be at 25 cycles and higher. The 25-cycle frequency, therefore, seems to be a reasonable compromise between the demands of the transmission line and the economical design of machinery.

364. Rise of Voltage in a Transmission Line.—A transmission line has inductance, resistance, and capacitance, these being uniformly distributed throughout its length about as suggested in Fig. 410. It would seem that in such a circuit resonance phenomena would be likely to occur at certain impressed frequencies, and such is actually the fact. Even when the total line capacity, combined with the total line inductance, is much lower than that required for resonance with the impressed frequency, a kind of resonance occurs when the line is disconnected from its load. The voltage between wires actually increases with increasing distance from the generator end, being a maximum at the open-circuited load end. The exact amount of this rise in voltage can be calculated by the use of hyperbolic functions, and the conditions for maximum rise of voltage, etc., can be accurately predicted.

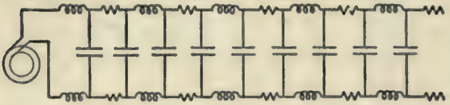

FIG. 410.—A transmission line can be reasonably well represented, electrically, by a large number of sections of resistance, inductance and capacitance, as shown here.

In a certain line 160 miles long, with an impressed voltage of 60,000 volts at the generator end, the voltage at the load end is 66,000 volts, 10 per cent higher than the generator voltage. This resonant rise in voltage occurs only with an open-circuited line; as soon as load is connected, even though only a small fraction of the rated capacity of the line, the voltage at the load end drops, becoming less than the generator end voltage, as we should expect.

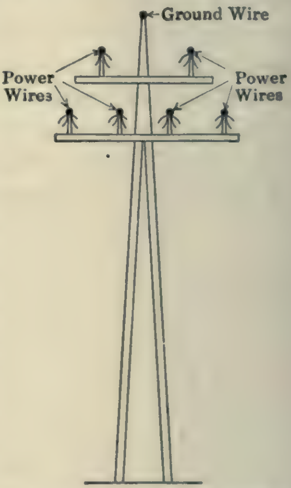

FIG. 411.—A typical arrangement of two three-phase lines with ground wire above.

365. Ground Wires.—Although there is at present some question as to the efficacy of the scheme, it seems to be good practice to install a ground wire above a transmission line, the idea being that it offers some protection against lightning disturbances. The wire is placed on the poles as shown in Fig. 411, where two symmetrical three-phase lines are shown on the cross arms, and the ground wire, solidly connected to the steel tower, is carried above both transmission lines. The presumption is that a lightning disturbance, which might otherwise hit the transmission line and put it out of operation by cracking insulators, breaking down the insulation of transformers, etc., will be carried to earth by the ground wire without creating any disturbance in the line itself. In some three-phase systems the ground wire serves as the neutral, carrying whatever unbalanced current may be demanded by the load. In such a system no wire can differ from ground potential by more than $1/\sqrt{3}$ of the line voltage, thus definitely limiting the possible strain on the insulators. A ground on any one wire of such a system, however, results in a short-circuit, with consequent interruption of the service; whereas with an ungrounded system a dead ground on one wire does not result in any excessive current at all, but it will put a strain, equal to full-line voltage, on the insulators of the other two lines. If any of these insulators happen to be slightly defective, this extra high voltage is very likely to break them down, even though they are sufficiently strong to stand their normal operating voltage.

366. Lightning Arresters.—An ideal lightning arrester is a device which instantaneously carries off to earth any induced charges which raise the voltage of the line appreciably above its normal operating value, and at once restores the normal insulation between the line and ground when the temporary high-voltage condition has been relieved. When a lightning discharge takes place in the vicinity of a transmission line, induced voltages are set up in the line, and these voltages, combined with the normal operating voltage, may result in a voltage high enough to crack insulators Moreover, these induced voltages are likely to be of high frequency, or to start high-frequency surges in the line, and these high-frequency surges traveling along the line are almost sure to do damage to apparatus connected to the line. A high-frequency traveling wave of this sort, entering a power house over the transmission-line wires, may be stopped, for example, by the high impedance of a transformer winding. But the stopping of such a traveling wave results in the setting up of even higher voltages in the system where it is stopped; this nearly always results in breaking down the insulation of the first few turns of the high-voltage winding of the transformer.

A suitably installed, reliable arrester will prevent this damage, by virtue of two actions: it or its auxiliaries offer a high impedance in the road to the power house and offer a comparatively low impedance path to ground. How such an action is set up may be seen by reference to Fig. 412. A pulse, traveling along the transmission line towards the power house, along either wire, or both wires, is blocked from the power house by the reactance of the choke coils, and the resultant building up of the potential at this point makes the current jump the gap and so escape to earth. The choke coils consist of a few turns of heavy conductor, which have so little reactance at line frequency as to be negligible in their effect on the normal operation of the line. The lightning pulse, however, being of high frequency, or being a wave with a "steep wave front," finds them to be of such reactance as to be impassable.

Such a simple installation of apparatus would make a very poor protection against lightning trouble, however, for every time the gap broke down the arcing gap would permit a large current from the power house to flow across the gap and so result in a short-circuited transmission line. The gap must therefore be specially constructed so that it will tend to prevent such a short-circuit. Such a form as the horn gap (Fig. 413) will accomplish this result. One side of the gap is connected to the ground and the other side to one wire of the transmission line. The narrow part of the gap is set just too wide for the normal line voltage to jump across, but a voltage slightly in excess of this will break across. The temporary high voltage is thus relieved, but, in addition, power-house current starts to follow the lightning discharge across the gap, and does so, establishing an arc across the gap at its lowest and narrowest part. In a very short time, however (perhaps a second or two) the arc travels up the horns, owing to the combined effects of heat and electromagnetic repulsion, and thus becomes longer and longer until it ruptures itself and the insulation of the line is again restored.

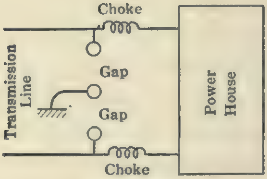

FIG. 412.

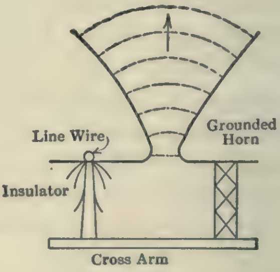

FIG. 413.

FIG. 412.—The principle involved in the lightning arrester is to prevent a surge from entering the power house and at the same time to offer a low resistance to ground.

FIG. 413.—A horn gap acts to extinguish very quickly the arc which follows a lightning discharge.

At present, the most reliable type of arrester for high-power transmission lines is the electrolytic form, the construction of which is indicated in Fig. 414. It is made up of a set of cone-shaped aluminum vessels, nested one within the other and held a short distance apart by suitable insulating spacers; the space between the cones is nearly filled with a solution of sodium or potassium hydroxide. This column of aluminum cups is held together by treated wood supports, and the whole is put into a steel tank which is afterwards filled with oil. The uppermost cup is connected to the line to be protected, either through a horn gap arrester or some equivalent device, and the lowest cup is connected to a suitable earthing connection, perhaps several long iron pipes driven into permanently moist earth.

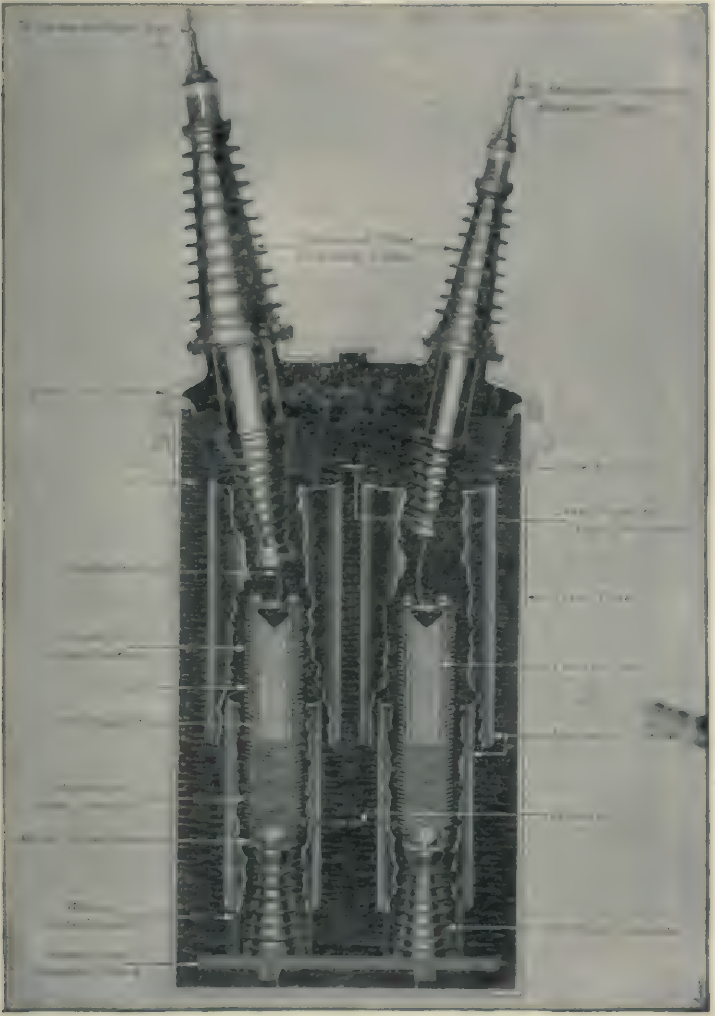

Fig. 414.—Showing the construction of a modern lightning arrester of the electrolytic type. In this one there are two columns of aluminum cups in series, a construction used in arresters designed for the higher voltages.

When such a series of aluminum cups is connected across a high-voltage line, a small current leaks through and forms, on each metal surface, a film of aluminum hydroxide, which is a fairly good insulator for two or three hundred volts. If more than a certain limiting voltage is put on such a pair of aluminum plates, however, the film breaks down completely and permits the flow of current with but little resistance. By putting a sufficient number of such hydroxide films in series (as is done by nesting one aluminum cup in the other) a very high line voltage may be impressed without much line current leaking through. When a lightning pulse comes along the transmission line, raising the voltage above normal, the resistance of the arrester breaks down and discharges the line of its high voltage. A horn gap is practically always connected in series with this type of arrester; as the power-house current follows the lightning discharge, the arc travels up the horn gap and ruptures itself, as stated before. Thus, after the arrester has discharged one pulse, it is immediately ready to take care of another; the film restores itself after the arc goes out.

After an arrester of this type has stood idle for some time, the hydroxide film tends to disappear, but it may be restored by connecting to the line voltage for a moment. The horn gap used in series with the aluminum arrester is generally arranged to permit one side of the gap to be moved into contact with the other; this allows the aluminum arrester to be connected from line to ground, and thus permits a small current to leak through and form the film.

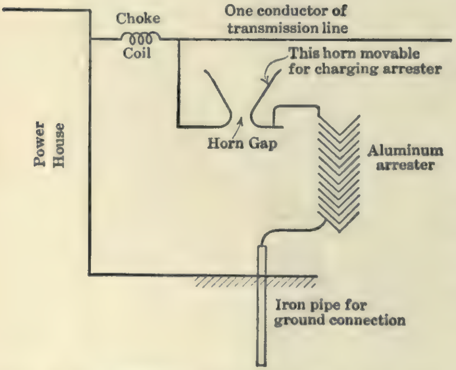
Choke
Coil
One conductor of transmission line
This horn movable for charging arrester
Horn Gap
Aluminum arrester
Power House
Iron pipe for ground connection

Fig. 415.—Showing how the choke coil, horn gap, and electrolytic arrester are combined to give effective protection to the transmission line. Each conductor of the transmission line is similarly equipped.

The horn gap not only helps to extinguish the power arc which is likely to follow the discharge of the arrester, but also serves to insulate the aluminum arrester from the line under normal voltage conditions. This is a useful service, because, although the hydroxide film is of high resistance, a small current will continually leak through if the arrester is directly connected to the line; moreover, because of the capacity of the arrester (the films acting as the condenser dielectric), a considerable charging current will flow and heat the electrolyte unnecessarily. The oil with which the steel tank is filled serves not only as an insulator but also to prevent the hydroxide electrolyte from evaporating, and to help carry off the heat developed by discharges.

The general scheme of installation of such an arrester is shown in Fig. 415, only one line of the three-phase system being shown; similar equipment is required for each wire.

Recently, another type of arrester, employing lead peroxide pellets, has been introduced as a reliable protector, its advantages, as compared with the aluminum arrester, being that it costs less and is less trouble to maintain. An insulating tube (of some material such as porcelain) is filled with peroxide (PbO_2) pellets about $\frac{1}{8}$ inch diameter, the pellets having been covered with a thin layer of some insulating powder. Such an arrester, properly proportioned for the voltage of the line to be protected, and consisting essentially of very many contacts between peroxide pellets, in series and in parallel, permits a steady current of only a very small fraction of one milliampere; when an excessive voltage is impressed the resistance drops to practically zero. After the condition of extra high voltage has passed, the contact surfaces at once regain their insulating qualities so that a discharge due to normal line voltage does not follow the lightning discharge. These pellet arresters apparently are completely self-restoring and self-maintaining, their leakage current actually decreasing as they are kept in service. Figure 416 gives an idea of the construction and appearance of one of these pellet arresters.

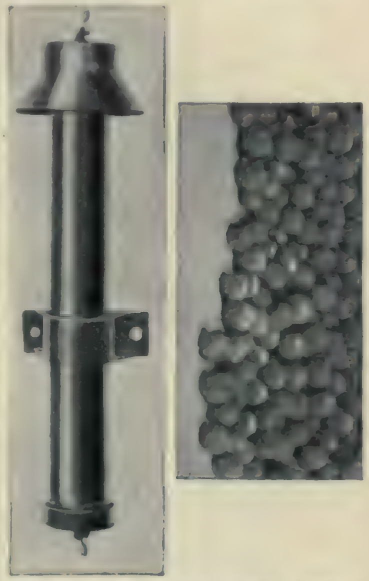

FIG. 416.—The lead pellet arrester, and enlarged view of some of the pellets.

367. Ground Detectors.—A three-phase system may be grounded at the power station or not; in case it is not grounded the neutral point of the system (*O* in Fig. 417) is referred to as a 'floating neutral.' In case the system is grounded no special ground detector is required, as will be evident by inspection of Fig. 417. Should a ground develop in line *A*, for example, the phase *O–A* of the generator would be short-circuited, and, by the excessive indication of the ammeter in line *A*, or by the blowing of the circuit-breaker, the operator would at once know that line *A* was in trouble.

With an ungrounded system the operator would have no direct evidence that line *A* was in trouble, however. All line ammeters would read practically the same as before; in fact, except for extra insulation strains the three-phase line would operate as well as it did before line A was grounded. It will be seen, however, that if line A grounds, the insulators of lines B and C will have impressed upon them full line voltage, instead of only $1/\sqrt{3}$ of the line voltage, for which they are intended. As the insulators in lines B and C are much more likely to break down under this excessive voltage than when the line is in normal condition, it is imperative that the operator know of the grounded condition of line A so that the ground may be located and removed.

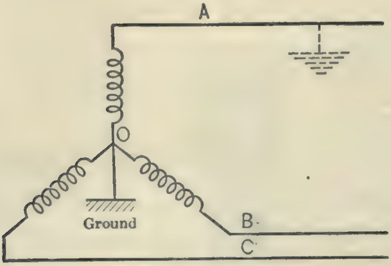

FIG. 417.

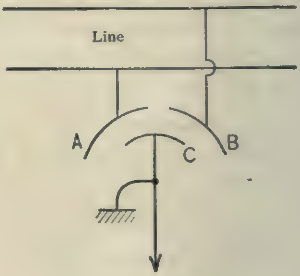

FIG. 418.

FIG. 417.—A three-phase line is sometimes grounded as the neutral, O, and sometimes not.

FIG. 418.—A ground detector for a single-phase circuit; the movable plate, C, will be closer to that line having the higher resistance with respect to ground.

Various kinds of ground detectors are possible, but practically all of those in use are essentially electrostatic voltmeters. In Fig. 418 is shown a common form of ground detector for use on a single-phase circuit. The vane C, connected to ground, will be most strongly attracted by the ungrounded side of the line, and its pointer will so indicate. In case one side of the line is completely grounded, as for example, the side to which A is connected, there will be no attraction between A and C, hence C will move completely over to B. The reading of such an instrument is therefore quantitative to a certain extent, indicating whether the line has a "dead ground" or only a "partial ground."

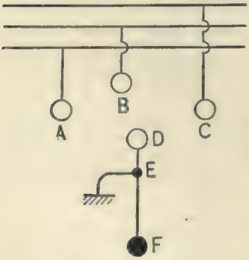

FIG. 419.—The ground indicator for a three-phase line acts similarly to that for the single-phase line, shown in Fig. 418; the movable plate, D, will tend to leave the grounded line.

For a three-phase line, three instruments of the type shown in Fig. 418 may be used, one connected between each pair of wires. Their relative readings indicate the condition of insulation on the different wires. Another type operates about as indicated by the conventional diagram given in Fig. 419. Three spheres arranged in an equilateral triangle, in a horizontal plane, are connected one to each line as shown at *A*, *B*, and *C*. Below them is another sphere *D*, which is electrically connected to ground. It is sufficiently separated from the other three to prevent a spark jumping between them at line potentials. This fourth sphere is pivoted at *E* so that it can swing in any direction, and is counterweighted at *F*. Normally sphere *D* is equally attracted by each of the others, but if one of the lines becomes grounded the attraction between its sphere and *D* is at once diminished so that *D* moves away from it. Hence the position of sphere *D* serves as an indication of the relative potentials of the three lines to ground, and this in turn is determined by their insulation from ground.

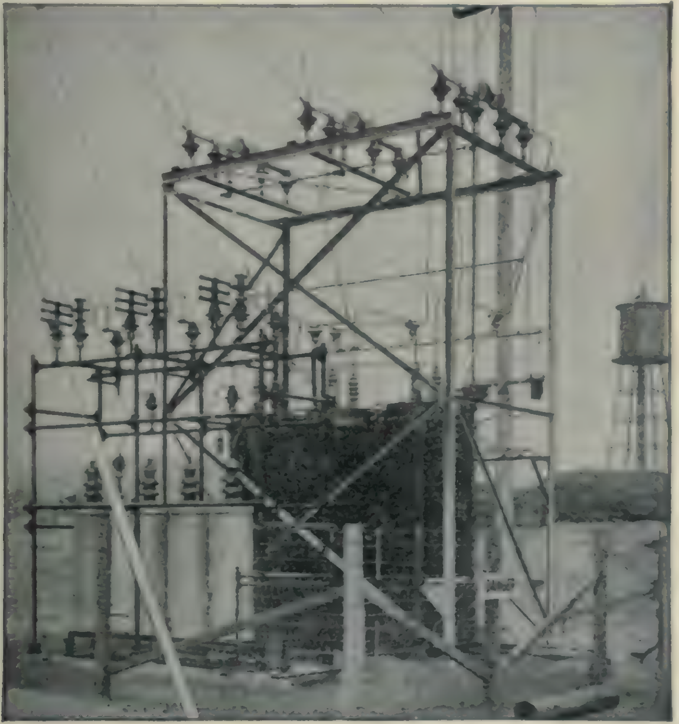

FIG. 420.—A typical outdoor substation showing transformers, electrolytic arresters, horn gaps, and choke coils (on uppermost frame work).

368. Types of Structures for Transmission Lines.—With the exception of low-voltage, low-capacity lines, which use wooden poles, practically all modern transmission lines are carried on steel towers, built up of standard structural forms. They are generally of the self-supporting type, having three or four supporting legs firmly bolted to concrete footings, these being cast in place and extending perhaps 6 feet below the surface of the ground.

The height of tower, spacing of towers, etc., vary much with the local conditions and local regulations. With a certain permitted clearance between wires and ground, it is evident that in situations where temperature changes and ice loading may be expected to cause much expansion, either higher poles or close spacing of towers is necessary. Special poles are required when the line departs from a straight line, because of the resulting side pull. Very high towers are necessary at the banks of streams and as the pull of wires in a long span is high, either steel cables or copper cables with steel cores are used.

More and more outdoor switching and transformer stations are being installed; the transmission line at these points becomes a maze of poles, cables, and insulators. The switches and transformers used are designed to function perfectly under extreme weather conditions, even with no protection of any kind. The outdoor transformer substations are generally installed near small towns, where the town requires say, 2200 volts for its distribution network; transformers to step the voltage down from perhaps 100,000 to 2200 are connected to the transmission line. Such an outdoor sub-station is shown in Fig. 420.

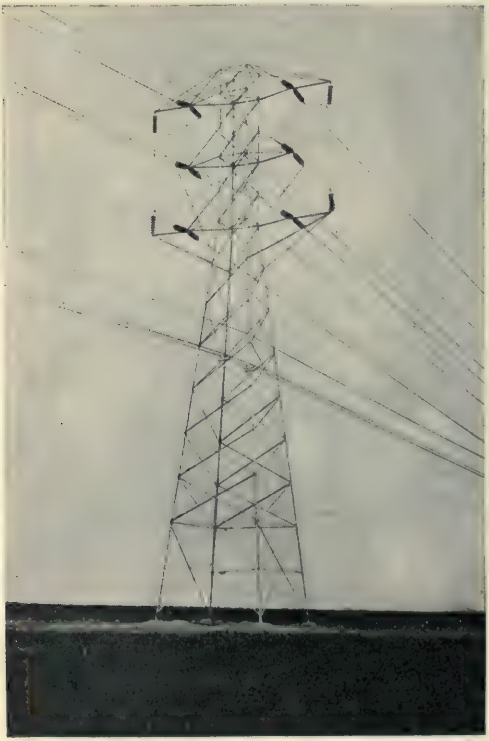

FIG. 421.—Showing how the special construction is carried out on a tower where the wires of a three-phase line are transposed.

369. Transposition.—The large currents carried by a high-power transmission line are likely to produce disturbances in neighboring telephone lines. The low-frequency current itself may not cause so much interference with the telephone lines, but the ripples in the voltage wave, which exist in many generators not especially designed to prevent them, have the right frequency to interfere seriously with voice currents. To reduce this effect of mutual induction, it is customary to transpose the wires of a three-phase system at regular intervals. In case an unsymmetrical arrangement of the wires of a three-phase line is employed (as for example, all three wires in the same plane, as is sometimes done), the transposition is also necessary to keep the regulation of the three phases alike, that is, to keep the system balanced at the load. At transposition points, specially designed towers are generally necessary; one form of transposition tower is shown in Fig. 421.

370. Power-factor Correction.—As the earning capacity of a transmission line is fixed by the kilowatts of power carried over it, we can see that the power factor of the load is a very important item in determining the economic success of the power company. Thus, if a line is designed with sufficient copper and sufficient generating apparatus to deliver 500 amperes per wire of a 200,000-volt, three-phase system, the *possible* delivery of power is $200 \times 500 \times \sqrt{3} = 173{,}000$ kw. If, however, there is a large number of under-loaded induction motors in the load, and many transformers for which the exciting current has to be furnished, the power factor of the load may be only 0.7 and the carrying capacity of the line only 70 per cent of its theoretical value, namely 120,000 kw. Thus the earning capacity of the line is fixed by the figure 120,000, instead of 173,000, and the possibility of paying dividends on the proposition is correspondingly diminished.

In many such cases it actually pays the power company to install, at its own expense, synchronous motors at the load end of the transmission line. These machines are run unloaded (doing no mechanical work) with fields sufficiently overexcited to draw full-load current from the line. This current will lead the voltage by nearly 90° (as was pointed out in section 291) and will therefore be in the right phase to neutralize the lagging current taken by the induction motor load.

Of course, these synchronous condensers (as synchronous motors so employed are designated) increase the amount of capital invested in the line, but the gain in carrying capacity of the line itself more than offsets the carrying charges on this added investment. If no special transformers are required to operate these synchronous condensers, i.e., if they can be operated from transformers already installed for other purposes, it pays to put in sufficient capacity in synchronous condensers to bring the power factor at the load end up to about 95 per cent. It becomes increasingly expensive to improve the power factor above this point, because of the slow rate of change of the cosine of an angle when the angle is small. For example, let us consider a load of 100,000 kw. at a power factor of 0.75, not an unreasonable one for average loading conditions. The amount of synchronous condenser required to increase the power factor of the line by increments of 5 per cent is as follows:

From 0.75 to 0.80	requires	13,000	kv.a.	of	synchronous	condenser
0.80 to 0.85	"	13,200	"	"	"	"
0.85 to 0.90	"	13,400	"	"	"	"
0.90 to 0.95	"	16,000	"	"	"	"
0.95 to 1.00	"	32,400	"	"	"	"

From this table it is quite evident that it cannot pay to improve the power factor by this method to a higher figure than 0.95 and possibly not quite as high.

Besides improving the earning capacity of the line, this use of synchronous condensers permits a much better regulation of the voltage at the load end of the line, thus giving the customer better service. This benefit to be derived by the use of synchronous condensers is of increasing importance with increasing distance of transmission; and as the distance of transmission is continually increasing, we may expect more and more machines to be installed to operate in this fashion.

It is to be noticed that if the current at the load end of the line lags behind the voltage by perhaps 15°, it will undoubtedly lead the voltage at the generator end, because of the effect of the charging current of the line. This charging current, which we have shown how to calculate in section 362, is a maximum at the generator end of the line and diminishes to zero at the load end; the actual current in the line therefore gradually advances its phase as the generator end of the line is approached.

371. Insulators.—For making the insulators used in transmission lines, porcelain is almost universally employed; glass is a possible substitute, having in general the same electrical properties as porcelain, a somewhat greater tensile strength than porcelain, and less tendency to absorb moisture. With the exception of low-voltage (2200) insulators, however, porcelain is always used.

A reasonable working voltage for porcelain is 10 to 20 kv. per inch of thickness, and it is not advisable to use it in pieces more than about one inch thick. For high voltages, therefore, the insulator must consist of many parts (electrically in series), the different parts being properly fastened together to stand the compressive or tensile strains under which the insulator is to be used.

An insulator may fail to function by actual puncture of the porcelain, by creepage over the surface, or by an arc jumping around the insulator from the line to the insulator support. A properly designed insulator will fail under excessive voltage by arcing, and by arcing in such a manner that the arc does not come in contact with the insulator at any part; that is, the arc must jump around the insulator, not come in contact with its surface. Should it do this the intense heat of the arc would at once crack the porcelain and the insulator would be ruined. In the average insulator design, about $\frac{3}{4}$ inch creepage surface is allowed per kilovolt; a 66,000-volt insulator would have about 4 inches of porcelain between wire and pin, and a creepage surface about 50 inches long. Such a design is shown in Fig. 422. The different sections of the insulator (in the case of a pin insulator) are cemented together, and the innermost section is shaped to fit the threaded steel pin which holds the insulator to the cross arm of the tower.

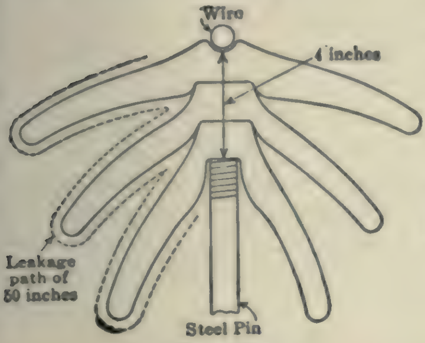

FIG. 422.

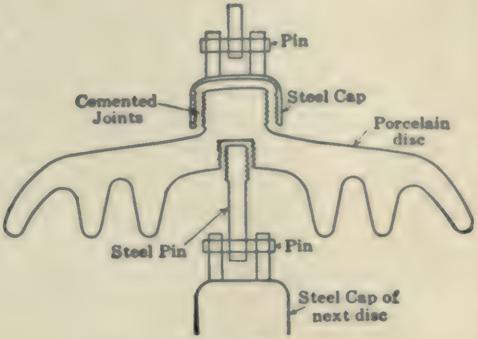

FIG. 423.

FIG. 422.—Cross-section through a 66,000-volt pin insulator, consisting of three parts cemented together.

FIG. 423.—Cross-section through a typical disc insulator.

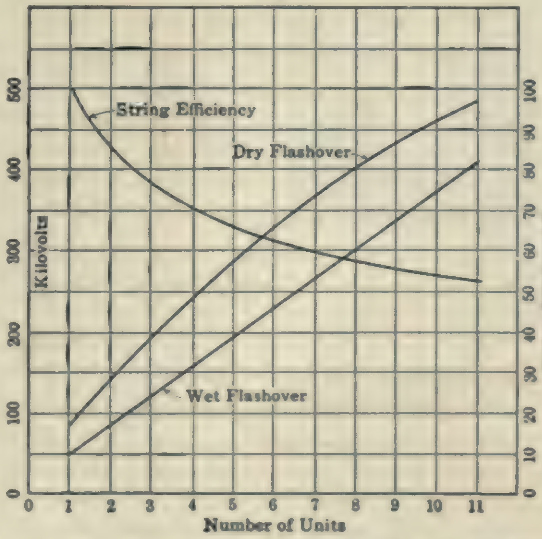

FIG. 424.—The strength of a string of disc insulators does not increase proportionally to the number of discs used; each disc added to the string increases its dielectric strength by an increasingly smaller amount.

In general, pin insulators are not economical for voltages much above 50 kilovolts; for higher voltages the disc, or suspension, type of insulator is always used. An idea of the construction of this type of insulator is given in Fig. 423. The saucer-shaped porcelain disc is about 12 inches across and has from one to one and a half inches of porcelain between the pin in its lower side and the metal cap on its upper side. The creepage path is considerably lengthened by the petticoats on the lower side of the disc. The steel cap cemented on the top, and the steel pin cemented in the lower side, have holes properly placed so that by steel pins as many discs can be fastened together as desired.

The insulating value of a string of insulators does not increase directly with the number of sections in the string, but increasingly less so; that is, a string of ten discs will not stand ten times as much voltage as one disc. This idea is brought out in Fig. 424, which shows how the strength of an insulator increases with the number of discs used in the string. By string efficiency is meant the ratio of the strength of the string to that of one disc multiplied by the number used.

By using different-sized discs in the string, those next to the wire having a greater electrostatic capacity than those nearer the pole, the line voltage is divided more evenly among the various units of the string and the string efficiency thereby raised.

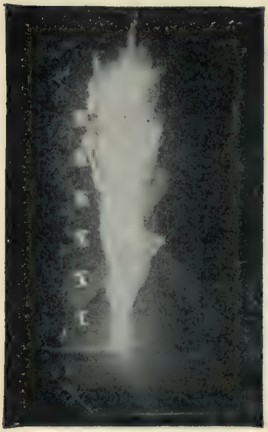
Fig. 425.—A "flash-over" on a string of disc insulators.

Insulators must be tested, of course, at voltages much higher than that of the line on which they are to be used, as excessive voltages are likely to occur on the transmission at any time. For example, the insulators to be used on a three-phase, 100,000-volt line should be able to stand at least 150,000 volts before breaking down. As an insulator will break down more easily when wet than when dry, the test is carried out for both the dry and the wet conditions; the latter is duplicated in the laboratory by spraying the insulator with water while the test voltage is applied. Figure 425 shows a breakdown test of a string insulator; it is arcing over, under a pressure of about 250,000 volts.

372. Superpower Transmission Network.—A study of the costs of generating electric power at once shows the advantage to be gained by substituting one large power plant for many small ones. All sorts of labor-saving devices and special efficiency-increasing schemes are applicable in plants of several hundred thousand kw. capacity, whereas they could not be economically installed in plants of a few thousand kw. capacity. Evidently also, the cost of generating electric power in steam plants will depend largely upon the cost of getting coal to the boilers; only those plants close to the mines, or those where a short, direct haul is possible, can have low transportation costs. It is evident, then, that for economy, in so far as cost per kw.-hour at the switch-board is concerned, a plant should be either a hydro-electric one or else should be located at the mine-head.

Either of these schemes, however, will generally put the plants far away from the industrial centers which they are to serve; hence the idea of a tremendous network of transmission lines, covering the industrial parts of the country, the network to be supplied from a few superpower plants.

One superpower plan, which seems likely to come into operation in the near future, covers the eastern part of the United States, about as shown in Fig. 426. Large power stations will be located in the mine fields, and will be designed to burn the lower grades of coal which it does not pay to haul at present. The plants could not be directly at the mines, because the prime factor determining their location is the availability of cooling water; but even so, the required coal haul would be short and direct.

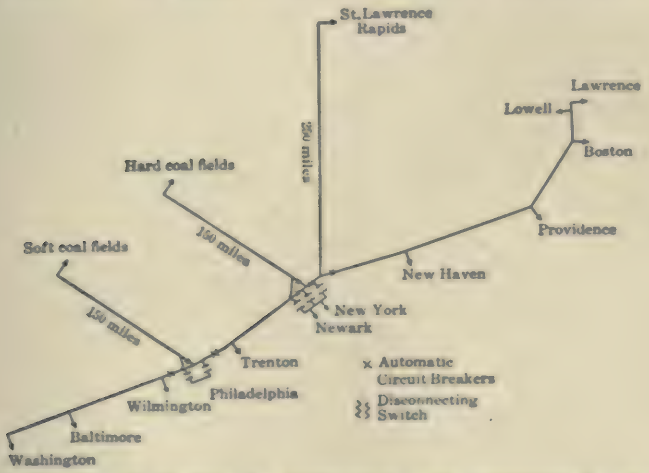

FIG. 426.—Showing the network of transmission lines recommended for economical power generation and distribution in the eastern part of the United States.

It is expected that many small water-power plants, as well as the great powers available in the mountains of New York State and the rapids of the St. Lawrence, will be tied into the network. Although the St. Lawrence power is about 300 miles away from the center of power consumption, it is not impossible for a line as long as this to function reasonably well. Three hundred thousand volts is not an impossible voltage; and by using several aluminum cables in parallel for one conductor the inductive reactance of the line can be kept to a reasonable figure.

In Fig. 426 is shown a plan of the contemplated transmission lines, only the trunk lines being shown. Very large steam plants in New York Philadelphia, and Boston, all of which can have their coal delivered directly at the plant by boat, will supplement the power plants in the mine fields and the hydro-electric plants; the water-power plants will run at full capacity all the time, the steam plants filling in the power demands which cannot be supplied by the water-power plants. It is estimated that such a network would save many millions of tons of coal a year, because of the increase in efficiency alone, exclusive of what the hydro-electric power would do to cut down the coal bill.

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